EUCLID'S ELEMENTS OF Geometry.

In XV. Books:
With a ſupplement of divers *PROPOSITIONS* and *COROLLARIES.*
To which is added, a *Treatiſe* of REGULAR SOLIDS,
By *CAMPANE* and *FLUSSAS.*
LIKEWISE

Euclid's DATA:

And MARINUS his Preface
thereunto annexed.

Alſo a Treatiſe *of the* Diviſions *of* Superficies, *aſcribed to* Machomet Bagdedine, *but publiſhed by* Commandine, *at the requeſt of* John Dee *of* London; *whoſe* Preface *to the ſaid* Treatiſe *declares it to be the* Worke of EUCLIDE, *the* Author *of theſe* ELEMENTS.

Publiſhed by the Care and Induſtry of
JOHN LEEKE and GEORGE SERLE, Students in the *MATHEMATICKS.*

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MDCLXI.

TO THE

Duke of YORK.

May it please your HIGHNESSE:



BEfore a PRINCE of so approved Conduct, I shall not enumerate the many wayes that the MATHEMATICKS are Auxiliary in WAR; nor trouble you with such Trite Stories to Court your belief thereof, as that of ARCHIMEDES; who sitting in his Chair, with his *Nugæ Mathematicæ* (as he called them) threw multiplicity of Deaths amongst the all-daring Legions of MARCELLUS: This Sir, is a taske needlesse, and most injurious to your Highnesse Merits; whose perfection in these

Sciences, could be only made queſtionable by ſuch exaggerations.

No, great Sir, The ſole reaſon for laying this Dedication: with ſo much preſumption, at your Highneſſe Feet, was our conſciouſneſs how particular an intereſt you have in the Prince of Mathematicians, whilſt all the world admires you, as the moſt abſolute Profcient both in *Mars*, and the *Muſes* Schools.

'Tis EUCLIDE, Sir, that implores your Princely Patrociny, the vouchſafe whereof, as it will procure him the favour of all; ſo it will be a Glorious Shield, to dazle the eyes of ſuch who would over-nicely cenſure the Lapſes of his Interpretors, and

SIR,

Your Highneſſes moſt humble and moſt obliged Servants,

JOHN LEEKE,
GEORGE SERLE.

13.4.4.20 28.4.10

TO THE READER.

ALthough it be uſual for Authors or Commentators to write ſomewhat in praiſe of the Subject of which they treat: Yet we ſuppoſe it neither neceſſary, or (we hope) will it be expected that we ſpeak much or little in praiſe of the Mathematicks, or of this Book, of the Elements of Geometry, *the ground and foundation of all Mathematical productions; the general uſe that hath been, and is ſtill made thereof, almoſt throughout the World, as well in reſpect of its Original Greek, as the Tranſlations thereof into all Languages, doth ſufficiently declare the excellency and worth of the Art, the Book, and the famous Author and Philoſopher* EUCLIDE: *An account of whoſe Original, and the time wherein he flouriſhed, we have thought good to inſert; as alſo that full and learned Preface of the famous Mathematician* John Dee, *then which nothing of that nature can be more ample or ſatisfactory.*

We ſhall only advertiſe the Reader what order and method hath been uſed and obſerved by us in this Book (which we now preſent to his favourable acceptance) which hath not been done by any other that have publiſhed theſe Elements in the Engliſh Tongue.

And first, to speak of the Schemes, he is to take notice that throughout the Book, the greater black lines stand for, and denote the Data, *or things given, the lesser black lines the things required, and the pricked lines serve for the Construction and Demonstration; the like method is also observed in the Circular lines given and required.*

Secondly, he is to take notice that the explications of the things given and required, are comprehended in the words of the Proposition, by meanes of the correspondent letters relating to the severall parts of the Schemes, in such sort, as the Propositions may be read with them, or without them, and so by this meanes the Demonstrations are much abreviated, yet our endeavours have been to render all things as plain, easie, and intelligible to the Reader as may be, and we hope they will be so found.

Thirdly, Whereas most Authors that have published the Elements of EUCLIDE, *do end at the fifth Proposition of the Fifteenth Book, we have in this Book added the rest of the Propositions belonging to the Fifteenth Book, as also the whole Sixteenth, according to* Flussas, *and also the Treatise of Regular Solids, which Propositions will be found of excellent use in the Geometrie of Solids: Those Propositions also we have reduced to the same method as the precedent, and so consequently they are much abreviated.*

Fourthly, He is to take notice, that at the end of the Second Book, we have added the Ten first Propositions of that Book according to Barlaam *a famous Greek Author, to shew the agreement and correspondency that is between Lines and Numbers, Geometry and Arithmetick, the said Propositions being proved true by both, they having the self same proprieties and passions in Numbers, which* EUCLIDE *in this Second Book demonstrates by magnitudes.*

Fifthly, We have added EUCLIDES Data, *a Book of excellent use in things relating to Algebra, as also a learned Preface thereon, written by the Philosopher* Marinus, *fully unfolding the signification of the Term, and the profit and utility of the Treatise; the same method of bringing the letters of the Scheme into the Propotsition, being observed here as in the other Bookes.*

Lastly, We have added two excellent Treatises of the Divisions

visions of Superficies; one of them written by our EUCLIDE, *though ascribed to* Machomet Bagdedine, *the other by* Frederick Commandine *of* Urbin, *touching the same matter: These two Treatises, together with* EUCLIDES Data, *were never before published in the English Tongue, nor is there any thing extant in English of that nature so full and ample, in all respects, as those are.*

There are some other Treatises ascribed to our Author, viz. *his* Opticks, Catoptricks, Phænomena, *and* Musick, *which we purposely omitted, because they are not Elementary, as also because those Subjects have been more fully treated on by other later Authors. And moreover it is questioned, by many whether they were the Workes of this our* EUCLIDE, *or some other.*

Thus as exactly as we can, we have given you a short view of what we thought fit to be premised concerning this Work, which if diligently read, will undoubtedly be attended with as much profit as delight: The greatest Errors (as it is almost impossible but some will passe, in a Work of this Nature) are denoted at the end of the Book: As for the lesse mistakes, we leave them to your goodnesse, either to correct them, or excuse them.

FAREWEL.

An Account of the Author according to Ancient Phylosophers.

THE famous *EUCLIDE* Instituter of these Mathematical Elements (to premise something concerning the Author, and something also of this his Noble Science) is variously reported of, both for his Person, and the Age and Time in which he flourished. Several Writers are of opinion; as *Campanus* and *Theon*, in their Edition of these Elements, have published that he was borne at *Megara*, a Town adjacent to *Isthmus*, a Hearer of *Socrates*, who denominated a Sect of Philosophers, his Disciples, from that place, which said Sect was otherwise called the *Dialectick*; for that their manner of Writing was by way of Question and Answer, the Property of Logick onely. Of this *EUCLIDE* *Diogenes Laertius* hath writ much, in his *Lives* of the Philosophers; as also *Cicero* in his Third Book of Academick Questions, where he saith that *EUCLIDE* was of *Megara*, a Disciple of *Socrates*, whose Scholars affirmed that to be the only good, which was *One*, *Alike*, the *Same*, and *Perpetual*. This relation of *EUCLIDE* is backt by *Valer. Max.* who in his Eighth Book saith, that *Plato* sent his Scholars to informe themselves of a certain Question, to *EUCLIDE* the Geometrician: But if credit may be given to *Proclus* a Noble Writer, and other Ancient Writers, This our *EUCLIDE* was younger then him of *Megara*, and flourished in the Reign of *Ptolomy* the First, who governed *Ægypt* after the death of *Alexander* the Great, in the 115th. Olympiad, and before the Birth of Christ 309 Yeares; which is the most probable opinion, from this Reason: For *Diogenes Laertius*, diligently enumerating the Works of *EUCLIDE* of *Megara*, makes no mention of this most renowned Volume, of the Geomerical Elements, by which this our *EUCLIDE* hath decreed to himself a never dying immortal Fame and Glory. Neither is it to be thought, that *Diogenes* a Person so well versed in the Moniments of Philosophers, would have wittingly past over this most eximious piece, or not take notice that it was made by his familiar *EUCLIDE*. Therefore certainly (though it be uncertaine from whence he had his Birth) this our *EUCLIDE* was younger then the other: A Philosopher he was, of great praise at first for his Learning in the Academicks, from which he addrest himself wholly to the Study of the Mathematicks, in which he so farre excelled, that by universal consent, and judgment, he is deservedly stiled the Prince of Mathematicians. He writ several Books concerning that Science, in which his accurateness, and admirable Doctrine, is very conspicuous; such are his *Opticks*, *Catoptricks*, *Elementary Institutions*, for the Attainment of Musick, *Phœnomena*, and a Book entituled *Data*, *A Treatise of Divisions*, which though ascribed to *Machomet Bagdedine*, is suspected to be his, being a very exact piece, and newly published by the endeavours of *John Dee* of *London*, and *Frederick Commandine* of *Urbin*. He writ also (as *Proclus* saith) the *Conick Elements*, which yet never saw the light, and some such other Tracts. But more especially he wrote this Book of *Geometrical Elements*, never enough (by universal vogue) to be commended, being wrought with such admirable method, and such immense Learning, that no man ever that yet writ of the same, came so much as neer him. In which as he shewed his acute and-piercing brain, so did he not publish all things belonging to that Science, but onely the chiefe and most necessary, supported by the most firme and solid Arguments. And what the profit of these his Elements are to the studious, hath by what we have already said, and by what we shall now say, plainly appear. They are called the Elements of Geometry, for this cause, for that no Mathematical Operation can be undertaken, nor no advantage be made, without them. For all Mathematical Writers, as *Archimedes*, *Appollonius*, *Theodosius*, and others, in their Demonstrations use these Elements, as Principles long since known, approved, and demonstrated. Therefore as he, who would learn to read, must first learn his A B C, and daily use them repeated in the forming and expressing of words, so he that would render the Science of the Mathematicks easie to him, must perfectly know these Geometrical Elements: For from these, as from a full Fountaine; all dimension and division of Latitude, Longitude, Altitude, Profundity, as of Fields, Mountains, Islands, take their rise and beginning; from hence the observation (by Instruments) of the Stars in Heaven, the making of all Scioterick Horologies, The force of Engines, Judgment of Weights, all diversity of various apparences (such as are seen in Glasses, in Pictures, in Water, and in the Aire variously radiated) have their Original. By these Principles (further) the Medium and Centre of this whole Fabrick of the World was found out, and the Poles or Hinges on which it is roled; and finally, the certain figure and quantity of it. By the strength of this alone, the rising, setting, depression, return, ascension, descension, and culmination of all the perpetual motions in the Heaven, the diversity of day and night, in all climates, and Latitudes throughout the Year, are infallibly known. The divers Conjunctions also, and Oppositions and Aspects of the Planets, are so easily known, that their places in the Heaven, the Eclipses and defects of the Sun and Moon are so certainly demonstrated, that they can be, and are predicted by the Mathematicks to all after Ages. To summe up all, this vast Work of God and Nature is presented to the eye of our understanding, by the benefit and help of Geometry. Adde hereto, that this Science renders things incredible, and which surpasse all beliefe, most perspicuous and evident to us. Such a Story is that which Historians testifie of *Archimedes* of *Syracusa*, For King *Hieron* having built a Ship (which he purposed to send to *Ptolomy* King of *Ægypt*) of such an immense bulk, that all the Syracusians with their uttermost strength, could not remove, *Archimedes* being a most skilful Geometrician, promised to launch it, which with little labour accordingly in the sight of them all he performed, to such an amazement of

of the King, that he expreſt with aſtoniſhment, *That what ever* Archimedes *ſhould from thenceforth ſay, ought indubitably to be believed.* Not much unlike this, was his other famous Operation before the ſame King, who had commanded a Crown to be made of Gold, *Archimedes* without melting it down, told the King the reſpective Weight of Gold and Silver, which the fraud and cunning of the Artificer had mixed and incorporated together. Nor may we paſſe by in ſilence that uſual Saying of his, *That (truſting to the Force and Efficacy of Geometrical Demonſtrations) if there were another World whereon he could ſet his foot, he would remove this whereon we are, from its place.* And by the ſame reaſon, with never ſo little ſtrength, remove never ſo ponderous and maſſie weights. Divers like things have been performed by him, and other induſtrious Geometricians, which are recorded at large. But ſuch a fame did this Science purchaſe to *Archimedes*, that *Marcellus* the Roman General, againſt whom he alone had long defended his Native City of *Syracuſa*, by certain Engines invented by Geometrical Demonſtrations, having taken the City, and given the plunder and ſpoil, and ſack of it to his Souldiers, forbidding any quarter to be given to the Citizens, did yet by his Proclamation command the ſaving of the life of *Archimedes*, whom when againſt that his command, he knew to be killed by the hands of a common Souldier, he did moſt paſſionately grieve, and beſtowed that Honour upon him being dead, which he could not confer upon him living. And *Cicero* highly pleaſes himſelf, and much glories in it, that when he was Quæſtor in *Sicily*, he had found out and ſeen his Monument and Sepulchre. Therefore none may wonder that Geometry was in ſuch eſteem among the Grecians. And this alſo much advances the excellency and utility of Geometry, becauſe its Demonſtrations are ſo clear, that no man can without it be a perfect Methodiſt; which *Galen* that famous Philoſopher and Prince of Phyſicians, in his Books ingeniouſly acknowledgeth: Who being every way accompliſhed in the Dialectick Learning; and having run through all the Schools both of the Stoicks and Peripateticks of his time, and with much intentneſſe of mind, conſidered and ſtudied their Precepts and Rules, declares that they contein nothing in them, which might conduce to the knowledge of Demonſtration; but rather moſt of them were yet in controverſie amongſt themſelves, others quite repugnant to natural reaſon; ſo that he was almoſt brought to the opinion of the *Pyrrhonian* Philoſophers, to doubt of every thing, and to determine nothing; but that by the knowledge of Geometry and Arithmetick (in which he was trained up by his Grand-father) he was recovered, and farther reclaimed from that errour. His counſel therefore is, that men apply their minds to ſtudy the Arithmetick Characters, and the Demonſtration of Lines. So alſo *Plato* is of this opinion; *Becauſe* (ſaith he) Geometry *conduceth to the acquiring and right underſtanding of all other Arts and Sciences.* To conclude, this is the chiefe Glory of Geometry, that it loyters not, or employes it ſelf about theſe inferiour Machines, from whence it had its Original, but hath ſoared up into Heaven, and reſetled humane minds, (groveling before in the duſt) in Cœleſtial Seats, and hath capacitated us to the underſtanding both of this whole Fabrick of the World, and the Adminiſtration and Government thereof.

THE

To the unfained lovers of Truth, and conſtant *Students* of Noble *Sciences*, JOHN DEE of *London*, heartly wiſheth grace from Heaven, and moſt proſperous ſucceſs in all their honeſt attempts and *exercises*.

DIvine PLATO, the great Maſter of many worthy Philoſophers, and the conſtant avoucher, and pithy perſwader of *Unum*, *Bonum*, and *Ens*: in his School and Academie, ſundry times (beſides his ordinary Scholers) was viſited of a certain kind of men allured by the noble fame of *Plato*, and the great commendation of his profound and profitable doctrine. But when ſuch Hearers, after long harkening to him, perceived, that the drift of his diſcourſes iſſued out, to conclude, this *Unum*, *Bonum*, and *Ens*, to the Spiritual, Infinite, Eternal, Omnipotent, &c. Nothing being alledged or expreſſed, How, worldly goods: how, worldly dignity: how, health, ſtrength, or luſtineſſe of body: nor yet the means, how a marvellous, ſenſible and bodily bliſſe and felicity hereafter, might be atteined: Straightway, the fantaſies of thoſe hearers were dampt, their opinion of *Plato* was clean changed: yea, his doctrine was by them deſpiſed; and his School, no more of them viſited. Which thing, his Scholer, *Ariſtotle*, narrowly conſidering, found the cauſe thereof, to be, For that they had no fore-warning and information, in general, "whereto his Doctrine tended. For, ſo, might they have had occaſion, either to have forborn his School haunting: (if they, then, had miſliked his Scope and purpoſe) or conſtantly to have continued therein: to their full ſatisfaction: if ſuch a final Scope and intent, had been to their deſire. Wherefore, *Ariſtotle*, ever, after that, uſed in brief, to "fore-warn his own Scholers and hearers, both of what matter, and alſo to what end, he "took in hand to ſpeak, or teach. While I conſider the diverſe trades of theſe two excellent Philoſophers (and am moſt ſure, both, that *Plato* right-well, otherwiſe could teach: and that *Ariſtotle* might boldly, with his hearers, have dealt in like ſort as *Plato* did) I am in no little pang of perplexity: Becauſe, that which I miſlike, is moſt eaſie for me to perform (and to have *Plato* for my example.) And that which I know to be moſt commendable: and (in this firſt bringing, into common handling, the *Arts Mathematical*) to be moſt neceſſary: is full of great difficulty and ſundry dangers. Yet, neither do I think it meet, for ſo ſtrange matter (as now is meant to be publiſhed) and to ſo ſtrange an audience, to be bluntly at firſt put forth, without a peculiar Preface: Nor (imitating *Ariſtotle*) well can I hope, that according to the ampleneſſe and dignity of the *State Mathematical*, I am able, either plainly to preſcribe the material bounds: or preciſely to expreſſe the chief purpoſes, and moſt wonderful applications thereof. And though I am ſure, that ſuch as did ſhrink from *Plato* his School, after they had perceived his final concluſion, would in theſe things have been his moſt diligent hearers (ſo infinitely might their deſires, in fine and at length, by our *Art Mathematical*, be ſatisfied) yet, by this my Preface and fore-warning, Aſwell all ſuch, may (to their great behoof) the ſooner, hither be allured: as alſo

The intent of this Preface.

the *Pythagorical*, and *Platonical* perfect Scholer, and the constant profound Philosopher, with more ease and speed, may (like the Bee) gather, hereby, both wax and honey.

" Wherefore, seeing I find great occasion (for the causes alledged, and farther, in respect " of my *Art Mathematick general*) to use a certain fore-warning and Preface, whose " content shall be, that mighty, most pleasant, and fruitful *Mathematical Tree*, with his " chief arms and second (grafted) branches: " Both, wl at every one is and also, what com- " modity, in general, is to be looked for, as well of grift as stock: And forasmuch as this " enterprize is so great, that, to this our time, it never was (to my knowledge) by any " atchieved: And also it is most hard, in these our dreiy dayes, to such rare and strange Arts, to win due and common credit: Nevertheless, if for my sincere endeavour, to satisfie your honest expectation, you will but lend me your thankful mind a while: and to such matter as, for this time, my pen (with speed) is able to deliver, apply your eye or ear attentively: perchance, at once, and for the first saluting, this Preface you will find a lesson long enough. And either you will, for a second (by this) be made much the apter: or shortly become, well able your selves, of the Lions claw, to conjecture his royal Symmetrie, and farther property. Now then, gentle, my friends and countrey-men, Turn your eyes, and bend your minds to that doctrine, which for our present purpose, my simple talent is able to yield you.

All things which are, and have being, are found under a triple diversity general. For, either, they are deemed Supernatural, Natural, or of a third being. Things Supernatural, are immaterial, simple, indivisible, incorruptible, and unchangable. Things Natural, are material, compounded, divisible, corruptible, and changable: Things Supernatural, are of the mind onely comprehended: Things Natural, of the sense exterior, are able to be perceived. In things Natural, probability and conjecture hath place: But in things Supernatural, chief demonstration and most sure Science is to be had. By which properties and comparisons of these two, more easily may be described, the state, condition, nature and property of those things, which, we before termed of a third being: which, by a peculiar name also, are called *Things Mathematical*. For, these, being (in a manner) middle, between things supernatural and natural: are not so absolute and excellent, as things supernatural; Nor yet so base and grosse, as things natural: But are things immaterial, and nevertheless, by material things able somewhat to be signified. And though their particular Images by Art, are aggregable and divisible: yet the general *Forms* notwithstanding, are constant, unchangeable, untransformable and incorruptible. Neither of the sense, can they, at any time, be perceived or judged. Nor yet, for all that in the royall mind of man, first conceived. But surmounting the imperfection of conjecture, weening and opinion; and comming short of high intellectual conception, are the Mercurial fruit of *Dianoetical* discourse, in perfect imagination subsisting. A marveilous newtrality have these things *Mathematical*, and also a strange participation between things supernatural, immortal, intellectual, simple and indivisible: and things natural, mortal, sensible, compounded and divisible. Probability and sensible proof, may well serve in things natural, and is commendable: In Mathematical reasonings, a probable Argument, is nothing regarded: nor yet the testimony of sense, any whit credited: But onely a perfect demonstration; of truths certain, necessary, and invincible: universally and necessarily concluded: is allowed as sufficient for an Argument exactly and purely Mathematical.

Number.

Note the word *Unite*, to express the Greek *Monas*, and not *Unity*, as we have all, commonly, till now, used.

Of *Mathematical* things, are two principal kinds, namely, *Number* and *Magnitude*. *Number*, we define to be a certain Mathematical Summe, of *Units*. And an *Unit* is that thing Mathematical, Indivisible, by participation of some likenesse of whose property, any thing, which is indeed, or is counted One, may reasonably be called One. We account an *Unit*, a thing *Mathematical*, though it be no *number*, and also indivisible because of it materially, Number doth consist: which principally, is a thing *Mathematical*. *Magnitude* is a thing *Mathematical*, by participation of some likeness of whose nature, any thing is judged long, broad, or thick. A thick *Magnitude* we call a *Solid*, or a *Body*. What *Magnitude* so ever is Solid or Thick, is also broad and long. A broad Magnitude we call a *Superficies* or a Plain. Every plain Magnitude, hath also length. A long Magnitude, we term a *Line*. A *Line* is neither thick nor broad, but onely long: Every certain Line, hath two ends: The ends of a Line, are *Points* called. A *Point* is a thing *Mathematical*, indivisible, which may have a certain determined situation. If a Point move from a determined situation, the way wherein it moved, is also a *Line*, Mathematically produced:

A Point.

produced: whereupon, of the ancient Mathematicians, a *Line* is called the race or course of a *Point*. A Point we define, by the name of a thing Mathematical: though it be no Magnitude, and indivisible: because it is the proper end, and bound of a *Line*: which is a true *Magnitude*. And *Magnitude* we may define to be that thing *Mathematical*, which is divisible for ever, in parts divisible, long, broad, or thick. Therefore though a Point be no *Magnitude*, yet *Terminatively* we reckon it a thing *Mathematical*, (as I laid) by reason it is properly the end and bound of a line.

A Line.

Magnitude.

Neither *Number* nor *Magnitude* have any Materiality. First, we will consider of *Number*, and of the Science *Mathematical*, to it appropriate, called *Arithmetick*, and afterward of *Magnitude*, and his Science called *Geometrie*. But that name contenteth me not: whereof a word or two hereafter shall be said. How Immaterial, and free from all matter, *Number* is, who doth not perceive? yea, who doth not wonderfully wonder at it? For, neither pure *Element*, nor *Aristoteles Quinta Essentia*, is able to serve for Number, as his proper matter. Nor yet the purity and simplenesse of Substance Spiritual or Angelical, will be found proper enough thereto. And therefore the great and godly Philosopher *Anitius Boëtius*, said: *Omnia quaecunque à primaeva rerum natura constituta sunt, Numerorum videntur ratione formata. Hoc enim fuit principale in animo Conditoris Exemplar.* That is: **All things (which from the very first original being of things, have been framed and made) do appear to be Formed by the reason of Numbers. For this was the principal Example or pattern in the mind of the Creator.** O comfortable allurement, O ravishing perswasion, to deal with a Science, whose Subject is so ancient, so pure, so excellent, so surmounting all creatures, so used of the Almighty and incomprehensible wisedome of the Creator, in the distinct creation of all creatures: in all their distinct parts, properties, natures, and vertues, by order, and most absolute number, brought from *Nothing*, to the *Formality* of their being and state. By *Numbers* property therefore, of us, by all possible means, (to the perfection of the Science) learned, we may both wind and draw our selves into the inward and deep search and view, of all creatures distinct vertues, natures, properties, and *Forms*: And also farther, arise, clime, ascend and mount up (with speculative wings) in spirit, to behold in the Glasse of Creation, the *Form* of *Forms*, the *Exemplar Number* of all things *Numerable*: both visible and invisible: mortal and immortal, Corporal and Spiritual. Part of this profound and divine Science, had *Joachim* the Propheser attained unto: by *Numbers Formal*, *Natural*, and *Rational*, foreseeing, concluding, and foreshewing great particular events, long before their comming. His books yet remaining, hereof, are good proof. And the Noble Earl of *Mirandula*, (besides that) a sufficient witnesse: that Joachim *in his propheses, proceeded by no other way, then by Numbers Formal.* And this Earl himself, in *Rome*, * set up 900 Conclusions, in all kind of Sciences, openly to be disputed of: and among the rest, in his Conclusions *Mathematical* (in the eleventh Conclusion) hath in Latine this English sentence. *By numbers, a way is had, to the searching out, and understanding of every thing, able to be known. For the verifying of which Conclusion, I promise to answer to the 74 Questions, under written by the way of Numbers.* Which Conclusions, I omit here to rehearse: aswell avoiding superfluous prolixitie: as, because *Joannes Picus*, *Works*, are commonly had. But, in any case, I would wish that those Conclusions were read diligently, and perceived of such, as are earnest Observers and Considerers of the constant law of Numbers: which is planted in things Natural and Supernatural: and is prescribed to all Creatures, inviolably to be kept. For, so, besides many other things, in those Conclusions to be marked it would appear, how sincerely, and within my bounds, I disclose the wonderful mysteries by numbers, to be atteined unto.

Anno 1488.

Of my former words, ease it is to be gathered, that *Number* hath a treble state: One, in the Creator: another is every Creature (in respect of his compleat constitution:) and the third, in Spiritual, and Angelical Minds, and in the Soul of man. In the first and third state, *Number*, is termed *Number Numbring*. But in all Creatures otherwise, *Number* is termed *Number Numbred*. And in our Soul, Number beareth such a sway, and had such an affinity therewith; that some of the old Philosophers taught, *Mans soul, to be a number moving it self*. And indeed, in us, though it be a very Accident, yet such an Accident it is, that before all Creatures it had perfect being, in the Creator, Sempiternally. *Number*

numbring therefore, is the discretion discerning; and distincting, of things. But in God the Creator, This discretion, in the beginning, produced orderly and distinctly all things. For, his *numbring*, then, was his Creating of all things. And his continual *numbring* of all things, is the Conservation of them in being. And, where and when he will lack an *Unit*: there and then, that particular thing shall be *Discreated*. Here I stay. But our Severalling, distincting, and *numbring*, createth nothing: but of Multitude considered, maketh certain and distinct determination. And albeit these things be waighty, and truths of great importance, yet (by the infinite goodnes of the Almighty *Ternarie*,) Artificial Methods and easie wayes are made, by which the zealous Philosopher may win neer this Riverish *Ida*, this Mountain of Contemplation: and more then Contemplation. And also, though *number* be a thing so immaterial, so divine, and eternal: yet by degrees, by little, and little, stretching forth, and applying some likenesse of it, as first, to things Spiritual: and then bringing it lower, to things sensibly perceived, as of a momentany sound iterated: then to the least things that may be seen, numerable: And at length (most grosly) to a multitude of any corporal things seen, or felt: and so of these grosse and sensible things, we are trained to learn a certain Image, or likenesse of numbers, and to use Art in them to our pleasure and profit. So grosse is our conversation, and dull is our apprehension, while mortal Sense, in us, ruleth the Common-wealth of our little world. Hereby we say, Three Lyons are three or a *Ternarie*. Which * *Ternaries*, are each, the *Union*, *knot* and *Uniformity* of three discreet and distinct *Units*. That is, we may in each *Ternary*, thrice severally, point, and shew a part, *One*, *One*, and *One*. Where, in Numbring, we say One, Two, Three. But how far these visible Ones, do differ from our Indivisible Units (in pure *Arithmetick* principally considered) no man is ignorant. Yet from these grosse and material things, may we be led upward, by degrees, so informing our rude Imagination, toward the conceiving of *numbers*, absolutely: (Not supposing, nor admixting any thing created, Corporal or Spiritual, to support, contein, or represent those *numbers* imagined:) that at lenght, we may be able, to find the number of our own name, gloriously exemplified and registred in the book of the *Trinity*, most blessed and eternal.

But farther understand, that vulgar Practisers, have Numbers, otherwise, in sundry Considerations: and extend their name farther, then to Numbers, whose least part is an *Unit*. For the common Logist, Reckenmaster, or Arithmetician, in his using of Numbers: of an *Unit*, imagineth lesse parts: and calleth them *Fractions*. As of an *Unit*, he maketh an half, and thus noteth it, $\frac{1}{2}$, and so of other (infinitely diverse) parts of an *Unit*. Yea, and farther, hath *Fractions of Fractions*, &c. And for asmuch as *Addition*, *Substraction*, *Multiplication*, *Division*, and *Extraction of Roots*, are the chief, and sufficient parts of *Arithmetick*: which is, the *Science that demonstrateth the properties of numbers, and all operations, in numbers to be performed.* "How often therefore, these five sundry "sorts of Operations, do, for the most part of their execution, differ from the five opera- "tions of like general property and name, in our Whole numbers practisable. So, often, "(for a more distinct doctrine) we, vulgarly account and name it, another kind of *Arithmetick*. And by this reason: the consideration, doctrine, & working in whole numbers onely: where, of an *Unit*, is no lesse part to be allowed: is named (as it were) an *Arithmetick* by it self. And so of the *Arithmetick of Fractions*. In like sort the necessary, wonderful, and Secret doctrine of Proportion, and proportionality hath purchased unto it self a peculiar manner of handing and working: and so may seem another form of *Arithmetick*. Moreover, the *Astronomers*, for speed, and more commodious calculation, have devised a peculiar manner of ordering numbers, about their circular motions, by Sexagenes and Sexagesimes. By Signes, Degrees, Minutes, &c. which commonly is called the *Arithmetick* of *Astronomical* or *Physical Fractions*. That have I briefly noted by the name of *Arithmetick Circular*. Because it is also used in Circles, not *Astronomical*, &c. Practise hath led *Numbers* farther, and hath framed them, to take upon them, the shew of *Magnitudes* property: Which is *Incommensurability* and *Irrationality*. (For in pure *Arithmetick*, an *Unit*, is the common Measure of all Numbers.) And here, Nnumbers are become, as Lines, Plains, and Solids, sometimes *Rational*, sometimes *Irrational*. And have proper and peculiar characters, (as $\surd 3$. $\surd \mathfrak{C}$. and so of other. Which is to signifie *Root Square*, *Root Cubick*: *and so forth*:) and proper and peculiar fashions in the five principal parts: Wherefore the Practiser esteemed this a diverse *Arithmetick* from the other.

Arithmetick Note.

1. 2. 3. 4.

other. Practise bringeth in, here, diverse compounding of Numbers: as sometime, two, three, four (or more) *Radical* Numbers diversly knit by signs, of More and Lesse: as thus $\surd 3\ 12 + \surd \mathfrak{C}\ 15$. Or thus $\surd 3\ 3\ 10 + \surd \mathfrak{C}\ 12 - \surd 3\ 2$, &c. And sometime with whole numbers, or fractions of whole Number, among them: as $20 + \surd 3\ 24$. $\surd \mathfrak{C}\ 16 + 13 - \surd 3\ 10$. $\surd 3\ 3\ 44 + 12\frac{1}{4} + \surd \mathfrak{C}\ 9$. And so infinitely, may hap the varietie. After this; both the one and the other hath fractions incident: and so is this *Arithmetick* greatly enlarged, by diverse exhibiting, and use of Compositions and mixtings. Consider how, I (being desirous to deliver the Student from errour and Cavillation) do give to this *Practise*, the name of the *Arithmetick of Radical numbers*: Not of *Irrational* or *Surd numbers*: which other while, are Rational: though they have the Signe of a Root before them, which *Arithmetick* of whole Numbers most usual, would say they had no such Root, and so account them *Surd numbers*, which, generally spoken, is untrue: as *Euclid's* tenth Book may teach you. Therefore, to call them, generally *Radical numbers*, (by reason of the sign $\surd$ prefixed) is a sure way, and a sufficient general distinction from all other ordering and using of Numbers: And yet (besides all this) Consider; the infinite desire of knowledge, and incredible power of mans Search and Capacity: how, they, joyntly have waded farther (by mixing speculation and practice) and have found out, and atteined, to the very chief perfection (almost) of *Numbers* practical use. Which thing is well to be perceived in that great Arithmetical Art of *Equation*: commonly called the *Rule of Cosi*, or *Algebra*. The Latines termed it, *Regulam Rei & Census*, that is, the **Rule of the thing, and his value.** With an apt name, comprehending the first and last points of the Work. And the vulgar names, both in Italian, French, and Spanish depends (in naming it,) upon the signification of the Latin word *Res*: **A thing**: unleast they use the name of *Algebra*. And therein (commonly) is a double errour. The one of them which think it to be of *Geber*, his inventing: the other of such as call it *Algebra*. For, first, though *Geber* for his great skill in Numbers, Geometry, Astronomy, and other marvellous Arts, might have seemed able to have first devised the said Rule: and also the name carrieth with it a very neer likenesse of *Geber* his name: yet true it is, that a *Greek* Philosopher and Mathematician, named *Diophantis*, before *Geber* his time, wrote 13 books thereof (of which six are yet extant: and I had them to * use, of the famous Mathematician, and my great friind, *Petrus Montaureus*:) *Anno* 1550. And secondly, the very name, is *Algiebar*, and not *Algebra*: as by the Arabian *Avicen*, may be proved: who hath these precise words in Latine, by *Andreas Alpagus* (most perfect in the Arabick tongue) so translated. *Scientia faciendi Algiebar & Almachabel.* 1. *Scientia inveniendi numerum ignotum, per additum numeri, & divisionem, & equationem.* Which is to say: **The Science of working Algiebar, and Almachabel**: that is: the **Science of finding an unknown Number, by adding of a Number, and Division, and Equation.** Here have you the name: and also the principal parts of the Rule, couched. To name it, *The rule, or art of Æquation*, doth signifie the middle part and the State of the Rule. This Rule hath his peculiar Characters, and the principal parts of *Arithmetick*, to it appertaining, do differ from the other *Arithmetical operations*. This *Arithmetick* hath Numbers Simple, Compound, Mixt, and Fractions accordingly. This Rule and *Arithmetick* of *Algiebar*, is so profound, so general, and so (in manner) conteineth the whole power of Numbers, Application practical: that mans wit, can deal with nothing more profitable about numbers: nor match, with a thing, more meet for the divine force of the Soul, (in humane studies, affairs or exercises) to be tried in. Perchance you looked for, (long ere now) to have had some particular proof, or evident testimony of the use, profit, and Commodity of Arithmetick vulgar, in the common life and trade of men. Thereto, then I will now frame my selfe: But herein great care I have, least length of sundry proofs, might make you deem, that either I did disdoubt your zealous mind to vertues school, or else mistrust your able wits, by some, to guesse much more. A proof, then four, five, or six, such, will I bring, as any reasonable man, therewith may be perswaded, to love and honour, yea learn and exercise the excellent Science of *Arithmetick*.

5.

And first: Who, neerer at hand, can be a better witnesse of the fruit received by *Arithmetick*, then all kind of Merchants? Though not alike, either need it, or use it. How could they forbear the use and help of the Rule, called the Golden Rule? Simple and

Compound: both forward and backward? How might they misse *Arithmetical* help in the Rules of Fellowship: either without time, or with time? and between the Merchant and his Factor? The Rules of Barterring in wares onely: or part in wares, and part in money, would they gladly want? Our Merchant Venturers, and Travellers over Sea, how could they order their doings justly and without losse, unlesse certain and generall Rules for Exchange of money, and Rechange, were, for their use, devised? The Rule of Alligation, in how sundry cases, doth it conclude for them, such precise veritics, as neither by natural wit, nor other experience they were able else to know? And (with the Merchant then to make an end) how ample and wonderful is the Rule of False positions? especially as it is now, by two excellent Mathematicians (of my familiar acquaintance in their life time) enlarged? I mean *Gemma Frisius* and *Simon Jacob*, Who can either in brief conclude, the general and Capital Rules? or who can Imagine the Myriades of sundry Cases, and particular examples in Act and earnest, continually wrought, tried and concluded by by the forenamed Rules only? How sundry other *Arithmetical practises*, are commonly in Merchants hands, and knowledge: They themselves can at large testifie.

The Mintmaster, and Goldsmith, in their mixture of Metals, either of divers kinds, or divers values; how are they, or may they, exactly be directed, and marvelously pleasured, if *Arithmetick* be their guide? And the honourable Physicians will gladly confesse themselves much beholding to the Science of *Arithmetick*, and that sundry wayes: But chiefly in their Art of Graduation, and compound Medicines. and though *Gallenus*, *Averrois*, *Arnoldus*, *Lullus*, and others have published their positions, aswel in the quantities of the Degrees above Temperament, as in the Rules, concluding the new *Form* resulting: yet a more precise, commodious, and easie *Method*, is extant: by a Countreyman of ours (above 200 years ago) invented. And forasmuch as I am uncertain, who hath the same: or when that little Latin treatise (as the Author writ it,) shall come to be printed: (Both to declare the desire I have to pleasure my Countrey, where in I may: and also, for very good proof of Numbers use, in this most subtile, and fruitful Philosophical Conclusion,) I intend in the mean while, most briefly, and with my farther help, to communicate the pith thereof unto you.

℞. B.

First, describe a circle: whose Diameter let be an inch. Divide the circumference into four equal parts. From the Center, by those 4 sections, extend 4 right lines: each of 4 inches and a half long: or of as many as you list, about 4 without the circumference of the circle: So that they shall be of 4 inches long (at the least) without the Circle. Make good evident marks at every inches end. If you list, you may sul divide the inches again into 10 or 12 smaller parts, equal. At the ends of the lines, write the names of the 4 principal Elemental qualities. *Hot*, and *Cold*, one against the other. And likewise *Moist* and *Dry*, one against the other. And in the circle write *Temperate*. Which *Temperature* hath a good Latitude: as appeareth by the Complexion of man. And therefore we have allowed unto it, the foresaid Circle: and not a point Mathematical or Physical.

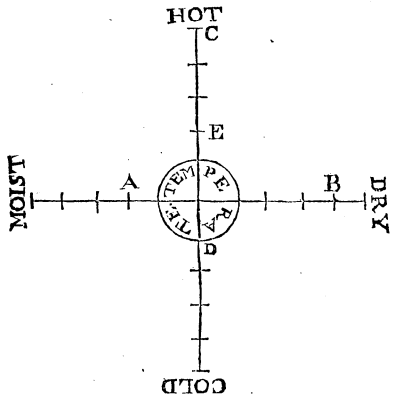
* Take some part of *Lullus* counsel in his Book *De Essentia*.

Now, when you have two things Miscible, whose degrees are * truly known; Of necessity, either they are of one quantity and waight, or of diverse. If they be of one quantity and waight: whether their forms, be contrary qualities, or of one kind (but of divers intentions and degrees) or a *Temperate*, and a contrary, *The form resulting of their Mixture, is in the Middle between the degrees of the forms mixt.* As for example, let *A*, be *Moist* in the first degree: and *B*, *Dry* in the third degree. Adde 1 and 3, that maketh 4: the half or middle of 4 is 2. This 2 is the middle, equally distant from *A* and *B* (for the * *Temperament* is counted none. And for it, you must put a Cypher, if at any time, it be in mixture.) Counting them from *B*, 2 degrees, toward *A*: you find it to be *Dry* in the first degree: So is the *Form resulting* of the mixture of *A*, and *B*, in our Example. I will give you another example. Suppose, you have two things, as *C*, and *D*: and of *C*, the Heat to be in the 4th. degree: and of *D*, the Cold, to be remisse, even unto the *Temperament*. Now, for *C*, you take 4: and for *D*, you take a Cypher: which, added unto 4, yeildeth onely 4. The middle, or half, whereof, is 2. Wherefore the *Form resulting* of *C*, and *D*, is *Hot* in the second degree: for, 2 degrees, accounted from *C*, toward *D*, end just in the second degree of heat. Of the third manner, I will give also an example: which let be this: I have a liquid Medicine whose Qualitie of heat is in the fourth degree exalted: as was *C*, in the example foregoing: and an other liquid Medicine I have: whose Qualitie, is heat, in the first degree. Of each of these, I mixt alike quantitie: Subtract here,

* Note.

Note.

here, the lesse from the more: and the residue divide into two equal parts: whereof the one part, either added to the lesse, or subtracted from the higher degree, doth produce the



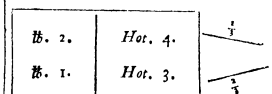
degree of the Form resulting, by this mixture of *C*, and *E*. As, if from 4, you abate 1, there resteth 3, the half of 3 is 1½: Add to 1 ½: you have 2 ½. Or subtract from 4 this 1½: you have likewise 2 ½ remaining. Which declareth, the *Form resulting*, to be *Heat*, in the middle of the third degree.

The second Rule.

"But if the quantities of two things Commixt, be divers, and the intensions, (of their "Forms miscible) be in divers degrees, and heigths. (Whether those Forms be of one "kind, or of Contrary kinds, or of a Temperate and a Contrary, *What proportion is of "the lesse quantity to the greater, the same shall be of the difference, which is between "the degree of the Form resulting, and the degree of the greater quantity of the thing "miscible to the difference, which is between the same degree of the Form resulting, and "the degree of the less quantity.* As for example. Let two pound of Liquour be given, hot in the fourth degree: and one pound of Liquour be given, hot in the third degree. I would gladly know the Form resulting, in the mixture of these two Liquours. Set down your numbers in order thus. Now by the Rule of Algiebar, have I devised a very easie, brief, and general manner of working in this case. Let us first suppose that *Middle Form resulting* to be 1 ☍: as that Rule teacheth. And because (by our Rule, here given) as the weight of 1, is to 2: So is the difference between 4, (the degree of the greater quantity) and 1 ☍: to the difference between 1 ☍ and 3: (the degree of the thing, in lesse quantity. And withal, 1 ☍, being always in a certain middle, between the two heigths or degrees. For the first difference, I set 4—1 ☍: and for the second, I set 1 ☍—3. And now again, I say, as 1 is to 2. so is 4—1 ☍ to 1 ☍—3. Wherefore, of these four proportional numbers, the first and the fourth multiplyed, one by the other, do make as much, as the second and the third multiplied the one by the other. Let these multiplications be made accordingly. And of the first and the fourth, we have 1 ☍— 3, and of the second and third, 8—2 ☍. Wherefore, your Æquation is between 1 ☍ — 3, and 8—2 ☍. Which may be reduced, according to the Art of Algiebar: as, here, adding 3, to each part, giveth the Æquation, thus, 1 ☍=11—2 ☍. And yet again, contracting or reducing it: Adde to each part, 2 ☍: Then have you 3 ☍ equal to 11 thus

lb. 2.	*Hot.* 4.
lb. 1.	*Hot.* 3.

thus represented 3 ☌ = 11. Wherefore, dividing 11, by 3: the quotient is 3 ⅓: the *Value* of our 1 ☌, *Cos*, or *Thing*, first supposed: and that is the heigth, or Intention of the *Form resulting*: which is *Heat*, in two thirds of the fourth degree: and here I set the shew of the work in conclusion thus. The proof hereof is easie, by substracting 3, from 3⅓, resteth ⅓. Substract the same heigth of the Form resulting, (which is 3 ⅓) from 4: then resteth ⅓. You see that ⅔ is double to ⅓: as 2 ℔. is double to 1 ℔. So should it be: by the Rule here given. Note. As you added to each part of the Æquation 3: so if you first added to each part 2 ☌, it would stand, 3 ☌ − 3 = 8: and now adding to each part 3: you have (as afore) 3 ☌ = 11.



Note.

And though, I, here, speak onely of two things miseible: and most commonly more then three, four, five or six, &c. are to be mixed: (and in one Compound to be reduced: and the Form resulting of the same, to serve the turn) yet these Rules are sufficient, duly repeated and iterated. In proceeding first, with any two: and then with the Form resulting and another, and so forth: For, the last work concludeth the Form resulting of them all: I need nothing to speak, of the mixture (here supposed) what it is. Common Philosophy hath defined it, saying, *Mixtio est miscibilium, alteratorum, per minima conjunctorum Unio.* Every word in the definition, is of great importance. I need not also spend any time, to shew, how, the other manner of distributing of degrees, doth agree to these Rules. Neither need I of the farther use belonging to the Crosse of Graduation (before described) in this place declare, unto such as are capable of that, which I have already said. Neither yet with examples, specifie the manifold varieties, by the foresaid two general Rules to be ordered. The witty and Studious, here, have sufficient: And they which are not able to attein to this, without lively teaching and more in particular: would have larger discoursing, then is meet in this place to be dealt withall: And other (perchance) with a proud snuffe will disdain this little: and would be unthankful for much more. I, Therefore conclude; and with such as have modest and earnest Philosophical minds, to laud God highly for this: and to marvail that the profoundest and subtilest point, concerning *Mixture of Forms, and Qualities Natural*, is so matcht and married with the most simple, easie, and short way of the noble Rule of *Algiebar*. Who can remain, therefore, unperswaded, to love, allow, and honour the excellent Science of *Arithmetick*? For, here, you may perceive that the little finger of *Arithmetick* is of more might and contriving, then a hundred thousands mens wits, of the middle sort, are able to perform, or truly to conclude, without help thereof.

Taxiuch.

Now, will we farther, by the wise and valiant Captain, be certified, what help he hath, by the Rules of *Arithmetick*: in one of the arts to him apperteining: and of the Greeks " named Taxiuch. That is, the skill of ordering Souldiers in Battle ray, after the best " manner to all purposes. This art so much dependeth upon Numbers use, and the Mathematicals, that *Ælianus* (the best writer thereof,) in his work to the Emperour *Hadrianus*, by his perfection, in the Mathematicals, (being greater then others before him had,) thinketh his book to passe all other the excellent work, written of that art, unto his dayes. For, of it, had written *Æneas*: *Cyneas of Thessaly*: *Pyrrhus Epirota*, and *Alexander* his sonne: *Clearchus*: *Pausanias*: *Evangelus*: *Polybius*, familiar friends to *Scipio*, *Eupolemus*, *Iphicrates*, *Possidonius*, and very many other worthy Captains, Philosophers and Princes of immortal fame and memory. Whose fairest flower of their garland (in this feat) was *Arithmetick*: and a little perceiverance, in *Geometrical* Figures. But in many other cases doth *Arithmetick* stand the Captain in great stead. As in proportioning of victuals for the army, either remaining at a stay; or suddenly to be encreased with a certain number of Souldiers: and for a certain time. Or by good arts to diminish his Company, to make the victuals, longer to serve the remanent, and for a certain determined time: If need so require. And so in sundry his other accounts, Reckonings, Measurings, and Proportionings, the wise, expert, and circumspect Captain will affirm the Science of *Arithmetick*, to be one of his chief Counsailors, directers and aiders. Which thing (by good means) was evident to the Noble, the Couragious, the Loyal, and Courteous *John*, late Earl

Earl of *Warwick*. Who was a young Gentleman, throughly known to very few. Albeit his lusty valiantnesse, force, and Skill in Chivalrous feats and exercises: his humbleness, and friendliness to all men, were things, openly, of the world perceived. But what roots (otherwise,) vertue had fastened in his brest, what Rules of Godly and honourable life he had framed to himself: what vices, (in some then living) notable, he took great care to eschew, what manly vertues, in other noble men, (flourishing before his eyes,) he Sythingly aspired after: what prowesses he purposed and meant to atchieve: with what feats of Arts, he began to furnish and fraught himself, for the better service of his King and Countrey, both in peace and war. These (I say) his Heroical Meditations, fore-castings, and determinations, no twain, (I think) betide my helf, can so perfectly, and truly report. And therefore, in Conscience, I count it my part, for the honour, preferment, and procuring of vertue, (thus briefly) to have put his Name, in the Register of *Fame Immortal*.

To our purpose. This *John* by one of his acts (besides many other: both in *England* and *France*, by me, in him noted) did disclose his hearty love, to vertuous Sciences: and his noble intent, to excell in Martial prowesse: when we with humble request, and instant Soliciting: got the best Rules (either in the time past, by Greek or Roman, or in our time used: and new Stratagems therein devised) for ordering of all Companies, summes, and Numbers of men, (many, or few) with one kind of weapon, or more appointed: with Artillery, or without: on horsback or on foot: to give or take on-set: to seem many, being few: to seem few, being many. To march in battail or journey: with many such feats, to foughten field, Skirmish or Ambush appertaining: and of all these lively designments (most curiously) to be in velame parchment described: with Notes and peculiar marks, as the art requireth: and all these Rules and descriptions Arithmetical, inclosed in a rich Case of Gold, he used to wear about his neck, as his Jewel most precious, and Counsailour most trusty. Thus, *Arithmetick*, of him, was shrined in gold: Of *Numbers* fruit, he had good hope. Now Numbers therefore innumerable, in *Numbers* praise, his shrine shall sinde.

This Noble Earle, died Anno 1552. scarce 24 years of age, having no issue by his wife, Daughter to the Duke *Somerset*.

What need I, (for farther proof to you) of the Schoolmasters of Justice, to require testimony: how needful, how fruitful, how skilful a thing *Arithmetick* is? I mean the Lawyers of all sorts. Undoubtedly, the Civilians can marvellously declare: how, neither the Ancient Romane Laws, without good knowledg of *Numbers Art*, can be perceived: Nor (Justice infinite Cases) without due proportion, (narrowly considered) is able to be executed. How justly, and with great knowledg of Art, did *Papinianus* institute a law of partition, and allowance, between man and wife after a divorce? But how *Accursius*, *Baldus*, *Bartolus*, *Jason*, *Alexander*, and finally, *Alciatus*, (being otherwise, notably well learned) do jumble, guesse and erre from the equity, art and intent of the Law-maker: *Arithmetick* can detect, and convince: and cleerly, make the truth to shine. Good *Bartolus* tired in the examining and proportioning of the matter; and with *Accursius* Glosse much cumbred: burst out, and said: *Nulla est in toto libro, hac glossa difficilior: Cujus computationem nec Scholastici, nec Doctores intelligunt, &c.* That is: **In the whole book, there is no glosse harder then this; Whose account or reckoning, neither the Scholers nor the Doctours understand, &c.** What can they say of *Julianus* law, *Si ita Scriptum*, *&c.* Of the Tettators will, justly performing, between the wife, sonne, and daughter? How can they perceive the equity of *Aphricanus*, *Arithmetical* Reckoning, where he treateth of *Lex Falcidia*? How can they deliver him, from his Reprovers: and their maintainers: as *Joannes*, *Accursius*, *Hypolitus*, and *Alciatus*? How justly and artificially, was *Africanus* reckoning made? Proportionating to the summes bequeathed, the Contributions of each part? Namely, for the hundred presently receiued, 17 ⅐. And for the hundred, received after ten moneths 12 ⅔: which make the 30: which were to be contributed by the legataries to the heir. For, what proportion 100 hath to 75: the same hath 17 ⅐ to 12 ⅔: Which is *Sesquitertia*. that is, as 4, to 3, which make 7. Wonderful many places in the Civil law, require an expert *Arithmetician*, to understand the deep Judgment and just determination of the Ancient Roman Law-maker. But much more expert ought he to be, who should be able to decide with equity, the infinite variety of Cases, which doe, or may happen, under every one of those laws and ordinances Civil. Hereby, easily, you may now conjecture: that in the Canon law: and in the laws of the Realm (which with us, bear the chief authority) Justice and equity might be greatly preferred, and skilfully executed through

Justice.

due skill of Arithmetick, and proportions apperteining. The worthy Philosophers and prudent law-makers (who have written many books *De Republica:* How the best state of Common-wealths might be procured and mainteined,) have very well determined of Justice: (which, not onely is the Base and foundation of Common weales: but also the total perfection of all our works, words, and thoughts:) "defining it, to be that vertue, "by which, to every one, is rendered, that to him appertaineth. God challengeth this at our hands to be honoured as God: to be loved, as a Father, and to be feared, as a Lord and Master. Our neighbours proportion is also prescribed of the Almighty Law-maker: which is, to do to others, even as we would be done unto. These proportions, are in Justice necessary: in duty, commendable: and of Common-wealths, the life, strength, stay, and flourishing. *Aristotle* in his *Æthicks* (to fetch the seed of Justice, and light of direction, to use, and execute the same) was fain to fly to the perfection, and power of Numbers: for proportions Arithmetical and Geometrical. *Plato* in his book called *Epinomis*, (which book is the Treasure of all his doctrine) where his purpose is to seek a Science, which, when a man had it, perfectly: he might seem, and so be, indeed, *Wise.* He briefly of other Sciences discoursing, findeth them, not able to bring it to passe: But of the Science of Numbers, he saith. *Illa, quæ numerum mortalium generi dedit, id profectò efficiet. Deum autem aliquem, magis quàm fortunam, ad salutem nostram, hoc munus nobis arbitror contulisse, &c. Nam ipsum bonorum omnium Authorem, cur non maximi boni, Prudentiæ dico, causam arbitramur?* **That Science, verily, which hath taught mankind number, shall be able to bring it to passe. And, I think, a certain God, rather than Fortune, to have given us this gift, for our bliss. For, why should we not judg him who is the Authour of all good things, to be also the cause of the greatest good thing, namely, Wisedome?** There, at length, he proveth *Wisedome* to be atteined by good skill of *Numbers.* With which great Testimony, and the manifold proofs, and reasons, before expressed, you may be sufficiently and fully perswaded; of the perfect Science of *Arithmetick*, to make this account: That of all Sciences, next to *Theologie*, it is most divine, most pure, most ample, and general, most profound, most subtile, most commodious, and most necessary. Whose next Sister, is the absolute Science of *Magnitudes:* of which (by the direction and aid of him, whose *Magnitude* is infinite, and of us incomprehensible) I now intend so to write, that both with the *Multitude*, and also with the *Magnitude* of marvellous and fruitful verities, you (my friends and Countreymen) may be stir'd up and awaked, to behold what certain Arts and Sciences (to our unspeakable behoof) our heavenly Father, hath for us prepared, and revealed by sundry *Philosophers* and *Mathematicians.*

BOth, *Number* and *Magnitude*, have a certain Original seed, (as it were) of an incredible property: and of man, never able, fully to be declared. Of *Number*, an Unit, and of *Magnitude*, a Point, do seem to be much like Original causes: But the diversity nevertheless, is great. We defined an *Unit* to be a thing Mathematical indivisible: A Point, likewise, we said to be a Mathematical thing indivisible. And farther, that a Point may have certain determined Situation: that is, that we may assign, and prescribe a Point, to be here, there, yonder, &c. Herein (behold) our Unit is free, and can abide no bondage, or to be tyed to any place, or seat: divisible or indivisible. Again, by reason, a Point may have a situation limited to him: a certain motion, therefore, (to a place, and from a place) is to a Point incident, and appertaining. But an *Unit* cannot be imagined, to have any motion. A point, by his motion, produceth Mathematically, a line, (as we said before) which is the first kind of Magnitudes, and most simple: An *Unit*, cannot produce any number. A line, though it be produced of a Point moved, yet it doth not consist of points: Number, though it be not produced of an *Unit*, yet doth it consist of Units; as a material cause. But formally, Number, is the Union and Unity of Units. Which uniting and knitting, is the workmanship of our mind: which, of distinct and discreet Units maketh a Number: by uniformity, resulting of a certain multitude of Units. And so, every number may have his least part given: namely, an Unit: But not of a magnitude, (no, not of a Line,) the least part can be given: because, infinitely, division thereof, may be conceived. All Magnitude, is either a Line, a Plain, or a Solid. Which Line, Plain or Solid, of no Sense, can be perceived, nor exactly by hand (any way) represented: nor of Nature

Number.

Nature produced: But, as (by degrees) Number did come to our perceiverance: So by visible forms, we are holpen to imagine, what our Line Mathematical is. What our Point, is. So precise, are our Magnitudes, that one Line is no broader than an other: for they have no breadth: Nor our Plains have any thicknesse: Nor yet our Bodies, any weight: be they never so large of dimension. Our Bodies, we can have smaller, than either Art, or Nature can produce any: and greater also, then all the World can comprehend. Our least Magnitudes, can be divided into so many parts, as the greatest. As, a Line of an inch long, (with us) may be divided into as many parts, as may the diameter of the whole World, from East to West: or any way extended: What priviledges, above all manual Arts, and Natures might, have our two Sciences Mathematical? to exhibit, and to deal with things of such power, liberty, simplicity, purity, and perfection? And in them, so certainly, so orderly, so precisely, to proceed: as excellent, is that workman Mechanical Judged, who neerest can approach to the representing of works, Mathematically demonstrated? And our two Sciences, remaining pure, and absolute in their proper terms, and in their own matter, to have, and allow onely such demonstrations; as are plain, certain, universal, and of an eternal verity? This Science of *Magnitude*, his properties, conditions, and appurtenances: commonly, now is, and from the beginning, hath of all Philosophers, been called *Geometry.* But verily, with a name too base and scant, for a Science of such dignity and ampleness. And, perchance, that name, by common and secret consent, of all wise men, hitherto eath been suffered to remain: that it might carry with it a perpetual memory of the first and notablest benefit, by that Science, to common people shewed: Which was, when Bounds and Meres of land and ground, were lost, and confounded, (as in *Egypt*, yearly, with the over-flowing of *Nilus*, the greatest and longest river in the World) or, that ground bequeathed, were to be assigned: or ground sold, were to be laid out: or (when disorder prevailed) that Commons were distributed into Severalties. For, where, upon these and such like occasions, Some by ignorance, some by negligence; Some by fraud, and some by violence, did wrongfully limit, measure, encroach, or challenge (by pretence of just content and measure) those lands and grounds: great losse, disquietnesse, murther, and war did (full oft) ensue: Till, by Gods mercy, and mans industry, The perfect Science of Lines, Plains, and Solids (like a divine Justicier,) gave unto every man his own. The people then, by this art pleasured, and greatly relieved, in their lands just measuring: and other Philosophers, writing rules for land measuring: between them both, thus, confirmed the name of *Geometria*, that is; (according to the very Etymologie of the word) Land-measuring. Wherein the people knew no farther of Magnitudes use, but in Plains: and the Philosophers of them had no fit hearers, or Scholers, farther to disclose unto, then of flat, plain *Geometry.* And though, these Philosophers, knew of farther use, and best understood the Etymologie of the word, yet this name *Geometria*, was of them applyed generally to all sorts of Magnitudes: unlesse otherwhile, of *Plato* and *Pythagoras.* When they would precisely declare their own doctrine. Then was * *Geometria*, with them, *Studium quod circa planum versatur.* But, well you may perceive by *Euclid's Elements* that more ample is our Science, then to measure Plains: and nothing lesse therein is taught (of purpose) then how to measure Land. Another name, therefore, must needs be had, for our Mathematical Science of Magnitudes: which regardeth neither clod nor turff: neither hill, nor dale: neither earth nor heaven: but is absolute *Megethologia*, not creeping on ground, and dasling the eye with pole, perch, rod, or line: but lifting the heart above the Heavens, by invisible lines, and immortal beams: "meeteth with the reflexions of the light inconprehensible, and so procureth joy, and perfection unspeakable. Of which true use of our *Megethica*, or *Megethologia*, Divine *Plato* seemed to have good taste and judgment: and (by the name of *Geometry*) so noted it: and warned his Scholers thereof: as, in his seventh *Dialogue*, of the Commonwealth may evidently be seen. Where (in Latin) thus it is; right well translated: *Profecto, nobis, hæc non negabunt, Quicunque vel paululum quid Geometriæ gustarunt, quin hæc Scientia, contrà, omnino se habeat, quàm de ea loquuntur, qui in ipsa versantur.* In English, thus: **Verily (saith *Plato*) whosoever hath (but even very little) talked, of Geometry, will not deny unto us, this: but that this Science, is of another condition, quite contrary to that, which they that are exercised in it, do speak of it.** And there it followeth of our *Geometry.* *Quòd quæritur cognoscendi*

Geometry.

Plato 7. de Rep.

cognoscendi illius gratia, quod semper est, non & ejus quod oritur quandóque & interit. Geometria, ejus quod est semper, Cognitio est. At tollet igitur (ô Generose Vir) ad Veritatem, animum: atque ita, ad Philosophandum preparabit cogitationem, ut ad supera convertamus: qua, nunc, contra quàm decet, ad inferiora dejicimus, &c. Quàm maximè igitur præcipiendum est, ut qui præclarissimam hanc habitant Civitatem, nullo modo, Geometriam spernant. Nam & quæ præter ipsius propositum, quodam modo esse videntur, haud exigua sunt, &c. It must needs be confessed (saith *Plato.*) That [Geometry] is learned, for the knowing of that which is ever: and not of that, which in time, both is bred and is brought to an end, &c. Geometry is the knowledge of that which is everlasting. It will lift up therefore (O Gentle Sir) our mind to the verity: and by that means, it will prepare the Thought, to the Philosophical love of wisedome, that we may turn or convert, toward heavenly things, [both mind and thought] which now, otherwise than becommeth us, we call down on base or inferiour things, &c. chiefly, therefore Commandement must be given, that such as doe inhabite this most honourable City, by no means, despise Geometry. For even those things [done by it] which, in manner seem to be, beside the purpose of Geometry: are of no smal importance, &c. And besides the manifold uses of *Geometry*, in matters apperteining to war, he addeth more, of second unpurposed fruit, and commodity, arising by *Geometry*, saying: *Scimus quin etiam ad Disciplinas omnes facilius per discendas interesse omnino, attigerit ne Geometriam aliquis, an non, &c. Hanc ergo Doctrinam, secundo loco discendam juvenibus statuamus.* That is. But also, we know, that for the more easie learning of all Arts, it importeth much, whether one have any knowledg in Geometry, or no, &c. Let us therefore make an ordinance or decree, that this Science of young men shall be learned in the second place. This was Divine *Plato* his judgment, both of the purposed, chief, and perfect use of *Geometry*: and of his second, depending, derivative commodities. And for us Christian men, a thousand thousand more occasions are to have need of the help of * *Megetheological* Contemplations: whereby to train our Imaginations and Minds by little and little to forsake and abandon, the grosse and corruptible Objects, of our outward senses: and to apprehend by sure doctrine demonstrative, Things Mathematical. And by them readily to be holpen and conducted, to conceive, discourse, and conclude of things Intellectual, Spiritual, Eternal, and such as concern our blisse everlasting: which, otherwise (without special priviledg of Illumination, or Revelation from Heaven,) No mortal mans wit (naturally) is able to reach unto, or to compasse. And, verily, by my small Talent (from above) I am able to prove and testifie that the literal Text, and order of our divine Law, Oracles and Mysteries, require more skill in Numbers and Magnitudes: then (commonly) the Expositors have uttered: but rather onely (at the most) so warned: and shewed their own want therein. (To name any, is needlesse: and to note the places, is, here, no place; But if I be duly asked, my answer is ready.) And without the literal, Grammatical, Mathematical, or Natural verities of such places, by good and certain Art, perceived no Spirituall sense, proper to those places, by Absolute *Theologie*) will thereon depend. "No man, therefore, can doubt, but toward the atteining of knowledg incom-
"parable, and Heavenly Wildome: Mathematical speculations both of Numbers and
"Magnitudes: are means, aids, and guides: ready, certain, and necessary. From henceforth, in this my Preface, will I frame my talk, to *Plato* his fugitive Scholers: or, rather, to such, who well can, (and also will,) use their outward senses, to the glory of God, the benefit of their Countrey, and their own secret contentation, or honest preferment on this earthly Scaffold. To them, I will orderly recite, describe, and declare a great Number of Arts, from our two Mathematical fountains, derived into the fields of *Nature*. Whereby such seeds, and roots, as lie deep hid in the ground of *Nature*, are refreshed, quickned, and provoked to grow, shoot up, flour, and give fruit, infinite and incredible. And these Arts shall be such, as upon magnitudes properties do depend more than upon Number. And by good reason we may call them Arts, and Arts Mathematical Derivative, for (at this time) I define An Art, to be a Methodical compleat Doctrine, having abun-

J. D. Herein I would gladly shake off, the earthly name of Geometry.

☞

An Art.

abundance of sufficient and peculiar matter to deal with, by the allowance of the Metaphysical Philosopher; the knowledge whereof, to humane state is necessary. And that I account an Art Mathematical derivative, which by Mathematical demonstrative Method, in Numbers or Magnieudes, ordereth and confirmeth his doctrine, as much and as perfectly as the matter subject will admit. And for that I intend, to use the name and propertie of a *Mechanician*, otherwise, then (hitherto it hath been used, I think it good (for distinction sake) to give you also a brief description, what I mean thereby. A Mechanician or a Mechanical workman, is he whose skill is without knowledg of Mathematical demonstration, perfectly to work and finish any sensible work, by the Mathematician principal or derivative, demonstrated or demonstrable. Full well I know, that he which inventeth or maketh these demonstrations, is generally called *A speculative Mechanician*, which differeth nothing from a *Mechanical Mathematician*. So, in respect of divers actions, one man may have the name of sundry Arts, as sometime of a Logician, sometimes, in the same manner otherwise handled) of Rethorician. Of these trifles, I make (as now, in respect of my Preface), small account: to file them for the fine handling of subtile curious disputers. In other places, they may command me, to give good reason: and here, I will not be unreasonable.

Art Mathematical Derivative.

A Mechanician.

First, then, from the purity, absolutenesse, and Immateriality of principal *Geometrie*, is that kind of *Geometrie* derived, which vulgarly is counted *Geometrie*, and is the Art of measuring sensible magnitudes, their just quantities and contents. This, teacheth to measure either at hand: and the Practiser to be by the thing Measured: and so, by due applying of Compasse, Rule, Square, Yard, Ell, Perch, Pole, Line, Gauging rod, (or such like Instrument) to the Length, Plain, or Solid measured, * to be certified, either of the length, perimetrie, or distance lineal: and this is called: *Mecometrie*. Or * to be certified of the content of any plain Superficies; whether it be in ground Surveyed, Board, or Glasse measured, or such like thing: which measuring, is named, *Embadometrie*. * Or else to understand the Solidity and content of any bodily thing: as of Timber and Stone, or the content of Pits, Ponds, Wels, Vessels, small and great, of all fashions. Where of Wine, Oil, Beer, or Ale Vessels, &c. the Measuring commonly hath a peculiar name and is called *Gauging*. And the general name of these Solid measures is *Stereometry*. Or else, this *Vulgar Geometry*, hath consideration to teach the Practiser, how to measure things, with good distance between him and the thing measured: and to understand thereby, either * how farre, a thing seen (on land or water) is from the measurer: and this may be called *Apomecometrie*: Or, how high, or deep, above or under the level of the measurers standing, any thing is, which is seen on land or water, called *Hypsometry*. * Or, it informeth the measurer, how broad any thing is, which is in the measurers view: so it be on land and water situated: and may be called *Platometry*. Though I use here to condition the thing measured, to be on land or water situated: yet, know for certain, that the sundry height of Clouds, blazing Stars, and of the Moon, may (by these means) have their distances from the earth: and, of the blazing Stars and Moon, the solidity (as well as distances) to be measured: but because, neither these things are vulgarly taught, nor of a common Practiser so ready to be executed. I, rather let such measures be reckoned incident to some of our other Arts, dealing with things on high, more purposely, than this vulgar Land measuring Geometry doth: as in *Perspective* and *Astronomy*, &c.

I.

Geometry Vulgar.

1.
2.
3.

2.

1.
2.
3.

Note.

OF these Feats (farther applyed) is sprung the Feat of *Geodesie*, or Land measuring: more cunningly to measure and surveigh Land, Woods, and Waters, afar off. More cunningly, I say: But God knoweth (hitherto) in these Realms of *England* and *Ireland*, (whether through ignorance or fraud, I cannot tell in every particular) how great wrong and injury hath (in my time) been committed by untrue measuring and surveying of Land or wood, any way. And, this I am sure: that the Value of the difference, between the truth and such Surveyes, would have been able to have found (for ever) in each of our two Universities, and excellent Mathematical Reader: to each allowing (yearly) a hundred Marks of lawful money of this Realm: which indeed, would seem requisite, here to be had (though by other wayes provided for) as well as the famous Universiey of *Paris* hath two

Note.

Mathematical Readers, and each two hundred French Crowns yearly, of the French Kings magnificent liberality onely. Now, again, to our purpose, returning: Moreover, of the former knowledg Geomegrical, are grown the skils of *Geography*, *Chorography*, *Hydrography*, and *Stratarithmetry*.

" GEOGRAPHIE teacheth ways by which, in sundry forms, (as *Spharike*, *Plain*, " or other) the Situation of Cities, Towns, Villages, Forts, Castles, Mountains, Woods, " Havens, Rivers, Creeks, and such other things, upon the out-face of the earthly Globle " (either in the whole, or in some principal member and portion thereof conteined) may " be described and designed, in commensurations Analogical to Nature and verity: and " most aptly to our view, may be represented. Of this Art how great pleasure, and how manifold commodities do come unto us daily and hourly: of most men is perceived. While some, to beautifie their Hals, Parlers, Chambers, Galeries, Studies, or Libraries with: other some, for things past, as battels fought, earth-quakes, heavenly firings, and such occurrents, in Histories mentioned: thereby lively, as it were, to view the place, the Region adjoyning, the distance from us: and such other circumstances. Some other presently to view the large dominion of the Turk: the wide Empire of the Moscovite: and the little morsel of ground, where Christendome (by profession) is certainly known. Little, I say, in respect of the rest, &c. Some either, for their own journeys directing into far lands: or to understand of other mens travails. To conclude, some for one purpose, and some for another, liketh, loveth, getteth, and useth Maps, Charts, and Geographical Globes. Of whose use, to speak sufficiently, would require a book peculiar.

CHOROGRAPHIE, seemeth to be an underling, and a twig of *Geographie*: " and yet nevertheless, is in practice manifold, and in use very ample. This teacheth " Analogically to describe a small portion or circuit of ground, with the contents: not " regarding what commensuration it hath to the whole, or any parcel, without it, con- " teined. But in the territory or parcel of ground, which it taketh in hand to make de- " scription of, it leaveth out (or undescribed) no notable, or odde thing, above the " ground visible. Yea, and sometimes, of things under ground, giveth some peculiar mark, " or warning: as of Metal-mines, Cole-pits, Stone quaries, &c. Thus a Dukedome, a Shire, " a Lordship, or lesse, may be described distinctly. But marvellous pleasant and profitable is it, in the exhibiting to our eye and commensuration, the plat of a City, Town, Fort, or Palace, in true Symmetry: not approaching to any of them: and out of Gun-shot, &c. Hereby the *Architect* may furnish himself, with store of what paterns he liketh; to his great instruction; even in those things which outwardly are proportioned: either simply in themselves, or respectively to Hils, Rivers, Havens, and Woods adjoyning. So also, term this particular description of places, *Topographie*.

" HYDROGRAPHIE, delivereth to our knowledg, on Globe or in Plain, the per- " fect Analogical description, of the Ocean, Sea-coasts, through the whole World, or " in the chief and principal parts thereof, with the Isles and chief particular places of " danger, conteined within the bounds and Sea-coasts described; as of Quick-sands, " Banks, Pits, Rocks, Races, Countertides, Whirle-pools, &c. This dealeth with the Element of the water chiefly: as *Geographie* did principally take the Element of the Earths description (with his appurtenances) to task. And besides this, *Hydrographie*, requireth a particular Register of certain Land-marks (where marks may be had) from the sea, well able to be skried, in what point of the Sea compasse they appear, and what apparent form, situation, and bigness they have, in respect of any dangerous place in the sea, or neer unto it, assigned: And in all Coasts, what Moon maketh full Sea, and what way, the Tides and Ebbs, come and go, the *Hydrographer* ought to record. The sounding likewise: and the Chanels wayes: their number, and depths ordinarily, at ebbe and floud, ought the *Hydrographer*, by observation and diligence of *Measuring*, to have certainly known. And many other points, are belonging to perfect *Hydrographie*, and for to make a *Rutter*, by: of which, I need not here speak: as of the describing, in any place, upon Globe or Plain, the 32 points of the Compasse, truly; (whereof, scarcely four, in *England*, have right knowledg, because, the lines thereof, are no strait lines, nor Circles.) Of making due projection of a Sphære in plain. Of the Variation of the Compasse, from true North. And such like matters (of great importance, all) I leave to speak of in this place, because, I may seem (already) to have enlarged the bounds and duty of an *Hydrographer*, much more than any man (to this day) hath noted or prescribed: yet am I well able to prove, all these

these things, to appertain, and also to be proper to the Hydrographer. The chief use and end of this Art, is the Art of Navigation, but it hath other divers uses, even by them to be enjoyed, that never lack sight of land.

STRATARITHMETRIE, is the skill (apperteining to the war,) by which a man can set in figure Analogical to any *Geometrical* figure appointed, any certain number or summe of men: of such a figure capable, (by reason of the usual spaces between Souldiers allowed: and for that, of men, can be made no Fractions: yet nevertheless, he can order the given summe of men, for the greatest such figure, that of them can be ordered) and certifie of the overplus (if any be) and of the next certain summe, which, with the overplus, will admit a figure exactly proportional to the figure assigned. By which skill also, of any army or company of men; (the figures and sides of whose orderly standing, or array is known,) he is able to expresse the just number of men, within that figure conteined: or (orderly) able to be conteined. * And this figure, and sides thereof, he is able to know: either by, and at hand, or a farre off. Thus farre stretcheth the description " and property of *Stratarithmetrie*: sufficient for this time and place. It differeth from " the feat *Tactical*, *De aciebus instruendis*, because, there is necessary the wisedome and " foresight, to what purpose he so ordereth the men, and skilful ability, also, for any occa- " sion, or purpose, to devise and use the aptest and most necessary order, array and " figure of his Company and sum of men. By figure, I mean, as either of a *Perfect Square*, " *Triangle*, *Circle*, *Ovale*, *long Square*, (of the Greeks it is called *Eteromekes*) *Rhombe*, *Rhomboid*, *Lunular*, *Ring*, *Serpentine*, and such other Geometrical figures: Which in wars, have been, and are to be used for commodiousnesse, necessity and advantage, &c. And no small skill ought he to have, that should make true report, or neer the truth of the numbers and summes, of Footmen or Horsemen, in the Enemies ordering. A farre off, to make an estimate, between neer terms of More and Lesse, is not a thing very rise, among those that gladly would doe it. Great policy may be used of the Captains, (at times fit, and in places convenient) as to use figures, which make greatest shew, of so many as he hath: and using the advantage of the three kinds of usual spaces: (between Footmen and Horsmen) to take the largest: or when he would seem to have few, (being many:) contrariwise in figure and space. The Herald, Pursuivant, Serjeant Royal, Captain, or whosoever is careful to come neer the truth herein, besides the judgment of his expert eye, his skill of ordering *Tractical*, the help of his Geometrical Instrument: Ring or Staffe Astronomical: (commodiously framed for carriage and use.) He may wonderfully help himself by perspective Glasses. In which, (I trust) our posterity will prove more skilful and expert, and to greater purposes; than in these dayes, can (almost) be credited to be possible.

Thus have I lightly passed over the Artificial Feats, chiefly depending upon vulgar *Geometry*: and commonly, and generally reckoned under the name of *Geometry*. But there are other (very many) *Methodical Arts*, which declining from the purity, simplicity, and Immateriality, of our Principal Science of *Magnitudes*: do yet, nevertheless, use the great ayd, direction, and method of the said principal Science, and have proper names, and distinct: both from the Science of *Geometry*, (from which they are derived) and one from the other. As PERSPECTIVE, ASTRONOMY, MUSICK, COSMOGRAPHIE, ASTROLOGY, STATIKE, ANTHROPOGRAPHIE, TROCHILIKE, HELICOSOPHIE, PNEUMATITHMIE, MENADRIE, HYPOGEIODIE, HYDRAGOGIE, HOROMETRIE, ZOGRAPHIE, ARCHITECTURE, NAVIGATION, THAUMATURGIKE, and ARCHIMASTRIE. I think it necessary, orderly, of those to give some peculiar descriptions: and withall, to touch some of their commodious uses, and so to make this Preface, to be a little sweet, pleasant Nosegay for you, to comfort your Spirits, being almost out of courage, and in despair, (through brutish brute.) Weening that *Geometrie*, had but served for building of an house, or a curious bridg, or the roof of Westminster hall, or some witty pretty divise, or engin, appropriate to a Carpenter, or Joyner, &c. That the thing is far otherwise, than the world, (commonly) to this day, hath deemed, by word and work, good proof will be made.

Among these Arts, by good reason, PERSPECTIVE ought to be had, e're of *Astronomical Apparences*, perfect knowledge can be attained. And because of the prerogative of *Light*, being the first of *Gods Creatures*: and the eye, the light of our body, and

* *Note.* The difference between Stratarithmetry; and Tracticie.

J. D. Friend, You will find it hard, to perform my description of this feat. But by Chorography, you may help your self somewhat: where the figures known (in Sides and Angles) are not regular: And where, Resolution, into Triangles can serve, &c. & yet you will finde it strange to deal thus generally with Arithmetical figures: and, that for Battail ray. Their Contents differ so much from like Geometrical figures.

and his sense most mighty, and his organ most artificial, and *Geometrical*. At *Perspective*, we will begin therefore. **Perspective is an Art Mathematical, which demonstrateth the manner and properties of all Radiations, Direct, Broken, and Reflected.** This Description, or Notation, is brief: but it reacheth so far, as the World is wide. It concerneth all Creatures, all Actions, and passions, by Emanation of beams performed. Beams or natural lines, (here) I mean, not of light onely, or of colour (though they, to eye, give shew, witnesse, and proof, whereby to ground the art upon) but also of other *Forms*, both *Substantial* and *Accidental*, the certain and determined active Radial Emanations. By this Art (omitting to speak of the highest points) we may use our eyes, and the light with greater pleasure, and perfecter judgment: both, of things, in light seen, and of other: which by like order of lights Radiations, work and produce their effects. We may be ashamed to be ignorant of the cause, why so sundry ways our eye is deceived, & abused: as, while the eye weeneth a round Globe or Sphere (being far off) to be a flat and plain circle, and so likewise judgeth a plain Square to be a round: supposeth walls parallels, to approach, afar off: roof and floure parallels, the one to bend downward, the other to arise upward, at a little distance from you. Again, of things being in like swiftnesse of moving to think the neerer to move faster, and the farther much slower. Nay, of two things, whereof the one (incomparably) doth move swifter than the other, to deem the slower to move very swift, and the other to stand: What an errour is this, of our eye? Of the Rainbow, both of his Colours, of the order of the colours, of the bigness of it, the place and height of it, (&c.) to know the causes demonstrative, is it not pleasant, is it not necessary? of two or three Suns appearing: of blazing Stars: and such like things: by natural causes, brought to passe, (and yet, nevertheless, of farther matter significative) it is not commodious for man to know the very true cause and occasion Natural? yea, rather is it not, greatly, against the Soveraignty of mans nature, to be so over-shot and abused with things (at hand) before his eyes? as with a Peacocks tail, and a Doves neck: or a whole ore, in water holden, to seem broken. Things far off to seem neer, and neer, to seem far off. Small things to seem great, and great to seem small. One man to seem an army. Or a man to be cuttily afraid of his own shadow. Yea, so much, to fear, that if you being (alone,) neer a certain Glasse, proffer with dagger or sword, to foyne at the glasse, you shall suddenly be moved to give back (in manner) by reason of an Image appearing in the air, between you and the glasse, with like hand, sword or dagger, & with like quicknesse foyning at your very eye, likewise as you doe at the glasse. Strange, this is, to hear off, but more mervalous to behold, than these my words can signifie. And nevertheless, by demonstration Optical, the order and cause thereof, is certified: even so as the effect is consequent. Yea, thus much more, dare I take upon me, toward the satisfying of the noble courage, that longeth ardently for the wisedome of causes Natural: as to let him understand, that, in *London*, he may with his own eye, have proof of that, which I have said herein. A Gentleman, (which, for his good service, done to his Countrey, is famous and honourable: and for skill in the Mathematical Sciences, and Languages, is the odde man of this land, &c.) even he is able: and (I am sure) will very willingly, let the glasse, and the proof be seen: and so I (here) requeste him: for the encrease of wisedome, in the honourable, and for the stopping of the mouths malicious; and repressing the arrogancy of the ignorant; ye may easily guesse, what I mean. This Art of *Perspective* is of that excellency, and may be led to the certifying, and executing of such things, as no man would easily believe: without Actual proof perceived. I speak not hing of Natural *Philosophy*, which, without *Perspective*, cannot be fully understood, nor perfectly atteined unto. Nor of *Astronomy*: which without *Perspective*, cannot well be grounded. Nor *Astrology*, naturally verified, and avouched. That part hereof, which dealeth with Glasses (which name Glasse, is a generall name, in this Art, for any thing, from which a Beam rebounderh) is called *Catoptrike*, and hath so many uses, both marvellous, and profitable: that, both, it would hold me too long, to note therein the principal conclusions, already known: And also (perchance) some things might lack due credit with you: And I, thereby to lose my labour: and you, to slip into light judgment. ☞ Before you have learned sufficiently the power of Nature and Art.

A marvellous glasse.

S. W. P.

Now

NOw to proceed: ASTRONOMIE, **is an Art Mathematical, which demonstrateth the distance, Magnitudes, and all natural Motions, apparences, and passions proper to the Planets and fixed Stars, for any time past, present, and to come: in respect of a certain Horizon, or without respect of any Horizon.** By this Art we are certified of the distance of the starry Skie, and of each Planet, from the Center of the Earth, and of the greatnesse of any fixed Star seen; or *Planet*, in respect of the earths greatnesse. As, we are sure (by this Art) that the Solidity, Massinesse, and Body of the *Sun*, conteineth the quantity of the whole Earth, and Sea, a hundred threescore and two times, lesse by $\frac{1}{8}$ one eight part of the earth, but the Body of the whole earthly Globe and Sea, is bigger than the body of the Moon, three and fourty times lesse by $\frac{1}{8}$ of the *Moon*. Wherefore the *Sun* is bigger than the *Moon* 7000 times, lesse by $59\frac{1}{44}$ that is, precisely $6940\frac{21}{44}$ bigger than the *Moon*. And yet the unskilful man, would judg them a like big. Wherefore, of necessity, the one is much farther from us than the other. The *Sun* when he is farthest from the earth (which, now, in our age is, when he is in the 8 degree, of *Cancer*) is 1179 Semidiameters of the Earth, distant. And the *Moon* when she is farthest from the earth, is 68 Semidiameters of the earth and $\frac{1}{2}$. The neerest, that the *Moon* commeth to the Earth, is Semidiameters $52\frac{1}{2}$. The distance of the starry skie is from us, in Semidiameters of the Earth $20081\frac{1}{2}$. Twenty thousand fourscore, one, and almost a half. Subtract from this, the *Moons* neerest distance, from the Earth: and thereof remaineth Semidiameters of the earth $20029\frac{1}{2}$. Twenty thousand nine and twenty and a quarter. So thick is the heavenly Palace, that the *Planets* have all their exercise in, and most mervailously perform the Commandement and Charge to them given by the Omnipotent Majesty of the King of Kings. This is that, which in *Genesis* is called *Ha Rakia*. Consider it well. The Semidiameter of the Earth, conteineth of our common miles $3436\frac{1}{4}$ three thousand, four hundred thirty six and four eleventh parts of one mile. Such as the whole Earth and Sea, round about, is 21600, one and twenty thousand six hundred of our miles. Allowing for every degree of the greatest circle, threescore miles. Now if you weigh well with your self but this little parcel of fruit *Astronomical*, as concerning the bignesse, distances of *Sun*, *Moon*, *Starry skie*, and the huge massinesse of *Ha Rakia* will you not find your consciences moved, with the Kingly Prophet, to sing the confession of Gods Glory, and say, **The Heavens declare the glory of God, and the Firmament [Ha Rakia] sheweth forth the works of his hands.** And so forth, for those five first staves of that kingly Psalm. Well, well, it is time for some, to lay hold on wisedom, and to judg truly of things: and not so to expound the Holy Word, all by Allegories: as to neglect the Wisdom, Power, and Goodnesse of God, in, and by his Creatures, and Creation to be seen and learned. By Parables and Analogies of whose natures and properties, the course of the Holy Scripture, also, declareth to us very many Mysteries. The whole Frame of Gods Creatures, (which is the whole world) is to us, a bright glasse: from which, by reflexion, rebounderh to our knowledg and perceiverance, Beams and Radiations: representing the Image of his infinite Goodnesse, Omnipotency, and Wisedom. And we thereby are taught and perswaded to glorifie our Creator, as God: and be thankful therefore. Could the Heathenists find these uses, of these most pure, beautiful and mighty Corporal Creatures: and shall we, after that the true *Sonne* of right wisenesse is risen above the *Horizon* of our temporal *Hemisphere*, and hath so abundantly streamed into our hearts the direct beams of his goodnesse, mercy, and grace: Whose heat all Creatures feel: Spiritual and Corporal: Visible and Invisible: Shall we (I say) look upon the *Heaven*, *Stars*, and *Planets*, as an Oxe, and an Asse doth: no further, careful or inquisitive, what they are: why were they created: how doe they execute that they were created for? Seeing, all Creatures were for our sake created: and both we, and they, created chiefly to glorifie the Almighty Creator & that by all means to us possible. *Nolite ignorare* (saith *Plato in Epinomis*,) *Astronomiam sapientissimum quiddam esse*. **Be ye not ignorant Astronomie to be a thing of excellent Wisedome.** *Astronomy* was to us from the beginning commended, and in manner commanded by God himself: In asmuch as he made the *Sun*, *Moon* and *Stars* to be to us for *Signes*, and knowledg of Seasons, and for distinctions of days and years. Many words need not. But I wish, every man should weigh this word *Signes*. And besides that, conferre it also with the tenth Chapter of *Jeremie*. And though

Note.

some think, that there they have found a rod: Yet modeſt reaſon, will be indifferent judg, who ought to be beaten therewith, in reſpect of our purpoſe. Leaving that: I pray you underſtand this: that without great diligence of Obſervation, Examination and Calculation, their periods and courſes (whereby Diſtinction of Seaſons, Years, and new Moons might preciſely be known) could not exactly be certified. Which thing to perform, is that Art, which we here have defined to be *Aſtronomie*. Whereby we may have the diſtinct courſe of Times, Days, Years & Ages: aſwel for conſideration of Sacred Prophecies, accompliſhed in due time, foretold: as for high Myſtical Solemnities holding: And for all other humane affairs, Conditions, and Covenants upon certain time, between man and man; with many other great uſes: Wherein, (verily,) would be great incertainty, confuſion, untruth, and brutiſh Barbarouſneſſe: without the wonderful diligence and skill of this Art: continually learning and determining Times, and periods of Time, by the Record of the heavenly book, wherein all times are written, and to be read with an *Aſtronomical Staffe*, inſtead of a feſcue.

MUSICK, of Motion, hath his Original cauſe. Therefore, after the Motions moſt ſwift, and moſt ſlow, which are in the Firmament, of Nature performed: and under the *Aſtronomers Conſideration:* now I will ſpeak of another kind of *Motion*, producing ſound, audible, and of man numerable. *Muſick* I call here that *Science*, which of the Greeks is called *Harmonice*. Not medling with the controverſie between the ancient *Harmoniſts* and *Canoniſts*. **Muſick is a Mathematical Science, which teacheth by ſenſe and reaſon, perfectly to judg, and order the diverſities of ſounds high and low.** *Aſtronomy* and *Muſick* are ſiſters, ſaith *Plato*. As for *Aſtronomy*, the eyes: ſo for *Harmonious Motion*, the ears were made. But as *Aſtronomy* hath a more divine contemplation and commodity, than mortal eye can perceive: ſo is *Muſick* to be conſidered, that the * Mind may be preferred before the ear. And from audicl ſound, we ought to aſcend, to the examination: which numbers are *Harmonious*, and which not. And why, either, the one are: or the other are not. I could at large, in the heavenly * motions and diſtances, deſcribe a marvellous Harmony of *Pythagoras* Harp, with eight ſtrings. Alſo ſomewhat might be ſaid of * *Mercurius* two Harps, each of four ſtrings Elemental. And very ſtrange matter, might be alledged of the *Harmony*, to our * ſpiritual part appropriate. As in *Ptolomeus* third book, in the fourth and ſixth Chapters may appear. * And what is the cauſe of the apt bond, and friendly fellowſhip, of the Intellectual and Mental part of us, with our groſſe and corruptible body, but a certain Mean, and *Harmonious Spirituality, with both participating, and of both (in a manner) reſulting? In the * Tune of Mans voice, and alſo * the ſound of Inſtrument*, what might be ſaid of *Harmonie:* No common Muſician would lightly believe. But of the ſundry mixture (as I may term it) and concourſe, diverſe collation and application of theſe *Harmonies:* as of three, four, five or more: marvellous have the effects been: and yet may be found and produced the like: with ſome proportional conſideration for our time and being: in reſpect of the ſtate, of the things then: in which, and by which, the wondrous effects were wrought. *Democritus* and *Theophraſtus* affirmed that by *Muſick* griefs and diſeaſes of the mind and body might be cured or inferred. And we find in Record, that *Terpander*, *Arion*, *Iſmenias*, *Orpheus*, *Amphion*, *David*, *Pythagoras*, *Empedocles*, *Aſclepiades*, and *Timotheus* by *Harmonical* Conſonancy, have done and brought to paſſe, things more than marvellous to hear of. Of them then, making no farther diſcourſe in this place. Sure I am, that common *Muſick*, commonly uſed, is found to the *Muſicians* and Hearers, to be ſo commodious and pleaſant, That if I would ſay and diſpute but thus much: That it were to be otherwiſe uſed, then it is, I ſhould find more reprievers, then I could find privy or skilful of my meaning. In things therefore evident, and better known, then I can expreſs: & ſo allowed & liked of, (as I would wiſh ſome other things had the like hap) I will ſpare to enlarge my lines any farther, but conſequently follow my purpoſe.

1.

2.
3. 4.

5.
6.

7. 8.
9. D.
Read in *Ariſtot.* his 8 book of Politicks the 5, 6, & 7 Chap. Where you ſhall have ſome occaſion farther to think of Muſick, then commonly is thought.

Of COSMOGRAPHIE, I appointed briefly in this place, to give you ſome intelligence. **Coſmographie is the whole and perfect deſcription of the Heavenly, and alſo Elemental part of the World, and their Pomological application, and mutual collation neceſſary.** This Art requireth *Aſtronomy, Geographie, Hydrographie*, and *Muſick*. Therefore, it is no ſmall Art, nor ſo ſimple, as in common practice, it is (ſlightly) conſidered. This matcheth Heaven, and

and the Earth in one frame, and aptly applyeth parts correſpondent: So, as, the Heavenly Globe, may (in practiſe) be duly deſcribed upon the Geographical and Hydrographical Globe. And there, for us to conſider an *Æquinoctial Circle*, an *Ecliptique line*, *Colours*, *Poles*, *Stars*, in their true Longitudes, Latitudes, Declinations, and Verticality: alſo Climes and Parallels: and by an *Horizon* annexed, and revolution of the Earthly Globe (as the Heaven, is by the *Primovant*, carried about in 24 equal hours) to learn the Riſings and Settings of Stars (of *Virgil* in his *Georgicks*, of *Heſiod*: of *Hippocrates* in his *Medicinal Sphere*, *to Perdicca* King of the *Macedonians*: of *Diocles*, to King *Antigonus*, and of other famous *Philoſophers* preſcribed) a thing neceſſary, for due manuring of the earth, for *Navigation*, for the Alteration of mans body: being whole, ſick, wounded, or bruiſed, By the Revolution, alſo, or moving of the Globe Coſmographical, the Riſing and Setting of the Sun: the lengths of days and nights: the Hours and times (both night and day) are known: with very many other pleaſant and neceſſary uſes: Whereof, ſome are known: but better remain, for ſuch to know and uſe: who of a ſpark of true fire, can make a wonderful bonfire, by applying of due matter, duly.

Of ASTROLOGIE, here I make an Art, ſeveral from *Aſtronomy*: not by new deviſe, but by good reaſon and authoritie: for, **Aſtrologie is an Art Mathematical, which reaſonably demonſtrateth the operations and effects, of the natural beams, of light, and ſecret influence: of the Stars and Planets: in every Element and elemental body: at all times in any Horizon aſſigned.** This Art is furniſhed with many other great Arts and experiences: As with perfect *Perſpective*, *Aſtronomy*, *Coſmographie*, *Natural Philoſophie*, of the four Elements, the Art of Graduation, and ſome good underſtanding in *Muſick:* and yet moreover, with another great Art, hereafter following, though I here, ſet this before, for ſome conſiderations me moving. Sufficient (you ſee) is the ſtuffe, to make this rare and ſecret Art, of: & hard enough to frame to the concluſion Syllogiſtical: yet both the manifold and continual travails of the moſt ancient and wiſe Philoſophers, for the atteinirig of this art: and by examples of effects, to confirm the ſame; hath left unto us ſufficient proof and witneſs, and we alſo daily may perceive, That mans body, and all other Elemental bodies are altered, diſpoſed, ordered, pleaſured and diſpleaſured, by the influential working of the *Sun*, *Moon*, and the other *Stars* and *Planets*. And therefore, ſaith *Ariſtotle*, in the firſt of his *Meteorological* books, in the ſecond Chapter; *Eſt autem neceſſario Mundus iſte ſupernis lationibus ferè continuus. Ut, inde, vis ejus Univerſa regatur. Ea ſiquidem cauſa prima putanda omnibus eſt, unde motus principium exiſtit.* That is: **This** [Elemental] **World is of neceſſity, almoſt, next adjoyning, to the Heavenly motions: That from thence all his vertue or force may be governed. For that is to be thought the firſt Cauſe unto all: from which, the beginning of motion, is.** And again, in the tenth Chapter. *Oportet igitur, & horum principia ſumamus, & cauſas omnium ſimiliter. Principium igitur ut moventis præcipuumque & omnium primum, Circulus ille eſt, in quo manifeſte Solis latio, &c.* And ſo forth. His *Meteorological* books, are full of arguments, and effectual demonſtrations; of the vertue, operation, and power of the heavenly bodies, in and upon the four Elements; and other bodies of them (either perfectly or unperfectly) compoſed. And in his ſecond book, *De generatione & corruptione:* in the tenth Chapter. *Quo circa & prima latio, Ortus & Interitus cauſa non eſt: Sed obliqui Circuli latio: ea namque & continua eſt, & duobus motibus fit.* In Engliſh thus. **Wherefore the uppermoſt motion is not the cauſe of Generation and Corruption, but the motion of the Zodiack, for that both is continual, and is cauſed of two movings.** And in his ſecond book, and ſecond Chapter of his *Phyſicks*. *Homo namque generat hominem, atque Sol.* **For man** (ſaith he) **and the Sun, are cauſe of mans generation.** Authorities may be brought very many; both of 1000, 2000, yea and 3600 years Antiquity; of great *Philoſophers*, *Expert*, *Wiſe*, and godly men, for that Concluſion: which, daily and hourly, we men may diſcern and perceive by ſenſe and reaſon. All beaſts doe feel, and ſimply ſhew by their actions and paſſions, outward and inward. All Plants, Herbs, Trees, Flowers and Fruits, All finally, the Elements, and all things of the Elements compoſed, do give

Testimony (as *Aristotle* saith) that their **Whole dispositions, vertues, and natural motions, depend of the Activity of the heavenly Motions and Influences. Whereby, besides the special order and form, due to every seed: and beside the Nature, proper to the individual Matrix, of the thing produced: What shall be the heavenly impression, the perfect and circumspect Astrologian hath to conclude:** Not onely (by *Aposteleſmes*) τὸ ὅτι, but by Natural and Mathematical demonstration τὸ διότι. Whereunto, what Sciences are requisite (without exception) I partly have here warned: and in my *Propædeumes* (besides other matter there disclosed) I have Mathematically furnished up the whole Method. To this our age, not so carefully handled by any, that ever I saw, or heard of, I was (for * 21 years ago) by certain earnest disputations, of the Learned *Gerardus Mercator*, and *Antonius Gogava*, (and other) thereto so provoked: and (by my constant and invincible zeal to verity) in observations of heavenly Influences (to the minute of time,) than so diligent: and chiefly by the Supernatural influence, from the Star of *Jacob*, so directed: That any modest and sober Student, carefully and diligently seeking for the Truth, will both find and confesse, therein, to be the Verity, of these my words: and also become a reasonable Reformer, of three sorts of people: about these influential operations, greatly erring from the truth. Whereof the one is **Light Believers**, the other **Light Despisers**, and the third **Light Practisers.** The first, and most common sort, think the Heavens and Stars, to be answerable to any their doubts or desires: which is not so: and, indeed, they, too much over-reach. The second sort think no influential vertue (from the heavenly Bodies) to bear any sway in Generation and Corruption, in this Elemental World. And to the *Sun*, *Moon* and *Stars* (being so many, so pure, so bright, so wonderful big, so far in distance, so manifold in their motions, so constant in their periods, &c.) they assign a slight, simple office or two, and so allow unto them (according to their capacities) as much vertue, and power influential, as to the Signe of the *Sun*, *Moon*, and seven *Stars*, hanged up (for Signs) in *London*, for distinction of houses, and such grosse helps, in our wordly affairs, and they understand not (or will not understand) of the other workings, and vertues of the Heavenly *Sun*, *Moon* and *Stars*: not so much, as the Mariner or Husbandman: no, not so much, as the *Elephant* doth, as the *Cynocephalus*, as the *Porpentine* doth: nor will allow these perfect and incorruptible mighty bodies, so much virtual Radiation and Force, as they see in a little piece of a *Magnes stone*: which, at great distance, sheweth his operation. And perchance they think, the Sea and Rivers (as the Thames) to be some quick thing, and so to ebbe and flow, run in and out, of themselves, at their own fantasies. God help, God help. Surely these men come too short: and either are to dull: or wilfully blind: or, perhaps too malicious. The third man is the common and vulgar *Astrologian*, or Practiser, who being not duly, artificially and perfectly furnished: yet, either for vain glory, or gain: or like a simple Dolt, and blind Bayard, both in matter and manner erreth: to the discredit of the *Wary* and modest *Astrologian*: and to the robbing of those most noble corporal Creatures, of their Natural Vertue: being most mighty, most beneficial to all elemental Generation, Corruption and the appurtenances: and most Harmonious in their Monarchie: For which things being known, and modestly used: we might highly and continually glorifie God, with the princely Prophet, saying. **The Heavens declare the Glory of God: who made the Heavens in his wisedome: who made the Sun for to have dominion of the day: the Moon and Stars to have dominion of the night: whereby day to day uttereth talk, and night to night declareth knowledge. Praise him, all ye Stars and Light. Amen.**

* *An.* 1548 *and* 1549 *in Lovain.*

Note. 1. 2. 3.

In order, now followeth, of STATIKE, somewhat to say what we mean by that name: and what commodity, doth, on such Art, depend. **Statike is an Art Mathematical, which demonstrated the causes of heavinesse of all things: and of motions and properties, to heavinesse and lightnesse belonging.** And forasmuch as, by the Bilanx, or Balance (as the chief sinsible Instrument) Experience of these demonstrations may be had: we call this Art *Statike*: that is, *the Experiments of the Balance.* Oh, that men wist, what profit (all manner of ways) by this Art might grow, to the able examiner, and diligent practiser. "Thou onely, knowest all "things precisely (ô God) who hast made Weight and Balance, thy Judgment; who "hast created all things in *Number*, *Weight* and *Measure*: and hast weighed the "Mountains and Hils in a Balance: who hast peysed in thy hand, both Heaven and "Earth. We therefore warned by the Sacred Word, to consider thy Creatures: and by "that consideration, to win a glimps (as it were,) or shadow of perceiverance, that thy "wisedome, might, and goodnesse is infinite, and unspeakable, in thy Creatures declared: "and being farther advertised, by thy mercisul goodnesse, that, three principal ways, "were of thee, used in Creation of all thy Creatures, namely, *Number*, *Weight*, and "*Measure*. And for as much as, of *Number* and *Measure*, the two Arts (ancient, famous, and to humane uses most necessary) are, already, sufficiently known and extant: This third key, we beseech thee (through thy accustomed goodnesse) that it may "come to the needful and sufficient knowledge, of such thy Servants, as in the work-"manship, would gladly find, thy true occasions (purposely of thee used) whereby we "glorifie thy name, and shew forth (to the weaklings in faith) thy wondrous Wisedome "and Goodnesse. *Amen.*

Marvel nothing at this pang (godly friend, you gentle and zealous Student.) Another day, perchance you will perceive, what occasion moved me. Here, as now, I will give you some ground, and withall some shew, of certain commodities, by this Art arising. And because this Art is rare, my words and practises might be too dark: unlesse you had some light holden before the matter: and that best will be, in giving you, out of *Archimedes* demonstrations, a few principal conclusions, as followeth.

I. The superficies of every Liquor, by it self consisting, and in quiet, is Spherical: the center whereof, is the same, which is the center of the Earth.

II. If Solid Magnitudes, being of the same bigness, or quantity, that any Liquor is, and having also the same Weight: be let down into the same Liquor, they will settle downward, so, that no part of them, shall be above the superficies of the Liquor: and yet, nevertheless, they will not sink utterly down, or drown.

III. If any Solid Magnitude being Lighter than a Liquor, be let down into the same Liquor, it will settle down, so far into the same Liquor, that so great a quantity of that Liquor, as is the part of the Solid Magnitude, setled down into the same Liquor: is in Weight, equal, to the Weight of the whole Solid Magnitude.

IV. Any Solid Magnitude, Lighter then a Liquor, forced down into the same Liquor, will move upward, with so great a power, by how much, the Liquor having equal quantity to the whole Magnitude, is heavier than the same Magnitude.

V. Any Solid Magnitude, heavier than a Liquor, being let down into the same Liquor, will sinke down utterly: and will be in that Liquor, Lighter by so much, as is the weight or heavinesse of the Liquor, having bignesse or quantity equal to the Solid Magnitude.

VI. If any Solid Magnitude, Lighter than a Liquor, be let down into the same Liquor, the weight of the same Magnitude, will be, to the Weight of the Liquor. (Which is equal in quantity to the whole Magnitude,) in that proportion, that the part, of the Magnitude setled down is to the whole Magnitude.

J. D. The cutting of a Sphere according to any proportion assigned, may by this propositiō be done Mechanically by tēpering liquor to a certain weight in respect of the weight of the Sphere therein swimming A Paradox. A common error noted. *N. T.* The wonderful use of these Propositions.

BY these verities, great errours may be reformed in opinion of the Natural Motion of things Light, and Heavy. Which errours are in Natural Philosophy (almost) of all men allowed: to much trusting to authority, and false Suppositions. As, **Of any two Bodies, the Heavier to move downward faster than the Lighter.** This errour is not first by me, Noted: but by one *John Baptist de Benedictis*. The chief of his Propositions, is this: which seemeth a Paradox.

If there be two Bodies of one form, and of one kind, equal in quantity or unequal, they will move by equal space, in equal time: So that both their movings be in air, or both in water: or in any one middle.

Hereupon, in the seat of GUNNING, certain good discourses (otherwise) may receive great amendment and furtherance, In the intended purpose, also, allowing somewhat to the imperfection of Nature: not answerable to the precisenesse of demonstration. Moreover, by the foresaid propositions (wisely used.) The Air, the Water, the Earth, the Fire, may be neerly known, how light, or heavy they are (Naturally) in their assigned parts: or in the whole. And then to things Elemental, turning your practise: you may deal

deal for the proportion of the Elements, in the things Compounded. Then to the proportions of the Humours in man: their weights, and weight of his Bones, and Flesh, &c. Then, by weight, to have consideration of the force of man, any manner of way: in whole, or in part. Then may you of Ships water drawing, diversly in the Sea and in fresh water, have pleasant consideration: and of weighing up of any thing, sunken in Sea, or in fresh water, &c. And (to lift up your head aloft:) by weight, you may as precisely, as by any Instrument else, measure the Diameters of *Sun* and *Moon*, &c. Friend, I pray you weigh these things with the just Balance of Reason. And you will find marvels upon marvels: and esteem one Drop of Truth (yea, in natural Philosophy) more worth than whole Libraries of Opinions undemonstrated: or not answering to Natures Law, and your experience. Leaving these things, thus: I will give you two or three light practises to great purpose: and so finish my Annotation *Statical*. In Mathematical matters, by the Mechanicians aid, we will behold here the Commodity of weight. Make a Cube of any one Uniform: and through like heavy stuffe: of the same stuffe make a Sphere or Globe precisely, of a Diameter equal to the Radical side of the Cube. Your stuffe, may be Wood, Copper, Tinne, Lead, Silver, &c. (being as I said of like nature, condition, and like weight throughout.) And you may by Say Ballance have prepared a great number of the smalest weights: which by those Balance can be discerned or tried: and so have proceeded to make you a perfect Pyle, company and number of weights: to the weight of six, eight, or twelve pound weight, most diligently tried, all and of every one, the Content known, in your least weight that is weighable. [They that cannot have these weights of of precisencesse: may by Sand, uniform, and well dusted, make them a number of weights, somewhat neer preciseneesse: by halfing ever the Sand: they shall, at length, come to a least common weight. Therein, I leave the farther matter, to their discretion, whom need shall pinch.] The *Venetians* consideration of weight may seem precisely enough: by eight descents progressional * halfing, from a grain: your Cube, Sphere, apt Balance, and convenient weights being ready: fall to work. * First, weigh your Cube. Note the number of the weight. Weigh, after that, your Sphere. Note likewise, the Number of the weight. If you now find the weight of your Cube, to be to the weight of the Sphere, as 21 is to 11: Then you see, how the Machanician and *Experimenter*, without Geometry and Demonstration, are (as neerly in effect) taught the proportion of the Cube to the Sphere: as I have demonstrated in the end of the twelfth book of *Euclid*. Often try with the same Cube and Sphere. Then, change your Sphere and Cube to another matter: or to another bigness: till you have made a perfect universal Experience of it. Possible it is, that you shall winne to neerer termes, in the proportion.

When you have found this one certain Drop of Natural verity, proceed on, to inferre, and duly to make assay, of matter depending. As because it is well demonstrated, that a Cylinder, whose height, and Diameter of his base, is equal to the Diameter of the Sphere, is *Sesquialter* to the same Sphere, (that is, as 3. to 2.) To the number of the weight of the Sphere, adde half so much, as it is: and so have you the number of the weight of that Cylinder. Which is also comprehended of our former Cube: so, that the base of that Cylinder, is a Circle described in the square, which is the base of our Cube. But the Cube and Cylinder, being both of one height, have their Bases in the same proportion, in the which, they are one to another in their massinesse or solidity. But before, we have two numbers, expressing their massinesse, solidities, and quantities, by weight: wherefore, we have * the proportion of the square, to the Circle, inscribed in the same square. And so are we fallen into the knowledge sensible and experimental of *Archimedes* great secret, of him, by great travail of mind, sought and found. Wherefore to any Circle given, you can give a square equal: * as I have taught in my Annotation, upon the first proposition of the twelfth book: and likewise to any square given, you may give a Circle equal: * If you describe a Circle, which shall be in that proportion to your Circle inscribed, as the square is to the same Circle: This, you may do, by my Annotations, upon the second proposition of the twelfth book of *Euclide*, in my third Problem there. Your diligence may come to a proportion of the square to the Circle inscribed neerer the truth, than is the proportion of 14 to 11: and consider, that you may begin at the Circle and Square, and so come to conclude of the Sphere and the Cube, what their proportion is: as now, you came from the Sphere to the Circle. For of Silver or Gold, or Latton Lamyns, or plates, (thorough one how drawn, as the manner is,) if you make a square figure, and weigh it: and then, describing thereon, the

The practise Statical, to know the proportion between the Cube and the Square.

J. D.

* For, so, have you 256 parts of a grain.

* The proportion of the Square, to the Circle inscribed.

* The Squaring of the Circle Mechanically.

* To any Square given, to give a Circle equal.

the Circle inscribed: and cut off, and file away, precisely (to the Circle) the overplus of the Square: you shall then, weighing your Circle, see, whether the weight of the Square, be to your Circle, as 14 to 11, as I have noted, in the beginning of *Euclids* twelfth book, &c. after this resort to my last propositions, upon the last of the twelfth: and there, help your self, to the end, and, here, Note this, by the way. That we may square the Circle, without having knowledge of the proportion of the Circumference to the Diameter: as you have here perceived. And otherwise also, I can demonstrate it. So that many have cumbered themselves superfluously, by travailing in that point first, which was not of necessity, first: and also very intricate. And easily you may, (and that diversly) come to the knowledge of the circumference: the Circles quantity, being first known. Which thing I leave to your consideration: making haste to dispatch another Magistial Problem: and to bring it neerer to your knowledge, and readier dealing with, than the world (before this day,) had it for you, that I can tell off. And that is, *A Mechanical doubling of the Cube, &c.*

Which may thus be done: **Make of Copper-plates, or Tin-plates a four square upright Pyramis or a Cone: perfectly fashioned in the hollow, within. Wherein, let great diligence be used, to approach as neer as may be) to the Mathematical perfection of those figures. At their bases: let them be all open: every where, else, most close, and just to. From the verter, to the Circumference of the base of the Cone: and to the sides of the base of the Pyramis: Let four straight lines be drawn, in the inside of the Cone and Pyramis: making at their fall, on the perimeters of the bases, equal angles, on both sides themselves, with the said perimeters. Then four lines (in the Pyramis, and as many in the Cone) divide one, in 12 equal parts; and another in 24, another in 60, and another in 100, (reckoning up from the verter.) Or use other numbers of division, as experience shall teach you. Then * set your Cone or Pyramis with the verter downward, perpendicularly, in respect of the base, (though it be otherwise, it hindereth nothing.) So let them most stedily be stayed.** Now if there be a Cube, which you would have doubled. Make you a prety Cube of Copper, Silver, Lead, Tinne, Wood, Stone, or Bone. Or else make a hollow Cube or Cubick coffen, of Copper, Silver, Tinne, or Wood, &c. These you may so proportion in respect of your Pyramis or Cone, that the Pyramis or Cone, will be able to contein the weight of them, in water three or four times at the least: what stuffe so ever they be made of. Let not your solid angle at the vertex, be too sharp: but that the water may come with ease, to the very vertex, of your hollow Cone or Pyramis. Put one of your solid Cubes in a Balance apt: take the weight thereof exactly in water. Poure that water, (without losse) into the hollow Pyramis or Cone quietly. Mark in your lines, what numbers the water cutteth: Take the weight of the same Cube again: in the same kind of water, which you had before: put that * also, into the Pyramis or Cone, where you did put the first. Mark now again, in what number or place of the lines, the water cutteth them. Two wayes you may conclude your purpose: it is to wit, either by numbers or lines. By numbers: as, if you divide the side of your Fundamental Cubes into so many equal parts, as it is capable of, conveniently, with your ease, and preciseneese of the division. For, as the number of your first and less line (in your hollow Pyramis or Cone) is to the second or greater (both being counted from the vertex) so shall the number of the side of your Fundamental Cube, be to the number belonging to the Radical side of the Cube, double to your Fundamental Cube: Which being multiplyed Cubick wise, will soon shew it self, whether it be double or no, to the Cubick number of your Fundamental Cube. By lines, thus: As your lesse and first line (in your hollow Pyramis or Cone) is to the second or greater, so let the Radical side of your Fundamental Cube, be to a fourth proportional line, by the 12 proposition of the sixth book of *Euclid*. Which fourth line, shall be the Root Cubick, or Radical side of the Cube, double to your Fundamental Cube: which is the thing we desired. For this, may I (with joy) say, EYPHKA, EYPHKA, EYPHKA: thanking the Holy and glorious Trinity, having greater cause thereto, then * *Archimedes* had (for finding the fraud used in the Kings Crown of Gold:) as all men may easily judge: by

Note. Squaring of the Circle, without knowing of the proportion between circumference and Diameter.

To double the Cube readily, by Art Mechanical, depending upon demonstration Mathematical.

J. D. The 4 sides of this Pyramis must be 4 Isosceles triangles alike & equal.

J. D. * In all workings with this Pyramis or Cone, let their Situations be in all points & conditions, alike, or all one: while you are about one work. Else you will erre.

J. D. * Consider well when you must put your waters together: & when you must empty your first water, out of your Pyramis, or Cone. Else you will erre.

* *Vitruvius* lib. 9. cap. 3.

God be thanked for this invention and the fruit ensuing. *Note.*

by the diversity of the fruit following of the one and of the other. Where I speak before of a hollow Cubick Coffin: the like use is of it, and without weight. Thus. Fill it with water precisely full, and pour that water into your Pyramis or Cone: and here note the lines cutting in your Pyramis or Cone. Again, fill your Coffin like as you did before. Put that water, also to the first. Mark the second cutting of your lines. Now, as you proceeded before, so must you here proceed. * And if the Cube, which you double, be never so great: you have, thus, the proportion (in small) between your two little Cubes: And then, the side of that great Cube (to be doubled) being the third, will have the fourth, found, to it proportional: by the 12 of the 6 of *Euclide.*

* Note, as concerning the Spherical superficies of the water.

Note, that all this while, I forget not my first Proposition Statical, he rehearsed: that, the Superficies of the water, is Spherical. Wherein use your discretion to the first line, adding a small hair breadth, more: and to the second half a hair breadth more, to his length. For you will easily perceive, that the difference can be no greater, in any Pyramis or Cone, of you to be handled. Which you shall thus try. *For finding the swelling of the water above level.* " Square the Semidiameter, from the Center of the Earth, to your " first waters superficies. Square then, half the subtendent of that watry superficies: " (which subtendent must have the equal parts of his measure, all one, with those of the " Semidiameter of the Earth, to your watry superficies:) subtract this square from the " first: Of the residue, take the Root square. That Root, subtract from your first Semi- " diameter of the Earth to your watry superficies: that which remaineth, is the height of " the water, in the middle, above the Level. Which you will find, to be a thing insensible: and though it were greatly sensible, * yet, by help of my sixth Theoreme upon the last Proposition of *Euclides* twelfth book, noted: you may reduce all to a true Level. But farther diligence of you is to be used, against accidental causes of the waters swelling: as by having (somewhat) with a moist sponge, before, made moist your hollow Pyramis or Cone, will prevent an accidental cause of swelling, &c. Experience will teach you abundantly: with great ease, pleasure, and commodity.

* *Note.*

Note this Abridgment of doubling the Cube, &c.

Thus, may you double the Cube Mechanically, Treble it, and so forth, in any proportion. Now will I abridg your pain, cost, and care herein. Without all preparing of your Fundamental Cubes: you may (alike) work this conclusion. For, that was rather a kind of Experimental Demonstration, than the shortest way; and all upon one Mathematical Demonstration depending. Take water (as much as conveniently will serve your turn, as I warned before of your Fundamental Cubes bigness.) Weigh it precisely. Put that water into your Pyramis or Cone. Of the same kind of water, then take again, the same weight you had before; put that likewise into the Pyramis or Cone; for in each time your marking of the lines how the water doth cut them, shall give you the proportion between the Radical sides, of any two Cubes, whereof the one is double to the other, working as before I have taught you: * saving that for your Fundamental Cube his Radical side: here, you may take a right line, at pleasure.

Note *

To give Cubes one to the other in any Proportion Rational or Irrational.

Yet farther proceeding with our Drop of Natural truth: **You may now give Cubes one to the other in any proportion given, Rational or Irrational.** On this manner. Make a hollow Parallelipipedon of Copper or Tinne: with one Base wanting, or open, as in our Cubick Coffin. From the bottom of that Parallelipipedon, raise up, many perpendiculars, in every of his four sides. Now if any proportion be assigned you, in right lines: " Cut one of your perpendiculars (or a line equal to it, or lesse " than it) likewise: by the 10 of the 6 of *Euclide.* And those two parts, set in two sundry " lines of those perpendiculars (or you may set them both, in one line) making their be- " ginnings to be, at the Base: and so their lengths to extend upward. Now, set your " hollow Parellelipipedon, upright, perpendicularly, steady. Pour in water handsomely, " to the height of your shorter line. Pour that water, into the hollow Pyramis or Cone. " Mark the place of the rising. Settle your hollow Parallelipipedon again. Pour water " into it: unto the height of the second line, exactly. Pour that water * duly into the " hollow Pyramis or Cone: mark now again, where the water cutteth the same line, " wh ich you marked before. For, there, as the first marked line, is to the second: So " shall the two Radical sides be, one to the other, of any two Cubes: which in their so- " lidity, shall have the same proportion which was at the first assigned: were it Rational " or Irrational.

* Emptying, the first.

Thus,

Thus, in sundry wayes you may furnish your self with such strange and profitable matter: which long hath been wished for. And though it be Naturally done, and Mechanically: yet hath it a good Demonstration Mathematical. Which is this: Alwayes, you have two like Pyramids: or two like Cones, in the proportions assigned: and like Pyramids or Cones, are in proportion, one to the other, in the proportion of their Homologal sides (or lines) tripled. Wherefore, if to the first, and second lines, found in your hollow Pyramis or Cone, you joyn a third and a fourth in continual proportion, that fourth line shall be to the first, as the greater Pyramis or Cone is to the lesse: by the 33 of the eleventh of *Euclid.* If Pyramis to Pyramis, or Cone to Cone, be double, then shall * Line to Line, be also double, &c. But as our first line, is to the second, so is the Radical side of our Fundamental Cube, to the Radical side of the Cube to be made, or to be doubled: and therefore, to those twain also, a third and a fourth line, in continual proportion, joyned: will give the fourth line in that proportion to the first, as our fourth Pyramidal, or Conick line was to his first: but that was double or treble, &c. as the Pyramids or Cones were, one to another (as we have proved) therefore, this fourth, shall be also double or treble to the first, as the Pyramids or Cones were one to another: But our made Cube, is described of the second in proportion of the four proportional lines: therefore * as the fourth line is to the first, so is that Cube to the first Cube: and we have proved the fourth line to be the first, as the Pyramis or Cone is to the Pyramis or Cone: Wherefore the Cube is to the Cube, as Pyramis to Pyramis, or Cone is to Cone. But we * suppose Pyramis to Pyramis, or Cone to Cone, to be double or treble, &c. Therefore Cube is to Cube, double or treble, &c. Which was to be demonstrated. And of the Parallelipipedon, it is evident, that the water solid Parallelipipedons, are one to another, as their heights are, seeing they have one base. Wherefore the Pyramids or Cones made of those water Parallelipipedons, are one to the other, as the lines are (one to the other) between which, our proportion was assigned. But the Cubes made of lines, after the proportion of the Pyramidal or Conick *Homologal* lines are one to the other, as the Pyramids or Cones are, one to the other (as we before did prove) therefore, the Cubes made, shall be one to the other, as the lines assigned, are one to the other: which was to be demonstrated. Note. * This my demonstration is more general, than onely in Square Pyramis or Cone: Consider well. Thus, have I, both Mathematically, and Mechanically, been very long in words, yet (I trust) nothing tedious to them, who, to these things, are well affected. And verily I am forced (avoiding prolixity) to omit sundry such things, easie to be practised: which to the Mathematician, would be a great Treasure: and to the Mechanician no small gain. * Now may you,

The Demonstrations of this doubling of the Cube; and of the rest.

J. D.

* Hereby help your self to become a precise practiser. And so consider how nothing at all, you are hindred (sensible) by the convexity of the water.

* By the 33 of the eleventh Book of *Euclid.*

J. D.

* And your diligence in practice can so (in weight of water) perform it: Therefore, now, you are able to give good reason of your whole doing.

* Note this Corollary.

* The great Commodities following of these new Inventions.

Between two lines given, find two middle proportionals in continual proportion: by the hollow Parallelipipedon, and the hollow Pyramis, or Cone. Now any Parallelipipedon rectangle being given: three right lines may be found proportional in any proportion assigned, of which shall be produced a Parallelipipedon, equal to the parallelipipedon given. Hereof, I noted somewhat upon the 36 proposition, of the 11 book of *Euclide.* Now, all those things, which *Vitruvius* in his Architecture, specified, able to be done, by doubling of the Cube. Or, by finding of two middle proportional lines, between two lines given, may easily be performed. Now, that Probleme, which I noted unto you, in the end of my Addition, upon the 34 of the 11 book of *Euclide*, is proved possible. Now may any regular body be transformed into another, &c. Now, any regular body, any Sphere, yea any mixt Solid: and (that more is) irregular Solids, may be made (in any proportion assigned) like unto the body first given. Thus, of a *Manneken*, (as the *Dutch* Painters term it) in the same *Symmetry*, may a Giant be made: and that, with any gesture, by the *Manneken* used: and contrariwise. Now, may you, of any Mould or Model of a Ship, make one, of the same Mould (in any assigned proportion) bigger or lesser. Now, may you, of any * Gun, or little piece of Ordnance, make another, with the same *Symmetrie* (in all points) as great, and as little, as you will. Mark that, and think on it. Infinitely, **may you apply this, so long sought for, and now so easily concluded: and withall, so willingly and frankly communicated to such, as faithfully deal with vertuous studies.** Thus can the Mathematical mind, deal Speculatively in his own Art: and by good means, mount above the clouds and stars: And thirdly, he can by order, descend, to frame Natural things, to wonderful uses: and when he list, retire

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Such is the fruit of the Mathematical Sciences and Arts.

home into his own Center: and there, prepare more means, to Ascend or Descend by: and all to the glory of God, and our honest delectation in Earth.

Although, the Printer, hath looked for this Preface, a day or two, yet could I not bring my pen from the paper, before I had given you comfortable warning and brief instructions, of some of the commodities by *Staticke*, able to be reaped: In the rest, I will therefore, be as brief, as it is possible: and withall describing them somewhat accordingly. And that, you shall perceive, by this, which in order commeth next. For whereas it is so ample and wonderful, that an whole year long, one might find fruitful matter therein, to speak of and also in practice is a Treasure endlesse: yet will I glanse over it, with words very few.

This do I call ANTHROPOGRAPHIE. Which is an Art restored, and of my preferment to your service. I pray you think of it, as of one of the chief points of Humane knowledg. Although it be but now first confirmed, with this new name: yet the matter, hath from the beginning, been in consideration of all perfect Philosophers. **Anthropographie is the description of the Number, Measure, Weight, Figure, Situation and Colour of every diverse thing, conteined in the perfect body of MAN with certain knowledge of the Symmetrie, Figure, Weight, Characterization, and due local motion of any parcel of the said Body, assigned: and of numbers, to the said parcel appertaining.** This is one part of the Definition, meet for this place: Sufficient to notifie, the particularity, and excellency of the Art: and why it is, here, ascribed to the Mathematicals. If the description of the heavenly part of the World had a peculiar Art, called *Astronomy*. If the description of the earthly Globe, hath his peculiar Art, called *Geographie*. If the matching of both hath his peculiar Art, called *Cosmographie*. Which is the description of the whole and universal frame of the World: Why should not the description of him, who is the lesse World and from the beginning called *Microcosmus*, (that is, **The lesse World**. And for whose sake and service, all bodily creatures else, were created: Who also participateth with Spirits and Angels, and is made to the Image and similitude of *God*, have his peculiar Art? and be called the *Art of Arts*: rather than either to want a name, or to have too base and improper a name? You must of sundry professions, borrow or challenge home, peculiar parts hereof: and farther proceed: as God, Nature, Reason and Experience shall inform you. The *Anatomists* will restore to you, some part: *Physiognomists* some: The *Chyromantists* some. The *Metaposcopists* some. The Excellent *Albert Durer*, a good part: the Art of Perspective, will somewhat, for the eye help forward: *Pythagoras*, *Hypocrates*, *Plato*, *Galenus*, *Meletius*, and many other (in certain things) will be Contributaries. And farther, the Heaven, the Earth, and all other Creatures, will each shew, and offer their Harmonious service, to fill up, that, which wanteth hereof: and with your own Experience, concluding: you may Methodically register the Whole, for the posterity: Whereby, good proof will be had, of our Harmonious, and Microcosmical constitution. The outward Image and view hereof, to the Art of *Zographie*, and painting, to Sculpture, and Architecture, (for Church, House, Fort, or Ship) is most necessary and profitable: for that, it is the chief base and foundation of them, Look in * *Vitruvius*, whether I deal sincerely, for your behoof, or no. Look in *Albertus Durerus de Symmetria humani Corporis*. Look in the 27 and 28 chapters, of the 2 Book, *De occulta Philosophia*. Consider the *Ark* of *Noe*. And by that, wade farther. Remember the *Delphical Oracle*, *NOSCE TE IPSUM* (know thy self) so long ago pronounced: of so many a Philosopher repeated: and of the *Wisest* attempted: And then you will perceive how long ago, you have been called to the School, where this Art might be learned. Well. I am nothing affraid, of the disdain of some such, as think Sciences and Arts, to be but seven, perhaps, those such, may, with ignorance, and shame enough, come short of them seven also: and yet nevertheless, they cannot prescribe a certain number of Arts: and in each certain unpassible bounds to God, Nature, and mans Industry. New Arts daily rise up: and there was no such order taken, that, all Arts, should in one age, or in one land, or of one man, be made known to the world. Let us embrace the gifts of God, and ways to wisedome, in this time of grace, from above, continually bestowed on them, who thankfully will receive them: *Et bonis omnia Co-operabuntur in bonum.*

MAN is the lesse World.

Microcosmus.

* *Lib.3.cap.1.*

TRO-

TROCHILIKE, is **that Art Mathematical, which demonstrateth the properties of all Circular Motions, Simple and Compound.** And because the fruit hereof vulgarly received, is the Wheels, it hath the name of *Trochilike*: as a man would say *Wheel Art*. By this Art, a Wheel may be given, which shall move once about, in any time assigned. Two Wheels may be given, whose turnings about in one and the same time (or equal times) shall have one to the other, any proportion appointed. By Wheels, may a streight line be described: Likewise, a Spiral line in plain, Conical Section lines, and other Irregular lines at pleasure may be drawn. These and such like are principal Conclusions of this Art: and help forward many pleasant and profitable Mechanical works: As Mils, to saw great and very long Deal-boards, no man being by. Such have I seen in *Germany*, and in the City of *Prague*, in the kingdome of *Bohemia*: Coyning Mils, Hand Mils for Corn grinding: And all manner of Mils and Wheel work: By Wind, Smoak, Water, Weight, Spring, Man or Beast moved. Take in your hand *Agricola de re Metallica*: and then shall you (in all Mines) perceive how great need is, of Wheel work. By Wheels, strange works and incredible are done: as will, in other Arts hereafter appear. A wonderful example of farther possibility, and present Commodity was seen in my time, in a certain Instrument: which by the Inventer and Artificer (before) was sold for twenty Talents of Gold: and then had (by misfortune) received some injury and hurt. And one *Janellus* of *Cremona* did mend the same, and presented it unto the Emperour *Charls* the Fifth. *Hieronymus Cardanus*, can be my witnesse, that therein, was one Wheel, which moved, and that in such rate, that, in 7000 years onely, his own period should be finished. A thing almost incredible: But how farre, I keep me within my bounds: very many men (yet alive) can tell.

Saw Mils.

HELICOSOPHIE, is neer Sister to *Trochilike*, and is **An Art Mathematical which demonstrateth the designing of all Spiral lines in Plain, on Cylinder, Cone, Sphere, Conoid, and Sphearoid, and their properties appertaining.** The use hereof in *Architecture*, and diverse Instruments and Engines, is most necessary. For, in many things, the Scrue worketh the feat, which else, could not be performed. By help hereof, it is * recorded, that, where all the power of the City of *Syracusa*, was not able to move a certain Ship (being on ground) mighty *Archimedes*, setting to his Scruish Engine, caused *Hiero* the King, by himself, at ease, to remove her, as he would. Whereat, the King wondring: Ἀπὸ τούτης τῆς ἡμέρας, περὶ παντὸς, Ἀρχιμήδει λόγου πιστεύειν. *From this day, forward*, (said the King) *Credit ought to be given to* Archimedes, *whatsoever he saith.*

* *Atheneus lib. 5. cap. 8.*

PNEUMATITHMIE, **demonstrateth by close hollow Geometrical Figures (regular and irregular) the strange properties (in motion or stay) of the Water, Air, Smoak, and Fire in their continuity, and as they are joyned to the Elements next them.** This Art, to the Natural Philosopher, is very profitable, to prove that *Vacuum*, or *Emptinesse*, is not in the world. And that, all Nature, abhorreth it so much, that, contrary to ordinary law, the Elements will move, or stand. As, water to ascend rather: than between him and Air, Space or place should be lest, more than (naturally) that quantity of Air requireth, or can fill. Again, water to hang, and to descend: rather than by descending, to leave emptinesse at his back. The like is of Fire and Air, they will descend: when, either their Continuity should be dissolved, or their next Element forced from them. And as they will not be extended to discontinuity: So will they not, nor yet of mans force, can be prest or pent in space, not sufficient and answerable to their bodily substance. Great force and violence will they use, to enjoy their natural right and liberty. Hereupon, two or three men togeather, by keeping Air under a great Cauldron, and forcing the same down, orderly, may without harm descend to the Sea bottome: and continue there a time, &c. Where, Note, how the thicker Element (as the water) giveth place to the thinner (as is the Air:) and receiveth violence of the thinner, in manner, &c. Pumps and all manner of Bellows, have their ground of this art: and many other strange devises: as *Hydraulica*, Organes going by water, &c. Of this Feat, called commonly *Pneumatica*) goodly works are extant, both in Greek and Latine. With old and learned Schoolmen, it is called *Scientia de pleno & vacuo.*

Proclus pag. 18

To go to the bottom of the Sea without danger.

MENADRIE, is an Art Mathematical, which demonstrateth, how, above Natures vertue, and power simple: vertue, and force may be multiplyed: and so, to direct, to lift, to pull to, and to put or cast fro, any multiplied or simple, determined vertue, Weight or Force; naturally, not, so, directible or moveable. Very much is this Art furthered, by other Arts, as in some points by *Perspective*, in some, by *Statick*, in some, by *Trochilike*, and in other by *Helicosophie*, and *Pneumatithmie*. By this Art, all Cranes, Gibbets, and Engines to lift up, or to force any thing any manner of way, are ordered, and the certain cause of their force is known. As, the force, which one man hath with the *Dutch* Waggen Rock: therewith, to set up again, a mighty waggen laden, being overthrown. The force of the Crossebow Rack, is certainly here demonstrated. The reason, why one man doth with a leaver, lift that, which six men, with their hands onely, could not so easily do. By this Art, in our common Cranes in *London*, where power is to crane up the weight of 2000 pound: by two wheeles more (by good order added) Art concludeth, that there may be craned up 200000 pound weight, &c. So well knew *Archimedes* this Art: that he alone with his devices and engines (twice or thrice) spoyled and discomfited, the whole Army and Host of the *Romans*, beseiging *Syracusa*, *Marcus Marcellus the Consul*, being their General Captain. Such huge stones, so many with such force, and so far, did he with his engines hail among them out of the City. And by Sea likewise: though there Ships might come to the wals of *Syracusa*, yet he utterly confounded the *Roman* Navy: what with his mighty Stones hurling: what with pikes * of 18 foot long, made like shafts: which he forced almost a quarter of a mile, what, with his catching hold of their ships, and hoysing them up above the water, and suddenly letting them fall into the Sea again: what with his * Burning Glasses: by which he fired their other ships a far off: what with his other policies, devices, and engines, he so manfully acquitted himself: that all the force, courage, and policy of the *Romans* (for a great season) could nothing prevail, for the winning of *Syracusa*: whereupon, the *Romans* named *Archimedes*, *Briareus*, and *Centimanus*. *Zonaras* maketh mention of one *Proclus* who so well had perceived *Archimedes* Art of *Menadrie*, and had so well invented of his own, that with his Burning Glasses, being placed upon the walls of *Bysance*, he multiplyed so the heat of the Sun, and directed the beams of the same against his enemies Navy with such force, and so suddenly (like lightening) that he burned and destroyed both man and ship. And *Dion* specifieth of *Priscus* a *Geometrician* in *Bysance*, who invented and used sundry Engines, of force multiplyed: which was cause, that the Emperour *Severus* pardoned him his life after he had wonne *Bysance*. Because he honoured the Art, wit, and rare industry of *Priscus*. But nothing inferiour to the invention of these Engines of Force, was the invention of Guns. Which, from an *English-man* had the occasion and order of first inventing: though in another land, and by other men, it was first executed. " And they that should see the record, where the occasion and order general of Gunning, " is first discoursed of, would think: that small things slight and common: comming to " wise mens consideration, and industrious mens handling, may grow to be of force in- " credible.

Plutarchus in Marco Marcello.
Syncsius in Epistola.
Polybius.
Plinius.
Quintilianus.
T. Livius.
* *Atheneus.*
* *Galenus.*
Athemius.

Burning Glass.

Guns.

HYPOGEIODIE, is an Art Mathematical, demonstrating, how under the Spherical Superficies of the Earth, at any depth, to any perpendicular line assigned (whose distance from the perpendicular of the entrance: and the Azimuth likewise, in respect of the said entrance is known) certain way may be prescribed and gone: And how any way above the Superficies of the Earth designed, may under Earth, at any depth limited, be kept: going always perpendicularly, under the way, on Earth designed: and contrariwise, any way, (straight or crooked,) under the Earth, be given: upon the outface, or superficies of the Earth, to Line out the same: so, as, from the Center of the Earth, perpendiculars drawn to the Spherical superficies of the Earth, shall precisely fall in the correspondent points of those two wayes. This, with all other cases and circum-

circumstances herein, and appurtenances, this Art demonstrateth. This Art is very ample in variety of Conclusions, and very profitable sundry wayes to the Common-wealth. The occasion of my inventing this Art, was at the request of two Gentlemen, who had a certain work (of gain) under ground, and their grounds did joyn over the work: and by the reason of the crookednesse, divers depths, and heights of the way under ground, they were in doubt, and at controversie, under whose ground, as then, the work was. The name onely (before this) was of me published, *De Itinere Subterraneo*. The rest be at Gods will. For Pioners, Miners, Diggers for Metals, Stone, Cole, and for secret passages under ground, between place and place (as this land hath diverse) and for other purposes, any man may easily perceive, both the great fruit of this Art, and also in this Art, the great aid of Geometry.

HYDRAGOGIE, demonstrateth the possible leading of Water, by natures law, and by artificial help, from any head (being standing or running Water) to any other place assigned. Long hath this Art been in use: and much thereof written; and very marvellous works therein performed: as may yet appear in *Italy*, by the Ruines remaining, of the *Aquæducts*. In other places of Rivers, leading through the Main land Navigable many a Mile: and in other places, of the marvellous forcings of water to ascend: which all declare the great skill to be required of him, who should in this Art be perfect, for all occasions of waters possible leading. To speak of the allowance of the Fall, for every hundred foot: or of the Ventils (if the waters labour be far and great) I need not: seeing, at hand (about us) many expert men can sufficiently testifie, in effect, the order: though the Demonstration of the Necessity thereof, they know not: Not yet, if they should be led, up and down, and about Mountains, from the head of the Spring: and then a place being assigned: and of them to be demanded, how low or high, that last place is, in respect of the head, from which (so crookedly, and up and down, they be come: Perhaps, they would not, or could not) very readily or neerly assoil that Question. *Geometry* therefore, is necessary to *Hydragogie*. Of the sundry ways to force water to ascend, either by *Tympane*, *Kettelmils*, *Scrue*, *Ctesibike*, or such like, in *Vitruvius*, *Agricola*, (and other,) fully, the manner may appear. And so, thereby, also be most evident, how the Arts of *Pneumatithmie*, *Helicosophie*, *Statike*, *Trochelike*, and *Menadrie*, come to the furniture of this in speculation, and to the Commodity of the Common-wealth in practice.

HOROMETRIE, is an Art Mathematical, which demonstrateth how at all times appointed, the precise usual denomination of time, may be known, for any place assigned. These words are smooth and plain case *English*, but the reach of their meaning is farther than you would lightly imagine. Some part of this Art, was called in old time *Gnomonice*: and of late, *Horologiographia*: and in *English*, may be termed *Dialling*. Ancient is the use, and more ancient is the Invention. The use, doth well appear to have been (at the least) above two thousand and three hundred years ago: in * King *Achaz* Dial, then, by the Sun, shewing the distinction of time. By Sun, Moon, and Stars, this Dialling may be performed, and the precise time of day, or night known. But the demonstrative delineation of these Dials, of all sorts, requireth good skill both of *Astronomie* and *Geometry* Elemental, Spherical, Phenomenal, and Conical. Then to use the grounds of the Art, for any regular Superficies, in any place offered: and (in any possible apt position thereof) thereon to describe (all manner of ways) how, usual hours, may be (by the *Suns* shadow) truly determined: will be found no sleight Painters work. So to paint and prescribe the Suns Motion, to the breadth of a hair. In this Feat (in my youth) I invented a way, How in any Horizontal, Mural, or Equinoctial, Dial, &c. at all hours (the Sun shining) the sign and degree ascendent, may be known. Which is a thing very necessary, for the rising of those fixed stars: whose Operation in the air, is of great might evidently. I speak no further, of the use hereof. But forasmuch as, mans affairs, require knowledge of Times and Moments, when neither Sun, Moon, or Star, can be seen: Therefore, by Industrie Mechanical, was invented first, how by Water, running orderly, the Time and Hours might be known: whereof, the famous *Ctesibius*, was Inventor: a man of *Vitruvius* to the skie (justly) extolled. Then, after that, by sand running, were hours measured: Then

* 2. *Kings* 20.

Then, by *Trochilike* with weight, and of late time, by *Trochilike* with Spring: without weight. All these by Sun or Stars direction, (in certain time) require oversight and reformation, according to the heavenly Æquinoctial Motion: besides the inequality of their own Operation. There remaineth (without parabolical meaning herein) among the Philosophers, a more excellent, more commodious, and more marvellous way, than all these: of having the motion of the Primovant (or first Æquinoctial motion) by Nature and Art, imitated: which you shall (by further search in weightier studies) hereafter, understand more of. And so it is time to finish this Annotation, of Times distinction, used in our common, and private affairs: the commodity whereof no man would want, that can tell, how to bestow his time.

A perpetual Motion.

ZOGRAPHIE, **is an Art Mathematical, which teacheth and demonstrateth, how, the intersection of all visual Pyramids, made by any plain assigned, (the Center, distance, and lights, being determined) may be, by lines, and due proper colours representated.** A notable Art, is this, and would require a whole Volume, to declare the propertie thereof: and the Commodities ensuing. Great skill of *Geometrie*, *Arithmetick*, *Perspective*, and *Anthropographie*, with many other particular Arts, had the Zographer, need of, for his perfection. For, the most excellent Painter, (who is but the proper Mechanician, and Imitator sensible, of the Zographer) hath atteined, to such perfection, that sense of man and beast, have judged things painted, to be things natural and not artificial: alive and not dead. This Mechanical Zographer (commonly called the Painter) is marvellous in his skill: and seemeth to have a certain divine power: as of friends absent, to make a friendly, present comfort, yea, and of friends dead, to give a continual, silent presence: not onely with us, but with our posterity for many ages. And so proceeding, consider, how in Winter, he can shew you, the lively view of Sommers joy, and riches: and in Sommer, exhibite the countenance of Winters doleful state and nakednesse. Cities, Towns, Forts, Woods, Armies, yea, whole Kingdomes (be they never so far, or great) can he with ease, bring with him, home (to any mans judgment) as patterns lively of the things rehearsed. In one little house, can he, enclose (with great pleasure of the beholders) the portraiture lively, of all visible Creatures, either on earth or in the earth, living: or in the waters lying, creeping, sliding or swimming: or of any fowl or fly, in the Air flying. Nay, in respect of the Stars, the Skie, the Clouds: yea, in the shew of the very light it self, that divine creature, can he match our eyes judgment, most neerly. What a thing is this? things not yet being, he can represent, so, as, at their being: the picture shall seeme (in manner) to have created them. To what Artificer, is not picture, a great pleasure and commodity? which of them all, will refuse the direction and aid of Picture? The Architect, the Goldsmith, and the Arras weaver: of Picture, make great account. Our lively Herbals, our portraitures of birds, beasts, and fishes: and our curious Anatomies, which way, are they most perfectly made, or with most pleasure, of us beholders? Is it not by Picture onely? and if Picture, by the industry of the Painter, be thus commodious and marvellous: what shall be thought of *Zographie*, the Schoolmaster of Picture, and chief Governour? Though I mention not *Sculpture*, in my Table of Arts Mathematical: yet may all men perceive, How, that *Picture* and *Sculpture*, are Sisters Germane: and both, right profitable, in a Commonwealth and of *Sculpture*, aswel as of *Picture*, excellent artificers have written great books in commendation. Witnesse I take, of *George Vasari*, *Pittore Aretino*: of *Pomponius Gauricus*: and others. To these two Arts, (with other,) is a certain odde art, called *Athalmasar*, much beholding: more, than the common *Sculptor*, *Entayler*, *Carver*, *Cutter*, *Graver*, *Founder*, or *Painter*, (*&c.*) know their art to be commodious.

An Objection.

ARCHITECTURE, to many may seem not worthy, or not meet, to be reckoned among the *Arts Mathematical*. To whom, I think good, to give some account of my so doing. Not worthy, (will they say,) because it is but for building of a House, Palace, Church, Fort, or such like grosse works: and you also, defined the *Arts Mathematical*, to be such as dealed with no Material or corruptible thing: and also did demonstratively proceed in their Faculty, by Number or Magnitude. First, you see, that I count here, *Architecture*, among those *Arts Mathematical*, which are derived from the principals: and you know, that such may deal with natural things, and sensible matter. Of which some draw

The Answer.

draw neerer, to the simple and absolute Mathematical Speculation, then others do. And "though the *Architect* procureth, informeth, and directeth the *Mechanician*, to hand-"work, and the building actual, of House, Castle, or Palace, and is chief Judge of the "same: yet with himself (as chief *Master* and *Architect*) remaineth the Demon-"strative reason and cause of the *Mechanicians* work, in Line, Plain, and Solid: by "*Geometrical*, *Arithmetical*, *Optical*, *Musical*, *Astronomical*, *Cosmographical*, (and "to be brief) by all the former derived *Arts Mathematical*, and other Natural arts "able to be confirmed and stablished. If this be so, then, may you think, that *Architecture* hath good and due allowance, in this honest company of *Arts Mathematical* Derivative. I will herein crave Judgment of two most perfect *Architects*, the one being *Vitruvius* the Roman: who did write the Books thereof, to the Emperour *Augustus*, (in whose days our Heavenly Arch-master was born) & the other *Leo Baptista Albertus*, a Florentine, who also published ten books thereof. *Architectura* (saith *Vitruvius*) *est Scientia pluribus disciplinis, & variis eruditionibus ornata: cujus judicio probantur omnia, quæ ab cæteris Artificibus perficiuntur opera.* That is: Architecture is a Science garnished with many doctrines and divers instructions: by whose Judgement, all works, by other workmen finished, are judged. It followeth. *Ea nascitur ex Fabrica, & Ratiocinatione, &c. Ratiocinatio autem est, quæ, res frabricatas, solertia ac ratione proportionis, demonstrare atque explicare potest.* Architecture groweth of Framing, and Reasoning, &c. Reasoning is that, which of things framed with fore-cast, and proportion: can make demonstration, and manifest declaration. Again. *Cùm, in omnibus enim rebus, tùm maximè etiam in Architectura, hæc duo insunt: quod significatur, & quod significat. Significatur proposita res, de qua dicitur: hanc autem significat Demonstratio, rationibus doctrinarum explicata.* Forasmuch, as in all things: therefore chiefly in Architecture, these two things are: the thing signified, and that which signifieth. The thing propounded, whereof we speak, is the thing signified. But Demonstration, expressed with the reasons of divers doctrines, doth signifie the same thing. After that. *Ut literatus, sit peritus Graphidos, eruditus Geometriæ, & Optices non ignarus: instructus Arithmetica: historias complures noverit, Philosophos diligenter audiverit: Musicam sciverit: Medicinæ non sit ignarus, responsa Jurisperitorum noverit: Astrologiam, Cœlique rationes cognitas habeat.* An Architect (saith he) ought to understand Languages, to be skilful of Painting, well instructed in Geometry, not ignorant of *Perspective*, furnished with *Arithmetick*, have knowledge of many Histories, and diligently have heard Philosophers, have skill of Musick, not ignorant of Physick, know the answers of Lawyers, and have Astronomie, and the courses Cœlestial, in good knowledge. He giveth reason, orderly, wherefore, all these Arts, Doctrines, and Instructions, are requisite in an excellent Architect. And (for brevity) omitting the Latin text, thus he hath. Secondly, it is behoofful for an Architect to have the knowledge of Painting: that he may the more easily fashion out, in patterns painted, the form of what work he liketh: and Geometrie giveth to Architecture many helps: and first teacheth the use of the Rule, and the Compasse: whereby (chiefly and easily) the descriptions of Buildings, are dispatched in Ground-plats: and the directions of Squares, Levels, and Lines. Likewise, by Perspective, the Lights of the Heaven, are well led, in the Buildings: from certain quarters of the World. By Arithmetick, the charges of Buildings are summed together: the measures are expressed, and the hard questions of Symmetries, are by Geometrical means and methods discoursed on, &c. Besides this, of the Nature of things (which in Greek is called φυσιολογία) Philosophy doth make declaration. Which it is necessary for an Architect, with diligence to have learned: because it hath many and divers Natural questions: is specially in Aqueducts. For in their courses, leadings about, in the level ground, and in the mountings, the natural spirits or breaths are ingendred divers wayes: the hindrances, which they cause, no man can help, but he, which out of Philosophy, hath learned the original causes of things. Likewise, whosoever shall read *Ctesibius*, or *Archimedes* books, (and of others, who have written such Rules) cannot think, as they doe: unlesse he shall have received of Philosophers, instructions in these things: and Musick he must needs know: that he may have understanding, both of Regular and Mathematical Musick: that he may temper well his Balists, Catapults, and Scorpions, &c. Moreover, the Brasen Vessels, which in Theatres, are placed by Mathematical order, in ambries, under the steps: and the diversities of the sounds (which the Grecians call ἠχεῖα) are ordered according to Musical Symphonies and Harmonies: being distributed

in

in the Circuits, by *Diateſſaron*, *Diapente*, and *Diapaſon*. That the convenient voice, of the players ſound, when it came to theſe preparations, made in order, there being increaſed: with that increaſing, might come more clear and pleaſant, to the ears of the lookers on, &c. And of Aſtronomy, is known the Eaſt, Weſt, South, and North. The faſhion of the Heaven, the Æquinox, the Solſticie, and the courſe of the Stars. Which things, unleſſe one know: he cannot perceive, any thing at all, the reaſon of Horologies. Seeing therefore, this ample Science, is garniſhed, beautified, and ſtored, with ſo many and ſundry skils and knowledges: I think, that none can juſtly account themſelves Architects of the ſodain. But they onely, who from their childs years aſcending by theſe degrees of knowledges, being foſtered up with the atteining of many Languages and Arts, have won to the high Tabernacle of Architecture, &c. And to whom Nature hath given ſuch quick Circumſpection, ſharpneſſe of Wit, and Memory, that they may be very abſolutely skilfull in Geometry, Aſtronomy, Muſick, and the reſt of the Arts Mathematical: Such ſurmount and paſſe the calling, and ſtate of Architects: and are become Mathematicians, &c. And they are found ſeldom: as in times paſt was, *Ariſtarchus Samius*, *Philolaus*, and *Archytas Tarentynes*: *Apollonius Pergeus*, *Eratoſthenes Cyrenens*: *Archimedes*, and *Scopas*, *Syracuſians*. Who alſo left to their poſterity, many Engines and Gnomonical works: by numbers, and natural means, invented and declared.

A Mathematician.

Thus much, and the ſame words (in ſenſe) in one onely Chapter of this incomparable *Architect Vitruvius*, ſhall you find. And if you ſhould, but take his book in your hand, and ſlightly look through it, you would ſay ſtraightway: This is *Geometry*, *Arithmetick*, *Aſtronomy*, *Muſick*, *Anthropographie*, *Hydragogie*, *Horometrie*, *&c.* and (to conclude) the ſtore-houſe of all workmanſhip. Now, let us liſten to our other Judg, our Florentine, *Leo Baptiſta*, and narrowly conſider, how he doth determine of *Architecture*. *Sed anteque ultra progrediar*, *&c.* But before I proceed any further (ſaith he) I think, that I ought to expreſſe, what man I would have to be allowed an Architect. For, I will not bring in place a Carpenter: as though you might compare him to the Chief Maſters of "others arts. For the hand of the Carpenter, is the Architects Inſtrument: But I will appoint "the Architect to be that man, who hath the skill, (by a certain and marvellous means "and way) both in mind and Imagination to determine: and alſo in work to finiſh: "what works ſo ever, by motion of weight and coupling and framing together of bodies, "may moſt aptly be commodious for the worthieſt uſes of Man. And that he may be able to perform theſe things, he had need of atteining and knowledge of the beſt and moſt worthy things, &c. The whole Feat of Architecture in building, conſiſteth in Lineaments and in framing. And the whole power and skill of Lineaments, tendeth to this: that the right and abſolute way may be had, of Coapting and joyning Lines and angles: by which, the face of the building, or frame may be comprehended and concluded. And it is the property of Lineaments, to preſcribe unto buildings, and every part of them, an apt place, and certain number: a worthy manner and a ſeemly order: that ſo the whole form and figure of the building, may reſt in the very Lineaments, &c. And we may preſcribe in mind and imagination the whole forms, * all material ſtuffe being ſecluded. Which point we ſhall attein, by noting and fore-pointing the angles, and lines, by a ſure and certain direction and connexion. Seeing then, theſe things are thus: Lineament ſhall be the certain and conſtant preſcribing, conceived in mind: made in lines and angles: and finiſhed with a learned mind and wit. We thank you Maſter *Baptiſt*, that you have ſo aptly brought your art and phraſe, thereof, to have ſome Mathematical perfection: by certain order, number, form, figure, and *Symmetrie* mental: all natural and ſenſible ſtuffe ſet apart. Now then it is evident, (Gentle Reader) how aptly and worthily I have preferred *Architecture*, to be bred and foſtered up in the Dominion of the peerleſſe *Princeſſe*, *Mathemaca*, and to be a natural Subject of hers. And the name of *Architecture*, is of the principality, which this Science hath, above all other Arts. And *Plato* affirmeth the *Architect* to be *Maſter* over all, that make any work. Whereupon, he is neither Smith nor Builder: nor, ſeparately, any Artificer: but the Head, the Provoſt, the Director, and Judge of all artificial works, and all Artificers. For, the true *Architect*, is able to teach, demonſtrate, diſtribute, deſcribe, and judge all works wrought. And he, onely ſearcheth out the cauſes and reaſons of all Artificial things. Thus excellent, is *Architecture*: though few (in our days) attein thereto: yet may not the art, be otherwiſe thought on, than in very deed it is worthy. Nor we may not of ancient Arts, make new and imperfect Definitions in our days: for

Vitruvius.

Who is an Architect.

* The Immaterialitie of perfect Architecture.

What Lineament is.

Note.

for scarcity of Artificers: No more than we may pinch in, the Definitions of *Wisedome*, or *Honesty*, or *Friendship*, or *Justice*. No more will I consent, to diminish any whit of the perfection and dignity, (by just cause) allowed to absolute *Architecture*. Under the Direction of this Art, are three principal, necessary *Mechanical Arts*. Namely, *Housing*, *Fortification*, and *Naupegie*. *Housing*, I understand, both for Divine Service, and Mans common usage: publick and private. Of *Fortification* and *Naupegie*, strange matter might be told you: But perchance, some will be tyred, with this Bed-rol, already rehearsed: and other some, will nicely nip my grosse and homely discoursing with you: made in poste haste: for fear you should want this true and friendly warning, and tast giving, of the *Power Mathematical*. Life is short, and uncertein: Times are perilous, &c. And still the Printer awayting, for my pen staying: all these things, with farther matter of Ingratefulnesse, give me occasion to passe away, to the other Arts remaining, with all speed possible.

The Art of NAVIGATION, demonstrateth how, by the shortest good way, by the aptest direction and in the shortest time, a sufficient Ship, between any two places (in passage Navigable) assigned: may be conducted: and in all stoms and natural disturbances chancing, how to use the best possible means, whereby to recover the place first assigned. What need the *Master Pilot* had of other Arts, here before recited, it is ease to know: as, of *Hydrographie*, *Astronomy*, *Astrologie*, and *Horometrie*. Presupposing continually, the common Base, and Foundation of all: namely, *Arithmeticke*, and *Geometry*. So that he be able to understand and judg his own necessary Instruments, and furniture necessary. Whether they be perfectly made or no: and also can, (if need be) make them himself. As Quadrants, the Astronomers Ring, the Astronomers Staffe, the Astrolabe Uuniversal. An Hydrographical Globe. Charts Hydrographical, true, (not with parallel Meridians.) The common Sea Compasse: The Compasse of variation: The Proportional and Paradoxal Compasses (of me invented, for our two Muscovy Master Pilots, at the request of the Company) Clocks with spring: hour, half hour, and three hour Sand-glasses: and sundry other Instruments: and also be able on Globe, or Plain to describe the Paradoxal Compasse: and duly to use the same, to all manner of purposes, whereto it was invented. And also, be able to Calculate the Planets places for all times. *An.* 1599.

Moreover, with Sun, Moon or Star (or without) be able to define the Longitude and Latitude of the place, which he is in: So that the Longitude and Latitude of the place, from which he sayled, be given: or by him, be known: whereto appertaineth expert means, to be certified ever, of the Ships way, &c. and by fore-seeing the Rising, Setting, Noonsteding, or Midnighting of certain tempestuous fixed Stars: or their Conjunctions, and Anglings with the Planets, &c. he ought to have expert conjecture of storms, tempests and spouts: and such like Meteorological effects, dangerous on Sea. For (as *Plato* saith,) *Mutationes opportunitatesque temporum presentire, non minus res militari, quàm Agricultura, Navigationique convenit.* **To foresee the alterations and opportunities of times is convenient, no lesse to the Art of War, than to husbandry and Navigation.** And besides such cunning means, more evident tokens in Sun and Moon, ought of him to be known: such (as the Philosophical Poet) *Virgilius* teacheth in his *Georgicks*, where he saith.

Sol quoque & exoriens & quum se condet in undas,
Signa dabit, Solem certissima signa sequuntur, &c.

——— Nam sæpè videmus,
Ipsius in vultu varios errare colores.
Cæruleus, pluviam denunciat, igneos Eurus.
Sin macula incipient rutilo immiscerier igni,
Omnia tum pariter vento, nimbisque videbis
Fervere: non illa quisquam me nocte per altum
Ire, neque à terra moveat convellere funem, &c.
Sol tibi signa dabit. Solem quis dicere falsum
Audeat? ——— &c. *Georgic.* 1.

And

And so of Moon, Stars, Water, Air, Fire, Wood, Stones, Birds, and Beasts and of many things else, a certain Sympathical forewarning may be had: sometimes to great pleasure and profit, both on Sea and Land. Sufficiently, for my present purpose, it doth appear, by the premisses, how *Mathematical* the *Art* of *Navigation* is, and how it needeth and also useth other *Mathematical Arts*: And now if I would go about to speak of the manifold Commodities, comming to this Land, and others, by ships and *Navigation*, you might think, that I catch at occasions, to use many words, where no need is.

Yet this one thing may I, (justly) say. In *Navigation*, none ought to have greater care, to be skilful, than our *English* Pilots. And perchance, some, would more attempt: and other some, more willingly would be aiding, if they wist certainly, what priviledge, God hath endued this Island with, by reason of Situation, most commodious for *Navigation*, to places most Famous and Rich. And though (of * late) a young Gentleman, a couragious Captain, was in great readinesse, with good hope, and great causes of perswasion, to have ventured for a **Discovery**, (either *Westerly*, by *Cape de Baramantia*: or *Easterly*, above *Nova Zemla*, and the *Cyrimisses*) and was at the very neer time of attempting, called and imployed otherwise (both then, and since,) in great good service to his Country, as the Irish Rebels * have tasted: Yet, I say, (though the same Gentleman, do not hereafter deal therewith) some one or other should listen to the matter: and by good advice, and discreet circumspection, by little and little, winne to the sufficient knowledge of that **Trade and Voyage**; which now I would be sorry, (through carelesness, want of skill and courage) should remain unknown, or unheard of. Seeing, also, we are, herein, half challenged by the learned, by half request published. Thereof, verily might grow commodity, to this Land chiefly, and to the rest of the Christian Common-wealth far passing all riches and worldly Treasure.

* *An.* 1567 *S. H. G.*

* *An.* 1569.

THAUMATURGIKE, is that Art Mathematical, which giveth certain order, to make strange works, of the sense to be perceived, and of men greatly to be wondred at. By sundry means, this *Wonder-work* is wrought. Some by *Pneumatithmie*: as the works of *Cresibius* and *Hero*: some by weight, whereof *Timæus* speaketh: some by strings strained, or Springs, therewith imitating lively Motions: some by other means, as the Images of *Mercury*: and the brazen head, made by *Albertus Magnus* which did seem to speak. *Boethius* was excellent in these feats. To whom *Cassiodorus* writing, saith. **Your purpose is to know profound things, and to shew marvels. By the disposition of your Art, Metals do low: Diomedes of Thrace, doth blow a Trumpet loud, a brasen Serpent hisseth: Birds made sing sweetly. Small things we rehearse of you, who can imitate the heaven, &c.** Of the strange self-moving, which at Saint *Denis*, by *Paris*, * I saw once or twice (*Orontius* being then with me, in company) it were to strange to tell. But some have written it: and yet (I hope) it is there, to be seen. And by *Perspective* also strange things, are done: as partly (before) I gave you to understand in *Perspective*: as, to see in the air aloft, the lively image of another man, either waking to and fro: or standing still. Likewise, to come into an house, and there to see the lively shew of Gold, Silver or precious Stones: and comming to take them in our hand, to find nought but Air. Hereby, have some men (in all other matters counted wise) souly over-shot themselves: misdeeming of the means. Therefore, said *Claudius Celestinus*, *Hodie magna lituratura viros, & magna reprobationis videmus, opera quædam quasi miranda, supra Naturam putare, de quibus in Perspectiva doctus causam faciliter reddidisset.* That is. **Now a dayes, we see some men, yea at great learning and reputation, to judge certain works as marvellous above the power of nature: of which works, one that were skilful in Perspective, might easily have given the cause.** Of *Archimedes Sphere*, *Cicero* witnesseth. Which is very strange to think on. **For when Archimedes (saith he) did fasten in a Sphere, the movings of the Sun, Moon, and of the five other Planets, he did, as the God, which (in Timæus of Plato) did make the World. That one turning should rule Motions most unlike in slowness and swiftness.** But a greater cause of marvelling we have by *Claudianus* report hereof. Who affirmeth this,

* *An.* 1551.

De his quæ Mundo mirabiliter eveniunt, cap. 8.

Archi-

Archimedes work, to have been of Glasse, and discourseth of it more at large: which I omit. The Dove of wood, which the *Mathematician Archytas* did make to flie, is by *Agellius* spoken of. Of *Dædalus* strange Images, *Plato* reporteth. *Homer* of *Vulcans Self-movers*, (by secret wheels,) leaveth in writing. *Aristotle* in his *Politicks* of both, maketh mention. Marvellous was the workmanship of late dayes, performed by good skill of *Trochilike*, &c. For in *Noremberg*, a Flie of Iron, being let out of the Artificers hand, did (as it were) flie about by the guests at the Table; and at length, as though it were wearie, return to his Masters hand again. Moreover, an artificial Eagle, was ordered, to flie out of the same Town, a mightie way, and that aloft in the air, toward the Emperour coming thither: and followed him, being come to the gate of the Town.* Thus you see what Art Mathematical can perform, when skill, will, industrie, and abilitie, are duly applied to proof.

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A Digression Apologetical.

AND for these, and such like marvellous Acts and Feats, Naturally, Mathematically, and Mechannically, wrought and contrived: ought any honest Student, and Modest Christian Philosopher, be counted, and called a **Conjurer**? Shall the folly of Idcots, and the Malice of the Scornful, so much prevail, that He, who seeketh no worldly gain or glory at their hands: But onely, of God, the treasure of heavenly wisdome, and knowledge of pure verity: Shall he (I say) in the mean space, be robbed and spoiled of his honest name and fame? He that seeketh (by S. *Pauls* advertisement) in the Creatures Properties, and wonderful vertues, to finde just cause, to glorifie the Æternal, and Almighty Creator by: Shall that man be (in hugger mugger) condemned, as a Companion of the Hel-hounds, and a Caller, and Conjurer of wicked and damned Spirits? He that bewaileth his great want of time, sufficiently (to his contentation) for learning of Godly wisdome, and Godly Verities in: and onely therein setteth all his delight: Will that man leese and abuse his time, in dealing with the Chief enemy of Christ our Redeemer: the deadly foe of all mankind: the subtile and impudent perverter of Godly Verity: the Hypocritical Crocodile: the Envious Basilisk, continually desirous, in the twinkling of an eie, to destroy all Mankind, both in Body, and Soul, eternally? Surely (for my part, somewhat to say herein) I have not learned to make so brutish, and so wicked a Bargain. Should I, for my xx, or xxv years Study: for two or three thousand Marks spending: seven or eight thousand Miles going and travelling, onely for good learning sake: And that, in all manner of weathers: in all manner of wayes and passages: both early and late: in danger of violence by man: in danger of destruction by wilde beasts: in hunger: and thirst: in perillous heats by day, with toil on foot: in dangerous damps of cold, by night, almost bereaving life: (as God knoweth): with lodgings, oft times, to small ease: and sometime to lesse security. And for much more (than all this) done and suffered, for Learning and atteining of Wisdome: Should I (I pray you) for all this, no otherwise, nor more warily: or (by Gods mercifulnesse) no more luckily, have fished, with so large, and costly, a Net, so long time in drawing and that with the help and advice of Lady Philosophy, and Queen Theology): but at length, to have catched, and drawn up, * a Frog? Nay, a Devil? For, so, doth the Common peevish Pratler Imagine and Jangle: And, so, doth the Malicious scorner, secretly, and bravely and boldly face down, behinde my back. Ah, what a miserable thing is this kinde of Men? How great is the blindnesse and boldnesse, of the Multitude, in things above their Capacity? What a Land: what a People: what Manners: what Times are these? Are they become Devils, themselves: and by false witnesse bearing against their Neighbour, would they also, become Murderers? Doth God, so long give them respite, to reclaim themselves in, from this horrible slandering of the guiltlesse: contrary to their own Consciences: and yet will they not cease? Doth the Innocent, forbear the calling of them, Juridically to answer him, according to the rigour of the Laws: and will they despise his Charitable patience? As they, against him, by name, do forge, fable, rage, and raise slander, by Word and Print: Will they provoke him by word and Print, likewise, to Note their Names to the World: with their particular devices, fables, beastly Imaginations, and unchristian-like slanders? Well: Well, O (you such) my unkinde Countreymen! O unnatural Countreymen! O unthankful Countreymen! O Brainsick, Rash, Spiteful and Disdainful Countreymen. Why oppresse you me thus violently, with your slandering of me: Contrary to Verity: and contrary to your own Consciences? And I, to this hour, neither by word, deed, or thought, have been, any way, hurtful,

* A Proverb; Fair fisht and caught a Frog.

damageable, or injurious to you or yours? Have I, so long, so dearly, so far, so carefully, so painfully, so dangerously sought and travelled for the learning of Wisdome, and attaining of Vertue: And in the end (in your judgement) am I become, worse, than when I began? Worse than a Mad man? A dangerous Member in the Common-wealth: and no Member of the Church of Christ? Call you this to be Learned? Call you this, to be a Philosopher? and a lover of Wisdome? To forsake the straight heavenly way: and to wallow in broad way of damnation? To forsake the light of heavenly Wisdome: and to lurk in the dungeon of the Prince of darkneſſe? To forsake the Verity of God, and his Creatures, and to fawn upon the impudent, craftie, obstinate Liar, and continual disgracer of Gods Veritie, to the uttermoſt of his power? To forsake the Life and Bliſſe Eternal: and to cleave unto the Author of Death everlasting? that murderous Tyrant, moſt greedily awaiting the Prey of Mans soul? Well: I thank God, and our Lord Jesus Christ, for the comfort, which I have by the Examples of other men, before my time: To whom, neither in godlineſſe of life, nor in perfection of learning, I am worthy to be compared: and yet they susteined the verie like injuries that I do: or rather greater. Patient *Socrates* his *Apologie* will testifie: *Apuleius* his *Apologies*, will declare the brutishneſſe of the Multitude: *Joannes Picus*, Earl of *Mirandula*, his *Apologie* will teach you, of the raging slander of the malicious Ignorant against him. *Joannes Trithemius* his *Apologie* will specifie, how he had occasion to make publike Proteſtation: as wel by reason of the Rude Simple: as also in reſpect of such, as were counted to be of the wiſeſt sort of men. "Many could I "recite: But I defer the preciſe and determined handling of this matter: being loth to de- * "test the Folly and Malice of my Native Countreymen. * Who, so hardly, can disgest "or like any extraordinarie course of Philosophical studies: not falling within the Com- "paſſe of their Capacitie: or where they are not made privie of the true and secret cause, "of such wonderful Philosophical Feats. These men, are of four sorts, chiefly. The first, I may name, *Vain pratling buſie-bodies*. The second, *Fond Friends*. The third, *Imperfectly Zealous*: and the fourth, *Malicious Ignorant*. To each of these (briefly, and in charitie) I will say a word or two, and so return to my Preface. *Vain pratling buſie-bodies*, 1. uſe your idle assemblies, and conferences, otherwiſe, than in talk of matter, either above your capacities, for hardneſſe: or contrarie to your consciences in veritie. *Fond Friends*, 2. leave off, so to commend your unacquainted friend upon blind affection. As, because he knoweth more, than the common Student: that, therefore he muſt needs be skilful, and a doer, in such matter and manner, as you term *Conjuring*. Weening thereby, you advance his fame: and that you make other men, great marvels of your hap, to have such a learned friend. Ceaſe to aſcribe Impietie, where you pretend Amitie. For, if your tongues were true, then were that your friend *Untrue*, both to God and his Sovereign. Such *Friends* and *Foudlings*, I shake off, and renounce you: Shake you off, your Folly. *Imperfectly* 3. *Zealous*, to you, do I say: that (perhaps) well, do you mean: but far you misſe the Mark: If a Lamb you will kill, to feed the flock with his bloud. Sheep, with Lambs bloud, have no natural sustenance: No more, is Christs flock, with horrible slanders, duly edified. Nor your fair pretenſe, by such rash ragged Rhetorique, any whit, wel graced. But such, as so uſe me, will finde a foul crack in their credit. Speak that you know: And know, as you ought: Know not, but Hear-say, when life lieth in danger. Search to the quick, and let charity be your guide. *Malicious Ignorant*, what shall I say to thee? *Prohibe linguam tuam à malo. A Detractione paretur linguæ.* 4. **Cause thy tongue to refrain from evil. Refrain your tongue from slander.** Though your tongues be sharpened Serpent-like, and Adders poiſon lie in your lips: yet take heed, and think, betimes, with your self, *Vir linguoſus non ſtabilietur in terra. Virum violentum venabitur* *Pſal. 140.* *malum, donec præcipietur.* For, sure I am, *Quia faciet Dominus Judicium afflicti, & vindictam pauperum.*

Thus, I require you my assured friends and Countreymen (you Mathematicians, Mechanicians, and Philosophers, charitable and discreet) to deal in my behalf, with the light and untrue tongued, my envious Adversaries or Fond friends. And farther, I would wiſh, that at leasure, you would consider, how *Baſilius Magnus* layeth *Moſes* and *Daniel* before the eyes of those, which count all such studies Philosophical (as mine hath been) to be ungodly or unprofitable. Weigh well Saint *Stephen* his witneſſe of *Moſes*. *Eruditus* *Act 7. 6.* *eſt Moſes omni Sapientia Ægyptiorum: & erat potens in verbis & operibus ſuis.*

Moſes

Moſes was instructed in all manner of wisdome of the Egyptians, and he was of power both in his words and works. You see this Philosophical Power and Wisdome which *Moſes* had, to be nothing misliked of the Holy Ghost. Yet *Plinius* hath recorded *Moſes* to be a wicked *Magician*. And that (of force) muſt be, either for this Philosophical wisdome, learned, before his calling to the leading of the Children of *Iſrael*: or for those his wonders wrought before King *Pharaoh*, after he had the conducting of the *Iſraelites*. As concerning the first, you perceive, how Saint *Stephen*, at his Martyrdome (being full of the Holy Ghost) in his Recapitulation of the Old Testament, hath made mention of *Moſes* Philosophy: with good liking of it: And *Baſilius* *Magnus*, also avoucheth it, to have been to *Moſes* profitable (and therefore I say, to the Church of God neceſſarie.) But as concerning *Moſes* wonders done before King *Pharaoh*: God himself said: *Vide ut omnia oſtenta, quæ poſui in manu tua, facias coram Pha-* *raone.* **See that thou do all those wonders before Pharaoh, which I have put in thy hand.** Thus, you evidently perceive, how rashly, *Plinius* hath slandered *Moſes*, of vain fraudulent *Magick*, saying: *Eſt & alia Magices Factio, à Moſe, Jamne,* *Lib. 30. c. 10.* *& Jotape, Judaïs pendens: ſed multo millibus annorum poſt Zoroaſtrem, &c.* Let all 1. such, therefore, who, in Judgement and Skill of Philosophie, are far inferiour to *Pliny*, take good heed, leſt they over-shoot themselves rashly, in judging of *Philosophers ſtrange acts*: 2. and the means how they are done. But much more ought they to beware of forging, devising, and imagining monstrous feats, and wonderful works, when and where no such were done: no, not any spark or likelihood, of such, as they, without all shame, do report. And (to conclude) moſt of all, let them be ashamed of Man, and afraid of the dreadful and just Judge: both fooliſhly or maliciously to deviſe: and then divelliſhly to father their new fond Monsters on me: Innocent in hand and heart: for treſpaſſing either against the 3. law of God or man, in any my Studies, or Exercises, Philosophical, or Mathematical: As in due time, I hope, will be more manifeſt.

NOw end I, with ARCHEMASTRIE. Which name, is not so new, as this Art is rare. For another Art, under this, a degree (for skill and power) hath been endued with this Englisſh name before. And yet this may serve for our purpoſe, sufficiently, at this present. **This art, teacheth to bring to actual experience, sensible, all worthy conclusions by all the Arts Mathematical purposed, and by true Natural Philosophy concluded: and both addeth to them a farther scope, in the terms of the same Arts, and also by his proper Method, and in peculiar terms, proceedeth, with help of the foresaid Arts, to the performance of compleat Experiences, which of no particular Art, are able (Formally) to be challenged.** If you remember, how we considered *Architecture*, in reſpect of all common handworks: some light may you have, thereby, to understand the Sovereignty and propertie of this Science. *Science* I may call it, rather, than an Art: for the excellencie and Maſterſhip it hath, over so many, and so mighty Arts and Sciences. And because it proceedeth by *Experiences*, and searcheth forth the causes of Conclusions, by *Experiences*: and also putteth the Concluſions themselves, in *Experience*, it is named of some *Scientia Experimentalis*. The **Experimental Science**, *Nicolaus Cuſanus* termeth it so, in his *Experiments Sta-* *R. B.* *tical*, and another *Philosopher* of this Land Native (the flower of whose worthy fame can never die nor wither) did write thereof largely, at the request of *Clement the Sixth*. The Art carrieth with it, a wonderful credit: by reason, it certifieth, sensibly, fully, and completly to the utmoſt power of Nature and Art. This Art certifieth by *Experiences* compleat and abſolute: and other Arts, with their arguments and demonſtrations, perſwade: and in words prove very well their Conclusions. * But words and arguments, are no sensible certifying: nor the full and final fruit of Sciences practicable. And though some Arts have in them *Experiences*, yet they are not compleat, and brought to the uttermoſt, they may be stretched unto, and applied sensibly. As for example: the natural Philosopher diſputeth and maketh goodly shew of reason: And the Aſtronomer, and the Optical Mechanician put some things in *Experience*: but neither, all, that they may: nor yet sufficiently, and to the utmoſt, those, which they do. There, then, the *Arch-maſter*, steppeth in,

n, and leadeth forth on, the *Experiences*, by order of his doctrine *Experimental*, to the chief and final power of Natural and Mathematical Arts. Of two or three men, in whom this Description of *Archimystry* was *Experimentally* verified, I have read and heard: and good record, is of their such perfection. So that this Art is no fantastical imagination: as some Sophister, might, *Cum suis Insolubilibus*, make a flourish: and dazle your imagination: and dash your honest desire and courage, from believing these things, so unheard of, so marvellous, and of such importance. Well: as you will. I have forewarned you. I have done the part of a friend: I have discharged my Dutie toward God: for my small Talent, at his most merciful hands received. To this Science, doth the *Science Alnirangiat*, great Service. Muse nothing of this name. I change not the name, so used, and in Print, published by others: being a name proper to the Science. Under this, commeth *Ars Sintrillia*, by *Artephius*, briefly written. But the chief Science, of the Archimystry, (in this world) as yet known, is another (as it were) OPTICAL Science: whereof, the name shall be told (God willing) when I shall have some, (more just) occasion, thereof, to discourse.

Here I must end, thus abruptly (Gentle friend, and unfeigned lover of honest and necessary verities.) For, they, how have (for your sake, and vertues cause) requested me, (an old foreworn Mathematician) to take pen in hand: (through the considence they reposed in my long experience: and tryed sincerity) for the declaring and reporting somewhat, of the fruit and commodity, by the **Arts Mathematical to be atteined unto**: even they, sore against their wils are forced, for sundry causes, to satisfie the workmans requeſt, in ending forthwith: He, so feareth this, so new an attempt, and so costly: and in matter so tenderly (hitherto) among the Common Sort of Students, considered or esteemed.

And where I was willed some what to alledge, why, in our vulgar speech, this part of the principal Science of *Geometry*, called *Euclids Geometrical Elements*, is published, to your handling: being unlatined people, and not Universitie Scholers: Verily, I think it needlesse.

1. For, the Honour, and Estimation of the **Universities and Graduates**, is, hereby nothing diminished. Seeing, from, and by their Nurse Children, you receive all this Benefit, how great soever it be.

2. Neither are their studies any whit hindred. No more than the Italian *Universities*, as *Academia Bononiensis*, *Ferrariensis*, *Florentina*, *Mediolanensis*, *Patavina*, *Papiensis*, *Perusina*, *Pisana*, *Romana*, *Senensis* or any one of them find themselves, any deal disgraced, or their Studies any thing hindred, by *Frater Lucas de Burgo*, or by *Nicolaus Tartalea*, who in vulgar Italian language, have published, not onely *Euclids Geometry*, but of *Archimedes* somewhat: and in Arithmetick, and Practical Geometry, very large Volumes, all in their vulgar speech. Nor in *Germany*, have the famous *Universities*, any thing been discontent with *Albertus Durerus*, his Geometrical Institutions in Dutch: or with *Guilielmus Xylander*, his learned translation of the first six books of *Euclide*, out of the Greek, into the High Dutch. Nor with *Gualterus H. Riffius*, his Geometrical Volume: very diligently translated into the High Dutch tongue, and published. Nor yet the *Universities* of *Spain* or *Portugal*, think their reputation to be decayed: or suppose any their studies to be hindred by the Excellent *P. Nonnius*, his Mathematical works, in vulgar speech by him put forth. Have you not, likewise in the French tongue, the whole Mathematical Quadrivie? and yet neither *Paris*, *Orleans*, or any of the other Universities of *France* at any time, with the Translators, or Publishers offended: or any mans studie thereby hin dred?

3. And surely, the common and vulgar scholer (much more the Grammarian) before his comming to the *Universety*, shall (or may) be, now (according to *Plato* his Counsel) sufficiently instructed in *Arithmetick* and *Geometry*, for the better and easier learning of all manner of *Philosophy Academical* or *Peripatetical*. And by that means, go more cheerfully, more skilfully, and speedily forward in his studies, there to be learned. And, so, in lesse time, profit more, than (otherwise) he should or could do.

4. Also many good and pregnant *English* wits, of young Gentlemen, & of other, who never intend to meddle with the profound search and studie of Philosophie (in the *Universities* to be learned) may nevertheless, now, with more ease and liberty, have good occasion, vertuously to occupier the sharpnesse of their wits: where else (perchance) otherwise, they would in found exercises, spent (or rather lose) their time: neither serving God: nor furthering the weal, common or private.

And great Comfort, with good hope, may the *Universities* have, by reason of this *English* **Geometrie and Mathematical Preface**, that they (hereafter) shall be the more regarded, esteemed, and resorted unto. For when it shall be known and reported, that of the *Mathematical Sciences* onely, such great Commodities are ensuing (as I have specified:) and that in deed, some of you unlatined students, can be good witnesse, of such rare fruit by you enjoyed (thereby:) as either, before this, was not heard of: or else not so fully credited: "Well, may all men conjecture, that far greater aid and better furniture, "to win to the perfection of all Philosophy, may in the Universities be had: being the "Store-house and Treasury of all Sciences and all Arts, necessary for the best and most "noble State of Common-wealths. 5. *Universities.*

Besides this, how many a common Artificer, is there in these Realms of *England* and *Ireland*, that dealeth with Numbers, Rule, and Compasse: Who, with their own skill and experience, already had, will be able (by these good helps and informations) to find out, and devise, new Works, strange Engines, and Instruments: for sundry purposes in the Common-wealths or for private pleasure? and for the better maintening of their own estate? I will not (therefore) fight against mine own shadow. For no man (I am sure) will open his mouth against this enterprize. No man (I say) who either hath charity toward his brother, (& would be glad of his furtherance in vertuous knowledge) or great hart, for rare and deep (to the bettering of the common state of this Realm. Neither any, that make accompt, what the wiser sort of men (*Sage* and *Stayed*) do think of them. To none (therefore) will I make any *Apologie*, for a vertuous act doing: and for commending, & setting forth, profitable Arts, to *English* men in their English tongue. "But, unto God our Creator, let us all be "thankful: for that, **As he, of his goodnesse, by his power, in his "wisedome, hath created all things in Number, Weight and Mea-"sure.** So, to us, of his great mercy, he hath revealed means, whereby to attein the "sufficient and necessary knowledge of the aforesaid his three principal Instruments: "Which means, I have abundantly proved unto you, to be the *Sciences* and *Arts Ma-"thematical*. 6.

And though I have been pinched with straightnesse of time: that, no way, I could so pen down the matter (in my mind) as I determined: hoping of convenient leasure: Yet, if vertuous zeal, and honest intent, provoke and bring you to the reading of this Compendious Treatise, I doe not doubt, but as the verity thereof (according to our purpose) will be evident unto you: So the pith and force thereof, will perswade you: and the wonderful fruit thereof: highly pleasure you. And that you may the easier perceive and better remember, the principal points, whereof my Preface treateth, I will give you the **Ground-plat** of my whole discourse, in a Table annexed: from the first to the last, somewhat Methodically contrived. *The ground Plat of this Preface in a Table.*

If haste, hath caused my poor pen, any where to stumble: You will (I am sure) in part of recompence, (for my earnest and sincere good will to pleasure you) consider the rockish huge mountains, and the perilous unbeaten ways, which (both night and day, for the while) it hath toyled and laboured through, to bring you this good News, and Comfortable proof of Vertues fruit.

So I commit you unto Gods merciful direction, for the rest: heartily beseeching him to prosper your Studies, and honest Intents: to his Glory, and the Commodity of our Country. *Amen.*

Written at my poor House,
At Mortlake

Anno 1570 *Febr.* 9.

Here followeth the GROUND-PLAT of the Mathematical Preface of Master JOHN DEE. By which a brief view of what hath been before at large delivered doth appear.

THE FIRST ELEMENT OF EUCLIDE.

THE ARGUMENT.

THis First Book comprehends the properties of the first and chiefest of right lined figures, *viz.* of *Triangles* and *Parallelograms*, as touching the equality, or inequality of their Area, and of their sides and angles, wherein first is declared the original and properties of Triangles compared among themselves: Then intermixing the properties of parallels, the nature of *Parallelograms* and the affections that are in them are demonstrated; after which is shewn the relation of Triangles and Parallelograms, and after what manner a Parallelogram may be made equal to a Triangle; Lastly, it is demonstrated that in a right angled Triangle, the Square that is made of the side subtending the right angle, is equal to the Squares that are made of the sides comprehending the right angle. And that all these things may the better be accomplished, here is also taught the division of a right lined angle, & of a right line into two equal parts, the construction of a perpendicular line, and after what manner a right lined angle may be made equal to a right lined angle; and other things of the like nature.

DEFINITIONS.

1 *A point is that which hath no part.* *As* A

IN Greek it is read σημεῖον, that is to say, a Sign; For seeing that it is void of all Magnitude, that which is externally made, is the signe of that which is conceived in the Minde; and it is (in a manner) the same as a Unite in Number, an Instant in Time, and a Sound in Musick.

2 *A Line is a Length without breadth.* *As* A——B.

THere is no such Line in any material thing, but as the point, so the line which we draw, is the sign of that which we conceive in Mind: For if the point which we conceive be moved, and leaveth an imaginary tract, that shall be a line, long by reason of the motion, but not broad, because the point from whence it proceedeth is void of all extention.

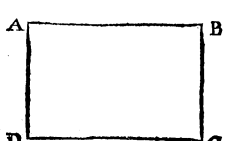
3 *The Extreams or Ends of a Line are points.*

THat is, the beginning and end of a Length, as it is a Length, is a point: because a Mathematician doth not consider Magnitude, but as it is finite: Therefore when *Euclide* speakes of an infinite line, he understandeth an indeterminate line, or a line of any Magnitude.



4 *A Right Line is that which lyeth equally between its points.*

OR whose extreams doth shadow all the middle parts, as saith *Plato*: or the least of all those which have the same extreams, as *Archimedes* will have it: For seeing that a line is conceived to be made by the flux of a point, if it floweth equally between its points, or by the shortest Space, it is called a right line, as A B; If a point be carryed with an uniform motion and distance from a certain point, it is called a circular line as C D; If it moves unevenly, in some place lower, and in others higher, and the extreams do not shadow all the middle parts, it is called a mixt line, as E F: From hence *Aristotle* saith ingeniously *Lib.* 1. *de Cœlo*, that according to this threefold distinction of a line, there can be only three motions, two simple, the right and circular, and the third mixt of both.



6 *A Superficies is that which hath only length and breadth.*

AS a line the first Species of continued quantity, is produced by the flux of a point, so a Superficies, the second Species, is conceived to be produced by the flux of a line transversly, or side-wayes, as if the line A D be supposed to move side-wayes to B C, it shall produce the Superficies A B C D, which may be divided in length as a line, and likewise

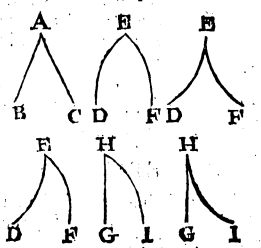
wise in breadth. Conceive a shadow, saith *Proclus*, and thou shalt conceive a Superficies long and broad, yet having no manner of thickenesse.

6 *The Extreams or Ends of a Superficies are lines.*

THis Definition is only to be understood of a plain or a mixt Superficies; and not of a circular; for the extream thereof is only a line, and not lines.

7 *A Plain Superficies is that which lyeth equally between his lines.*

WHat I have said of a right line, the same is to be understood of a Plain Superficies.

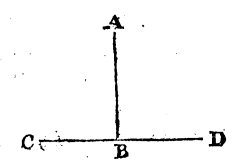


8 *A Plain angle* B A C, *is an inclination or bowing of two lines* A B *and* A C *in a Plain, the one to the other, the one touching the other, and not lying streight forth at length.*

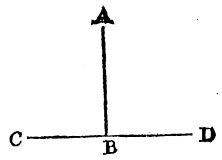
HEre the Causes of an angle are explained, the Material are two lines which touch one another: The Formal is an Inclination of the one to the other. From whence it followeth, First, that those two lines ought not so to touch, that they may lye joyned in a direct line the one to the other, that is, so as they make one right line, but the one ought to incline to the other: Secondly, it followeth that the quantity of an angle consists in a greater or lesser inclination of the lines, and not in the length of the lines: Thirdly, it followeth that it is not necessary that the two lines being produced, should intersect one another after the contact, as *Pelletarius* would have it, for that is only true in right lined angles, but it sufficeth that they touch one another, and be inclined to one another. Lastly, if the angle be in a Plain Superficies, it is called a Plain angle. And in every figure although we call every angle by three letters, yet the middlemost is alwayes to be understood for the Angle.

9 *And when the lines which contein the angle are right lines, it is called a right lined angle.*

AND if both of them be curved as D E F, it is a curve lined angle; and if one be curved, and the other right as G H I, it is a mixt angle.



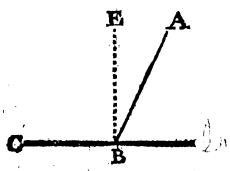
10 *When a right line as* A B, *standing upon a right line as* C D, *maketh the angles on either side,* A B C *and* A B D *equal, then*



either of thoſe angles is a right angle, and the right line A B, *that ſtandeth erected, is called a perpendicular line to that upon which it ſtandeth* C D.

THen either angle is ſaid to be equal, when the right line A B inclineth not more to C then to D.

That which the Greeks call κάθετος, is rendred in Latine *Perpendicularis*, but the Mathematicians do more frequently uſe the Greek word then the Latine, chiefly in the Opticks, where there is nothing more uſed by them then πρὸς κάθετον alſo in Latine they render it *ad Calhetum*.



11 *An Obtuſe angle as* C B A, *is that which is greater than a right angle* E B C.

BEcauſe the right line A B, doth more recline from the baſe line C D, then the perpendicular E B.

12 *An Acute angle, as* A B D, *is that which is leſſer than a right angle* E B D.

13 *A Term is that which is the extream of any thing.*

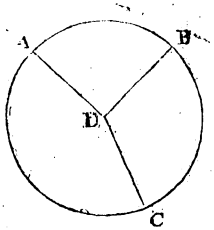
SUch are a point, a line, a ſuperficies; that is to ſay, a point is the Term of a line, a line of a ſuperficies, and a ſuperficies of a Body.

14 *A Figure is that which is comprehended by one Term, or many.*

IT is ſaid by one Term, becauſe one Term comprehendeth a Circle and an Ellipſis, that is to ſay, a Circular, or an Elliptical line: but there are alwayes more Terms required to comprehend right lined figures.

Moreover it is to be obſerved, the Terms ought to encompaſſe and comprehend the quantity which is called a Figure, and not only terminate it. From whence it followeth, firſt, that a line properly is no Figure, ſeeing that points do not encompaſſe a line, but only terminate it: It followeth, ſecondly, that there, can be no figure of an infinite Superficies, or an infinite body, if any ſuch may be given; firſt becauſe every figure ought to encompaſſe and comprehend the thing figured: Secondly, becauſe it is encompaſſed with Terms, but a Term is the extream of a thing. And how can that which hath an end and extreams be infinite?

15 *A*



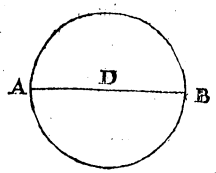
15 *A Circle is a plain figure, comprehended by one line* A B C, *which is called a Circumference: unto which all right lines drawn from one point within the figure, as* D A, D B, D C, *and falling upon the Circumference thereof, are equal the one to the other.*

16 *And that point is called the center of the Circle.*

THeodoſius hath the ſame in his firſt Book of *Sphericks*, (concerning a Sphere) Definition the firſt, and ſecond, but in the fifth Definition he thus deſcribeth a Pole.

The Pole of a Circle in a Sphere, is a point in the Superficies of the Sphere, from whence all right lines drawn to the Circumference of the Circle are equal the one to the other.

From which it may be gathered that there is only this difference between the Center and the Pole of a Circle on the Sphere, becauſe the Center is conceived to be within the figure: But the Pole is in the Superficies of the Sphere.

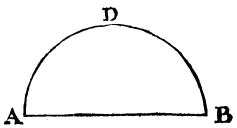


17 *The Diameter of a Circle is a certain right line, as* A B, *drawn by the center* D, *and being terminated by the circumferenee on either ſide; as* A *and* B, *divides the circle into two equal parts.*

HEre you ſhall obſerve three things, firſt, that all the Diameters of the ſame Circle are equal to one another, ſeeing that their halves are equal by the 15th. Definition, the ſecond which followeth from the firſt is, that although there may be infinite right lines drawn in a Circle not paſſing by the Center, yet only thoſe right lines which are drawn by the Center and terminated in the Circumference, are called Diameters; becauſe the Diameter only can meaſure the Circle: ſeeing that they only be all equal the one to the other, and of a determinate length; but the others are alwayes unequal and uncertain: For *Ptolomie* ſaith in his *Analemma* that the meaſure of any thing ought to be certain, and determinate, not indefinite; Wherefore let not beginners wonder if it be put in the Feminine Gender by the Mathematicians, For Diameter is the ſame as the diametient line, or a line dividing into two equal parts.

The

The third is that the Diameter doth divide the Circle into two equal parts, which *Thales* doth thus demonstrate in *Proclus*; Conceive in mind the portion of a Semicircle, so to be fitted to another portion, so as the Diameter may be the Base of both of them: If the Circumference of one do altogether agree to the Circumference of the other, it is manifest that those two portions made by the Diameter are equal the one to the other; seeing that neither of them doth exceed the other: But if one Circumference do not agree with the other, but falleth either without or within it, or partly without, and partly within, then the right lines drawn from the Center to the Circumference shall be equal, and not equal, which is absurd.



18 *A Semicircle is a figure which is conteined by the Diameter* A B, *and by that line* A D B. *which is taken away from the circumference of the circle.*

19 Right lined Figures, *are those which are conteined under right lines.*

AFter the Definition of the Circle, *Euclide* intending to give the Description of divers figures, teacheth first of all what figures are called Right lined Figures; for of those he principally treats in the following Books.

Therefore all plain figures enclosed on all sides by right lines, are called Right lined Figures; from whence it is manifest that the plain figures environed by Curve lines, are called Curvilined: but those which are comprehended partly by Right lines, and partly by Curve lines, are called Mixt: Now divers kinds of Figures are here described by *Euclide*.

20 Trilateral, *or three sided figures, are those which are conteined under three right lines.*

WHen *Euclide* here saith, that Right lined Trilateral figures are such as are environed by three right lines, he shews us clearly how a Triangle ought to be defined: For seeing that in Right lined figures there are as many angles as there are sides, or of right lines of which they are constituted, the Triangle shall be a figure conteined under three right lines, whose Species shall be hereafter declared.

21 Quadrilateral, *or four sided figures, are those which are conteined under four right lines.*

IN like manner, the Quadrangle, or Quadrilateral figure, is a figure conteined of four right lines, whose divers Species and Kinds, shall be hereafter declared.

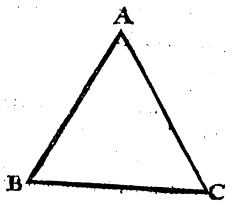
22 Mul-

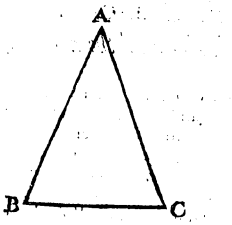
22 Multilateral, *or many sided figures, are those which are conteined under more than four right lines.*

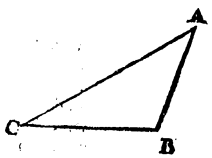
FOrasmuch as the Species or Kinds of Right lined figures are innumerable, by reason of the infinite progression of numbers: For three right lines environing a figure, do constitute the first Kind, under which are conteined all Triangles, four right lines constitute the second Kind, which compriseth all quadrangular or quadrilateral figures, five right lines compound the third Kind, and six the fourth, and so *ad infinitum*: Therefore seeing that right lined figures cannot be infinitely defined, *Euilde* calleth all other right lined figures, comprehended by more than four right lines, by this General Name [*Multilateral*] contenting himself with the Denomination of the Trilateral and Quadrilateral, it may be for this reason, because he treateth principally of those in his Books, and by the knowledg of those the Definition of the rest may be easily understood.

23 *Now of* Trilateral *or three sided figures, that which hath three equal sides is called an* Equilateral *Triangle; as* A B C.

Equilateral. Isosceles. Scalenum.







24 *But that which hath only two sides equal, is called an* Isosceles *Triangle; as* C A B.

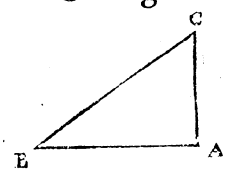
25 *And that which hath the three sides unequal, is called a* Scalenum, *as* B A C.

BY these three Definitions he exposeth the three Species or Kinds of Triangles, the first and most simple of which is the *Equilateral* Triangle, or Triangle of three equal sides, the second is the *Isosceles* Triangle, which hath onely two sides equal, and the third is the *Scalenum*, which hath all the three sides unequal, the Equilateral Triangles are alwayes uniform, forasmuch as their angles are alwayes equal: But the Isosceles and Scalene Triangles are diversified in infinite manners.

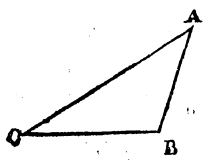
26 *More-*

26 *Moreover, of three sided, or* Trilateral *figures, the* Orthogonal, *or Rectangled Triangle, is that which hath one right angle, as* B A C.

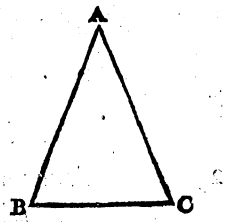
Orthogon.



Ambligon.



Oxigon.



27 *The* Ambligonium, *or obtuse angled Triangle, is that which hath one angle obtuse, as* A B C.

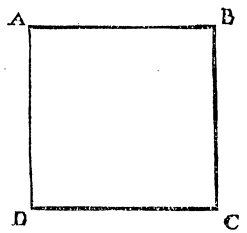
28 *And the* Oxigonium, *or acute angled Triangle is that which hath all the angles acute; as* B C A.

EUCLIDE expoſeth Triangles in conſidering them; firſt, according to their ſides; and he doth it now according to the conſideration of their angles, and foraſmuch as there are onely three varieties of Right lined angles, as we have declared, alſo there are three Kinds of Triangles, to wit, the Rectangle, which is that which hath one angle a right angle, which may be either Scalene, or Iſoſceles: The Ambligon Triangle may be alſo in like manner, either Iſoſceles or Scalene, (but not equilateral,) but every Oxigonium, or acute angled Triangle, may be either Equilateral, Iſoſceles, or Scalene.

Here it is to be noted, that in every Triangle, two of the lines being taken for two ſides, the third remaining ſide is uſually called by the Mathematicians the [Baſe] of the Triangle, whether it be the loweſt ſide of the Triangle or not, in ſuch ſort, as each of the lines which encloſeth the Triangle, may be taken for the Baſe.

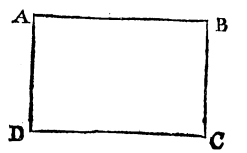
29 *But of* Quadrilateral, *or four ſided figures, the Square is that which is equilateral, and right angled; that is to ſay, which hath the four ſides equal, and the angles right.*

After



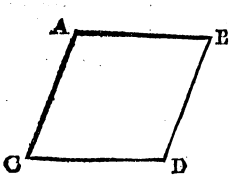
After the Trilateral and three ſided figures he expoſeth the Quadrilateral and four ſided figures, ſpeaking only of five ſorts, of which the firſt four are Regular, but the fifth and laſt Irregular: Therefore the firſt figure is called a Square, of which all the ſides are equal to one another, and all the angles right angles, as is the Figure A B C D; and therefore the Quadrangle which is equilateral, and not right angled, or contrarily rectangled, and not equilateral, ſhall not be called a Square.

30 *An* Oblong, *or Long Square is a figure rectangled, but not equilateral.*

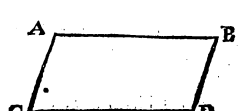


The ſecond Quadrilateral Figure is called by the Greeks ετερόμηκες, that is to ſay, longer of one part, which is that which hath the four angles right angles, but hath not the four ſides equal to one another, although the oppoſite ſides are equal to one another, as appears by this Figure A B C D, where the ſides A B and D C oppoſite to one another, are equal, and in like manner, the ſides A D and B C.

31 *A* Rhombus, *is a figure equilateral, but not right angled.*



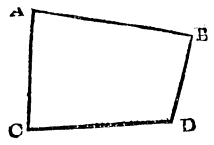
This Figure which is the third kind of Quadrilateral, or four ſided figures, hath the four ſides equal, but the four angles are not right angles, although that the oppoſite angles are equal to one another, two to two, the angles A and D being equal to one another, but not right angles; and in like ſort, the angles B and C.



32 *A* Rhomboides *is a Figure which hath the oppoſite ſides, and angles equal, but is neither equilateral, nor right angled.*

THis Figure called Rhomboides, neither hath the sides equal, nor any angle a right angle, but only hath the opposite sides and opposite angles equal to one another, as the sides A C and B D, are equal to one another, and in like manner, A B and D C, and the opposite angles A and D, and likewise B and C equal to one another; therefore these four Quadrilateral figures may be termed regular, but all other Quadrilaterals are irregular.

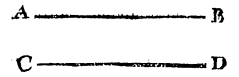
33 *But besides these figures, all other Quadrilateral Figures are called* **Trapeziums.**



ALL other Quadrilateral Figures, which differ from those before mentioned; that is to say, which are neither Squares, nor long Squares, Rhombs, nor Rhomboides, are called Trapeziums, and are Irregulars, considering that they may be of divers manners and formes.

34 *Parallels are right lines, which being in one and the same Plain, and prolonged infinitely on both parts, neither meet on the one side, nor on the other, as the lines* A B *and* C D.

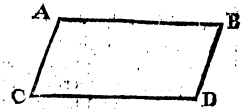
TO the end the right lines be parallel, or equi-distant, it sufficeth not, that being prolonged infinitely on both parts, that they meet not at a point, or never interfect, but it is necessary that they be also in one and the same Plain Superficie: For divers right lines being not in one and the same Plain Superficie, prolonged infinitely, will never interfect at one point; yet nevertheless are not said to be Parallels, such as would be two lines posited transversly, and understood to be in the air, and touch not one another; those I say, can never interfect; Therefore the right lines that are in one and the same Plain Superficie, and which being prolonged infinitely on both sides, (as for Example, the lines A B and C D,) will never interfect; those shall be termed Parallels, or lines equally distant, *Posidonius* thus defineth Parallels.



Parallels, (saith he) are such lines as are neither elevated nor depressed in a Plain, but have all the perpendiculars equal, which are drawn from all the points taken in the one or the other line.

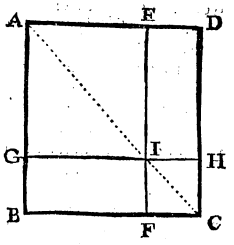
¶ *EUCLIDE* here endeth the Definitions of his First Book, but forasmuch as mention is made therein of Figures, Parallelograms, and of their Complements, we have judged it convenient to explain the two following Definitions, to render the Demonstrations more intelligible.

35 *A* Parallelogram *is a Quadrilateral, or four sided figure, whose opposite sides are parallel, or equidistant.*



AS if in the Figure A B C D, the side A B be parallel to the side opposite C D, and A C parallel to B D, it shall be called Parallelogram; that is to say, a Figure made of lines which are parallels, and of these there are four sorts, to wit, the Square, the long Square, which are also called rectangled Parallelograms; the Rhombe and the Rhomboides, Parallelograms not rectangled, from whence may be collected, that there are only four sorts of Parallelograms.

36 *But when in a Parallelogram there is drawn a Diameter or Diagonal, and two right lines parallel to the sides, cutting the Diameter in one and the same point, in such sort as the Parallelogram be divided by those parallel lines, into four Parallelograms, those two through which the Diameter doth not passe, are called Complements, but the two others, through which the Diameter doth not passe, are said to be about the Diameter.*

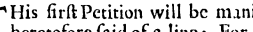


LEt A B C D, be a Parallelogram, in which having drawn the Diameter A C, and the line E F, cutting the same in the point I, and in like manner, H G cutting the same Diameter in the same point I, and parallel to A D and BC, it is manifest, that the whole Parallelogram is divided into four Parallelograms, two of which, to wit, D I and I B, through which the Diameter doth not passe, are by Geometricians called Complements, or Supplements of the Parallelograms; the remaining Parallelograms E G and F H, are said to be about the Diameter; forasmuch as the Diameter doth passe through them.

Petitions or Requests.

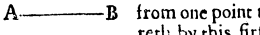
1 *It is required that it be granted, that a right line may be drawn from any point to any point.*

THis first Petition will be manifest, if you consider what hath been heretofore said of a line; For seeing that a line is an imaginary flowing of a point, and by this means a right line is a flowing, proceeding from the direct way, it will happen that if some point be understood to move to another point, a right line will be drawn from one point to another, which *EUCLIDE* requireth by this first Petition; as you may see by the line AB, drawn from the point A, to the point B.

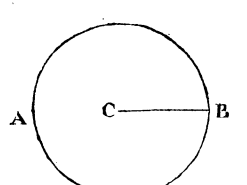


2 *And that a terminated right line may be produced streight forth continually.*

AS if it were understood that this point were moved yet further forth directly, the terminated right line shall be continued; and this production may be continued *ad infinitum*; seeing that we may imagine this point to move infinitely; as if the right line AB were continued to the point C, and then to the point D, &c.



3 *And that upon any Center, and at any distance, a Circle may be described.*



NOw if we conceive in the understanding, some terminated right line, applyed according to one of the extremities at a point fixed; to move about the same, until it return where it began its motion, it will have described a Circle, which is what is required by this Petition, as may be seen by Example in the line CB, which drawn about the Center C, hath described the Circle BA, acording to the quantity, distance, or interval CB.

Others add this fourth Petition.

4 *That any Magnitude being given, it is possible to take another greater or lesser.*

FOr all continued quantity may be augmented by addition, and by division may be infinitely diminished: Therefore there cannot be given a continued quantity so great, but a greater may yet be given; nor one so little, but a lesser may be given; This holds as true in Numbers, by addition, for each number may be augmented infinitely; by the continual addition of Unity, although in the diminishing thereof; you arrive to Unity indivisible.

Common Sentences, or Axioms.

1 *Things that are equal to one and the same, are also equal to one another.*

FRom this first Axiom it is evident, that a thing greater, or lesse than one of the equal things, shall be also greater or lesse than the other; and if one of the equal things be greater or lesse than some Magnitude, the other shall be in like manner greater or lesse than the same Magnitude.

2 *And if to equal things, equal things be added, the whole shall be equal.*

3 *And if from equal things, equal things be taken, the remainders shall be equal.*

4 *And if to unequal things, equal things be added, the whole shall be unequal.*

IN like manner, it is manifest, that if to unequal things, things unequal be added, the greatest to the greatest, and the least to the least, that the whole shall be unequal, the one greater, the other lesse.

5 *And if from unequal things, equal things be taken, the remainders shall be unequal.*

IT follows from hence, that if from unequal things, unequal things be taken; to wit, from the greatest, lesse, and from the least more, the remainders shall be unequal; that is to say, this shall be greater, and that other lesse.

Now in all these Axioms, excepting only the first, by these words, *equal quantities*, you ought also to understand one and the same, common to divers: for if to equal things there be added one common thing, the whole shall be equal; and if from equal things, a common thing be taken, the remainders shall be also equal: and if to unequal things, there be added one and the same common thing, or to one and the same common thing, there be added unequal things, the whole shall be unequal, and if from unequal things there be taken one and the same common thing, or from one and the same common thing, there be taken unequal things; the remainders shall be unequal.

6 *And the things that are double to one and the same thing, are also equal to one another.*

Whence

WHence it followes, that, that which is double to one of the equal things, is alſo double to the other: Likewiſe thoſe things which are triple, quadruple, quintuple, &c. to one and the ſame thing, are equal to one another, as is manifeſt.

7 *And the things which are the one half of one and the ſame thing, are alſo equal to one another.*

COntrariwiſe it follows, that thoſe things which are equal to one another, are the halves of one and the ſame; likewiſe the things which are the third, fourth, or fifth parts, &c. are alſo equal to one another.

In thoſe two Maximes, by one and the ſame quantity, ought alſo to be underſtood, equal quantities, for things double, triple, &c. of things which are equal, are in like manner equal to one another. Alſo the things which are the halves, third, or fourth parts, &c. of equal things; are alſo equal to one another.

8 *And the things which agree to one another, are alſo equal to one another.*

THat is to ſay, that two quantities, the one being poſited on the other, the one exceedeth not the other, but both agree together, thoſe are equal; as two right lines ſhall be ſaid to be equal to one another, when the one being poſited on the other, that which is placed above, totally agrees with the other, in ſuch ſort, as that it neither exceedeth the other, nor is exceeded by it: The ſame alſo of two right lined angles, equal to one another, when the one being poſited on the other, the uppermoſt exceedeth not the other, nor is exceeded by it, but the lines which conſtitute the one, do totally fall upon thoſe which conſtitute the other; for that being ſo, the inclination of the lines are equal; although, nevertheleſſe, thoſe lines are unequal to one another: but it is to be underſtood, that the Magnitudes which agree to one another, are equal, according to that only in which they agree. Now length agrees only to length, a Superficie to a Superficie, and the inclination of lines, to the inclination of lines, &c.

Clavius willing to convert this Axiom, ſaith thus, [and contrariwiſe the things which are equal to one another, agree with one another, if the one be poſited on the other;] which nevertheleſſe, cannot be, for if a ſquare be deſcribed, and on the Diameter of that Square there be deſcribed another ſquare, the firſt ſquare ſhall be equal to the half of the laſt; that is to ſay, to an Iſoſceles Triangle, and nevertheleſſe, will not agree, being poſited the one on the other: There may be alſo made a Curvilined angle, equal to a right angle, an obtuſe angle, or acute angle, as *Proclus* hath demonſtrated, which nevertheleſſe will not agree, being poſited on one another. Laſtly, a right lined Triangle may be made equal to another right lined Triangle, which may not agree, being poſited on one another, as is manifeſt by the 37 and 41 of the firſt Book: Therefore this Axiom cannot be converted univerſally, but thus it may. Equal right lines agree with one another, being poſited the one on the other: in like manner, equal right lined angles, as alſo plain Superficies, alike and equal, &c.

9 *And*

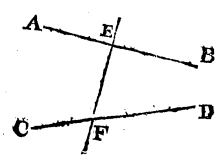
9 *And the whole is greater than its part.*

10 *All right Angles are equal to one another.*

THis Axiom is manifeſt by the 10 Definition, *Pappus* ſhews on this place that this Axiom cannot be converted; for every angle equal to a right angle is not a right angle, as we have ſhewn in the 8th. Common Sentence; but only, that every right lined angle equal to a right angle, is alſo a right angle; and this may be taken for an Axiom.

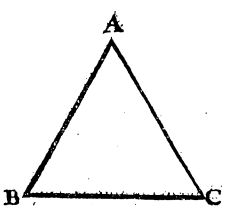
11 *And if on two right lines there fall another right line, making the interior angles on the ſame part leſſe than two right angles; thoſe two right lines being infinitely prolonged, will interſect and cut one another, on that part where the two angles are leſſe than two right angles.*

AS if a right line E F, falling on the two right lines A B and C D, maketh the two interior angles B E F, and D E F on the ſame part B D, leſſe than two right angles; *EUCLIDE* ſaith, that thoſe lines being prolonged on the part B D, where the angles are made leſſe than two right angles, that finally they ſhall interſect at a point, which is manifeſt, for when the two interior angles on the ſame part are equal to two right angles; then the two lines cannot interſect in any part, but are alwayes equally diſtant the one from the other, as ſhall be demonſtrated in the 28th. Prop. of this Book; wherefore if the interior angles on the ſame part be leſſe than two right angles, it is neceſſary that the Space between the lines ſtreighten more and more, and grow leſſe and leſſe, and on the other part that they open and enlarge themſelves more and more: Therefore thoſe lines at laſt will interſect at a point.



12 *Two right lines contein no Space.*

THis Axiom hath no difficulty in it, for if two lines make an angle at a point, they ſpread and dilate themſelves ſtill more and more both wayes, if they be prolonged, as is manifeſt by this Figure, for having drawn a right line from A to B; from the ſame point A, cannot be drawn another right line to the point B, by another way, in ſuch ſort as that between A B and the other line, there ſhould remain a Space incloſed; ſo as that another right line drawn from the point A,



as

as A C, ſhall make an angle with A B, in ſuch ſort as that another line ought yet to be drawn, as B C, to encloſe the Space: but a right line and a Curviline, or elſe two curved lines, may well contein or encloſe a Space, as is manifeſt.

After theſe Axioms of EUCLIDE, *Clavius* and other Interpreters, have yet added theſe following Axioms.

1 *Two right lines, have not one and the ſame Common Section.*

2 *Two right lines meeting at a point, if they be prolonged, they will neceſſarily interſect.*

There are yet added theſe ſix.

1 *If to equal things, unequal things be added, the exceſſe or difference of the whole ſhall be equal, or the ſame with the difference of the things added.*

2 *If to unequal things, equal things be added, the exceſſe, or difference of the whole ſhall be equal, or the ſame with the exceſſe or difference of thoſe which were at the beginning.*

3 *If from equal things, unequal things be taken, the exceſſe or difference of the remainders ſhall be equal, or the ſame with the exceſſe or difference of the things taken away.*

4 *If from unequal things, equal things be taken, the exceſſe or difference of the remainders ſhall be the ſame with the exceſſe or difference of the whole.*

5 *The whole is equal to all its parts taken together.*

6 *If the whole be double to the whole, and the part cut off, double to the part cut off, the reſt ſhall be double to the reſt.*

NOw theſe ſix pretended Axioms are alſo hereafter demonſtrated by *EUCLIDE*, as ſhall (God aſſiſting) be made appear. Wherefore ſeeing that they are proved elſwhere, following the Demonſtrations of

of *EUCLIDE*, they ought not to be taken for Axioms: We ſhall demonſtrate according to *EUCLIDE* the five firſt, at the Second Propoſition of this Book, and the Sixth, at the fifth Propoſition of the Fifth Book.

From what hath been before ſaid, we may gather, with *Proclus* and *Geminus*, this difference between Requeſts, and Maximes or Axioms; that ſeeing both the one and the other, are known of themſelves, and indemonſtrable, that Requeſts are of the nature of Problemes; foraſmuch as by them there is required ſome thing to be done; but Axioms reſemble Theoremes; ſeeing that by them it is not required that any thing ſhould be done, but only there is propoſed ſome Sentence well known. But the Requeſt or Petition differeth from the Problem, in that the Conſtruction of the Requeſt ſtandeth in need of no Demonſtration, but no one will conſent to the conſtruction of a Problem, without Demonſtration, for that ſome difficult thing is propoſed to be conſtrued. In like manner, there is difference between Axiom and Theorem, for the Axiom ought not to be demonſtrated; but the Theorem ought in no wiſe to be conſented to, if not demonſtrated: For no one will require the proof or Demonſtration of this Propoſition, [Things which are equal to one and the ſame thing, are likewiſe equal to one another;] but of this there will be required Demonſtration, [Of every Triangle, the interior angles are equal to two right angles;] and the ſame is to be underſtood of the other Axioms and Theorems, as alſo of Petitions and Problems.

It is likewiſe very certain, that of Requeſts or Petitions, ſome of them are more proper to Geometry, as thoſe three which *EUCLIDE* hath propoſed to us: other ſome are common to Geometry and Arithmetick, as is this, [A quantity may be infinitely augmented;] for as well the number as the Magnitude, may be augmented by Addition, ſo as that no end can be found in this augmentation; the ſame may be ſaid of the Axioms for the Eighth, Tenth, Eleventh, and are proper to Geometry only, but all the others are applyed to the Demonſtrations both Geometrical and Arithmetical, for even ſo as equal Magnitudes taken from equal Magnitudes, do leave equal Magnitudes, be the Magnitudes conſiſting of Lines, Superficies, or of Solid Bodies; So likewiſe equal numbers, taken from equal numbers, leave equal numbers; and having ſaid this of theſe three Kinds of Principles, let us now come to the Demonſtrations, by which the nature of theſe Principles will be more perfectly known; for there are divers principles among Mathematicians, and ſuch as are not eaſily underſtood, before their uſe be known in their Demonſtrations, which will be found by experience.

¶ To the end the courſe of the Demonſtrations be not interupted, we have cited the Principles and Propoſitions of *EUCLIDE* in the Margent, with a Letter of the Alphabet; the ſame Letter being alſo put in the Diſcourſe, there where the Propoſition ought to be cited, written in the Margent with the ſame Letter, and may thus be underſtood;

1 Def. The first Definition, and so of the rest.
1 Pet. The First Petition, or Request.
1 c.s. or 1. ax. The first Common Sentence, or Axiom.
1. 1. The first Proposition of the first Book.
1. 2. The first Proposition of the second Book, and so of the rest, in this manner.
Color. 32. 1. The Corollary of the 32 Proposition of the first Book.

What a Proposition is.

THe principles thus placed and ended, now follow the Propositions, which are Sentences set forth to be proved by reasoning and Demonstrations; and therefore they are again repeated in the end of the Demonstration: For the Proposition is ever the conclusion, and that which ought to be proved.

Propositions are of two sorts, the one is called a Problem, the other a Theorem.

What a Problem is.

A Problem is a Proposition which requireth some action or doing, as the making of some figure, or to divide a figure or line, to apply figure to figure, to adde figures together, or to subtrah one from another, to describe, to inscribe, to circumscribe one figure within or without another, and such like: As of the first Proposition of the first Book is a Problem, which is thus, *Upon a given terminated right line, to make an Equilateral triangle.* For in it, besides the Demostration, and Comtemplation of the mind, is required somewhat to be done: namely, to make an Equilateral triangle upon a line given. And in the end of every Problem, after the Demonstration, is concluded after this manner: *Therefore, &c. Which was to be done.*

What a Theorem is.

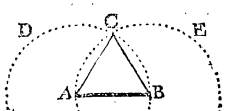
A Theorem is a Proposition which requireth the searching out and demonstration of some property or passion of some figure: Wherein is only Speculation and Contemplation of Mind, without doing or working of any thing, As the seventeenth Proposition of the first Book, which is thus, *Of every triangle two angles are lesse than two right angles, after what manner soever they be taken*; is a Theorem. For in it is required only to be proved and made plain by reason and Demonstration, that two angles are lesse than two right angles, &c. without further working or doing. And in the end of every Theorem, after the Demonstration, is concluded after this manner: *Therefore, &c. Which was to be demonstrated.*

PRO-

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. PROBLEM 1.

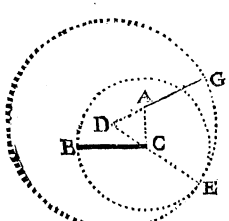


Upon a given terminated right line A B, *to make an Equilateral Triangle* A B C.

Construction ON the Center A, at the distance A B, [a] describe the Circle B C D; and on the Center B, at the same distance B A, [b] describe the Circle A C E, and from the point C, where the Circles intersect one another, [c] draw the two right lines C A and C B: Then I say, that the Triangle A B C is Equilateral. a) 3. pet. 1. b) 3. pet. 1. c) 1. pet. 1.

Demonstration FOr the right line A C is equal to the right line A B [d], and the right line C B is equal to the same right line A B [e]: Therefore [f] the right lines A C and B C, are also equal the one to the other: And the three right lines C A, A B, and B C, being equal, [g] the Triangle A B C is equilateral: Wherefore upon a given terminated right line A B, is made an Equilateral triangle A B C; Which was required to be done. d) 15. d. 1. e) 15. d. 1. f) 1. ax. 1. g) 24. d. 1.

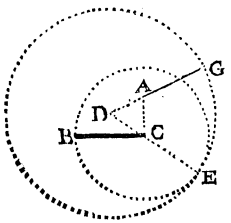
PROP. 2. PROBL 2.



From a point given A, *to draw a right line* A G, *equal to a right line given* B C.

Construction DRaw [a] from A to C, the right line A C, upon which make [b] the equilateral triangle C A D, then on the Center C, at the distance B C, describe [c] the Circle B E, a) 1. pet. 1. b) 1. 1. c) 3. pet. 1.

d) 2. pet. 1.
e) 3. pet. 1.



and d produce the side D C to E, and on the Center D, at the distance D E, e describe the Circle E G, and produce the side D A to G. Then I say, that the right line A G, (drawn from the given point A) is equal to (the given line) B C.

f) 15. d. 1.
g) 3. ax. 1.

Demonstration FOr the right lines D E and D G are f equal: Therefore taking away the equal right lines D A and D C, g there shall remaine the equal right lines g A G and C E: But C E is equal to C B: Therefore also A G is equal to C B.

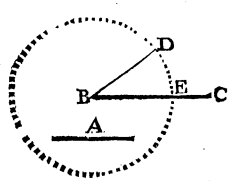
Wherefore from a point given A, is drawn a right line A G, equal to a right line given B C; Which was required to be done.

The position of the point A, in or without the given line B C doth varie the Case, but yet there is alwayes the same Construction and Demonstration.

SCHOLIUM.

The line A G *may be taken equal to* B C *with the Compasses, but in so doing, it answers to no Petition, as* Proclus *well observes.*

PROP. 3. PROBL. 3.



Two unequal right lines A, *and* B C, *being given, to take away from the greater* B C, *a right line* B E, *equal to the lesser* A.

a) 2. 1.
b) 3. pet. 1.

Construction FRom the given point B, a draw the right line B D, equal to the given right line A, and on the Center B, b at the distance B D describe the Circle D E: Then I say, that the part cut off is equal to A.

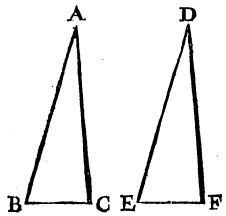
c) 15. d. 1.
d) 1. ax. 1.

Demonstration FOr the right line B E is equal to the right line B D c, and B D is equal to A by Construction; Therefore, the part cut off B E, is equal to the given line A d: Which was required to be done.

PROP. 4. THEOR. 1.

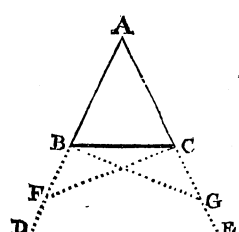
If two triangles B A C *and* E D F, *have two sides* B A *and* A C *of the one, equal to two sides* E D *and* D F, *of the other, each side to his correspondent side, (that is,* B A *equal to* E D, *and* A C *equal to* D F,) *and the angle* B A C, *conteined by the equal right line of the one, be equal to the angle*

angle E D F, *conteined by the equal right lines of the other: The base also* B C *of the one, shall be equal to the base* E F *of the other; and the one triangle* B A C, *shall be equal to the other triangle* E D F, *and the other angles remaining, shall be equal to the other angles remaining, the one to the other; that is, the angle* A B C *to the angle* D E F, *and the angle* A C B *to the angle* D F E, *by which equal sides are subtended.*



Demonstration FOr the triangle A B C being put upon the triangle D E F, and the point A upon the point D, and the right line A B upon the right line D E the point B shall fall upon the point E, because the right line A B, is equal to the right line D E: Seeing therefore that the right line A B doth agree with the right line D E, and the right line A C with the right line D F, because the angle B A C is equal to the angl E D F: Therefore, also the point C shall agree with the point F, wherefore also the base B C shall agree with the base E F. For if (B agreeing with E, and C with F,) the base B C doth not agree with the base E F: Then two right lines shall include a Superficies, which is absurd. Therefore the base B C, agrees with the base E F, and is equal to it: Wherefore the whole triangle A [illegible], shall agree with the whole triangle D E F, (for all the angles and sides do agree) and is equal to it, and the remaining angles of the one, shall agree with the remaining angles of the other, and shall be equal the one to the other, *viz.* the angle A B C to the angle D E F, subtended by the equal sides A C and D F, and the angle A C B to the angle D F E, subtended by the equal sides A B and D E.

PROP. 5. THEOR. 2.



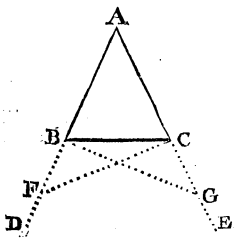
The angles A B C *and* A C B, *which are at the base of* Isosceles *triangles* A B C, *are equal the one to the other, and the equal right lines* A B *and* A C, *being produced, (as to* D *and* E,) *the angles* C B D *and* B C E, *that are under the base, are equal the one to the other.*

Con-

Construction LEt any point as F be taken in the line B D, and from the greater line A E, [a] let the leſſer A G, be taken equal to the line A F, and [b] draw the right lines F C and G B.

a) 3. 1.
b) 1. pet. 1.

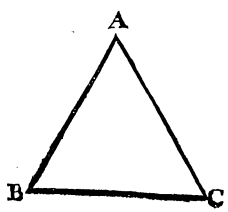
Demonſtration SEeing that A F is equal [c] to A G, and A B to A C [d]: Therefore theſe two F A and A G are equal to theſe two G A and A B, the one to the other, and they contein a common angle F A G: Therefore [e] the baſe F C is equal to the baſe G B, and the triangle A F C is equal to the triangle A G B, and the remaining angles ſhall be equal to the remaining angles, the one to the other, by which equal ſides are ſubtended, that is, A C F to A B G, and A F C to A G B, *and becauſe* A F *is equal to* A G, *and* A B *to* A C. *Therefore*, B F is equal [f] to C G, and it is proved that F C is equal to G B: Therefore theſe two B F and F C are equal to theſe two C G and G B, the one to the other; and the angle B F C is equal to the angle C G B, and they have one common baſe B C: Therefore [g] the triangle B F C ſhall be equal to the triangle C G B, and the remaining angles of the one equal to the remaining angles of the other, the one to the other, by which equal ſides are ſubtended: Therefore the angle F B C is equal to the angle G C B, and the angle B C F to the angle C B G: And becauſe the whole angle A B G is equal to the whole angle A C F, of which the angle C B G is equal to B C F; the remaining angle A B C ſhall be equal to the remaining angle [h] A C B, and they are at the baſe of the triangle, and it is proved that the angles under the baſe F B C and G C B are equal: Therefore the angles A B C and A C B, at the baſe of Iſoſceles Triangles are equal, &c. Which was to be demonſtrated.



c) by Con.
d) by Supp.
e) 4. 1.
f) 3. ax. 1.
g) 4. 1.
h) 3. pet. 1.

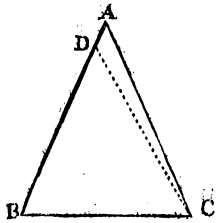
COROLLARIE.

From this Fifth Propoſition it alſo followeth, that every equilateral triangle is equiangled; that is to ſay, that the three angles of whatſoever equilateral triangle are equal to one another: For let A B C *be an equilateral triangle: Therefore foraſmuch as the two ſides* A B *and* A C *are equal, the two angles* B *and* C, *upon the baſe* B C, *ſhall be alſo equal. Likewiſe becauſe the two ſides* A B *and* B C *are equal, the two angles* A *and* C *upon the baſe* A C *ſhall be equal: Therefore the three angles* A, B, *and* C *ſhall be equal: Which was to be demonſtrated.*



PROP.

PROP. 6. THEOR. 3.



If two angles A B C *and* A C B, *of a triangle* A B C, *be equal to one another, the ſides* A B *and* A C *ſubtending thoſe equal angles ſhall be alſo equal to one another.*

Construction FOr if they be unequal, one of them ſhall be greater than the other; Let A B therefore be the greater, (if poſſible,) and let D B [a] be cut off, equal to A C, and let D C be drawn.

a) 3. 1.

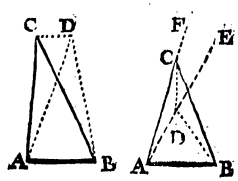
Demonſtration FOraſmuch as D B is equal to A C, and B C common, the two ſides D B and B C of the triangle D B C, ſhall be equal to the two ſides A C and C B, of the Triangle A C B, each ſide to his correſpondent ſide, and the angle A B C is equal to the angle A C B by Suppoſition; Therefore [b] the baſe A B ſhall be equal to the baſe D C, and the whole triangle D B C, equal to the whole triangle A C B; that is, the part equal to its whole; which is abſurd; therefore A B ſhall not be unequal to A C: Therefore if two angles, &c. Which was to be demonſtrated.

b) 4. 1.

COROLLARIE.

It follows from this Propoſition, that every equiangled triangle (that is to ſay, all whoſe angles are equal to one another,) is equilateral; for ſeeing it hath been demonſtrated, that the angles B *and* C *being equal, the ſubtending ſides* A B *and* A C *ſhall be alſo equal, it may be concluded in the ſame manner, that ſeeing the angles* A *and* B *are equal, the ſubtending ſides* A C *and* C B *ſhall be alſo equal, and ſo of the reſt: Therefore the three ſides ſhall be equal, and therefore the triangle is equilateral.* As in the preceding Corollary of the Fifth Propoſition.

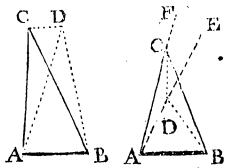
PROP. 7. THEOR. 4.



If two right lines A C *and* B C, *drawn from the extremities* A *and* B, *of one and the ſame right line* A B, *do meet in a point* C, *there cannot be drawn from the ſame extremities* A *and* B, *to another point, and on the ſame ſide, two other right lines equal to the firſt, each line to his correſpondent line.*

It

IT is most certain, that from the point A, towards C, to another point as D, a line may be drawn equal to AC, but the line drawn from B to the same point D, shall not be equal to BC: Likewise from B there may be drawn a line equal to BC, on the same side to some other point: but the line drawn from A to the same point, shall not be equal to AC: Let therefore [a] AD be drawn equal to AC, on the same side, and let BD be joyned: I say, that BD cannot be equal to BC; for having drawn CD, the figure ACD shall be an Isosceles triangle, AC and AD being equal by construction [b]: Therefore [c] the angle ACD shall be equal to the angle CDA, on the base CD; But [d] the angle BCD is lesse than the angle ACD, the part then its whole; it [e] shall be therefore also lesse than CDA, which is shewn to be equal to ACD; and therefore yet much lesse than the whole CDB: But if the line BD were also equal to the line BC, the triangle BCD would be an Isosceles triangle; and therefore [f] the angle BCD should be equal to the angle CDB, on the base CD. But BCD is shewn to be much lesse than CDB, it should be then equal, and much lesse, which is impossible: Therefore BD shall not be equal to BC. Which was to be demonstrated.



a) 2. 1.
b) 5. 1.
c) 5. 1.
d) 9. c. s.
e) 9. c. s.
f) 5. 1.

And if from the point B there be drawn a line equal to BC, to some other point then C, and on the same side, it will be shewn by the same reason, that the line drawn from the point A, to the same point, shall not be equal to AC.

Otherwise, (if it be possible,) from the extremities A and B, let there be drawn AD and BD, to another point D, towards C, equal to the first AC and BC, each to his correspondent, to wit AD, equal to AC, having the same extremity A, and BD equal to BC, having the same extremity B, and let DC be joyned; and if the point D be within the triangle, (as in the second figure;) Let AC and AD be prolonged directly towards E and F, as much as shall be expedient.

For asmuch as the line AD is put equal to AC, the angle EDC alone, shall be equal to DCB and BCF, taken for one only angle, [g] seeing they are both of them under the base DC, of the triangle ADC in the first figure, or as in the second figure: But [h] the whole angle DCF is greater than the angle DCB alone, [h] the whole, than the part: Therefore the angle EDC shall be also greater than the angle DCB; again seeing that the line BC is put equal to the line BD, the angle BCD alone shall be equal to EDC and EDB, taken for one only angle: Seeing [i] that these angles BCD, EDC, and EDB, are on the base DC of the triangle BDC: but we have shewn that the angle EDC is greater than the angle BCD: Therefore the same angle EDC, shall be also greater than the angle EDC and EDB, the part than the whole, which is impossible: Therefore, If two, &c. Which was to be demonstrated.

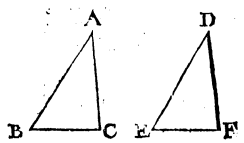
g) 5. 1.
h) 9. c. s.
h) 9. c. s.
i) 5. 1.

COROLLARIE.

Hence it follows, that having drawn two lines from the extremities of a right line to

to one and the same point, as AC and BC, if from the same extremities there be drawn two others, equal to the two first, each to his correspondent, on the same side, they will meet together in one and the same point, being inclined the one to the other, is manifest by the aforegoing figures.

PROP. 8. THEOR. 5.



If two triangles ABC, *and* DEF, *have two sides* AB *and* AC, *equal to two sides* DE *and* DF, *each to his correspondent side, and the base* BC *be equal to the base* EF, *they shall also have the angle* A, *conteined of those sides* AB *and* AC, *equal to the angle* D, *conteined of the sides* DE *and* DF.

Demonstration FOr if it be understood that the triangle ABC be put on the triangle DEF; to wit, the point B on the point E, and the base BC on the base EF, the point C will agree with the point F: [a] Seeing that BC and EF are equal, and the two right lines BA and CA will agree on the two others ED and FD, [b] being equal to them, each to his correspondent, and the point A on the point D; and therefore the angle A shall be equal to the angle D, otherwise they would meet together in another point, which is impossible; [c] Seeing that if two right lines drawn from the extremities of another right line, do meet in one point, &c. Therefore A shall be equal to D: Therefore, If two triangles, &c. Which was to be demonstrated.

a) 8. c. s.
b) 8. c. s.
c) 7. 1.

COROLLARIE.

It may be gathered from this Proposition that the other angles are also equal to the other angles, each to his correspondent, and the whole triangle, to the whole triangle; for seeing that the angle A, *is shewn to be equal to the angle* D, *the sides* AB *and* AC *being equal to* DE *and* DF, *each to his correspondent, it follows* [d] *that the triangle is equal to the triangle, and the other angles equal to the other angles, each to his correspondent.*

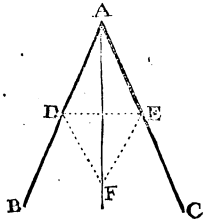
d) 4. 1.

PROP. 9. PROBL 4.

To divide a given right lined angle BAC, *into two equal parts.*

Construction LEt there be taken in the line AB, a point at pleasure D; and from the greatest line AC prolonged infinitely, let [a] there be cut off AE equal to AD, and let DE be drawn, on which [b] let there be described the equilateral triangle DFE, and let AF be drawn; I say that the angle BAC is divided into two equal angles.

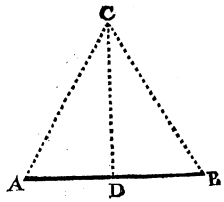
a) 2. 1.
b) 1. 1.



Demonſtration FOraſmuch as E A hath been taken equal to A D and A F is common, the two ſides D A and A F are equal to the two ſides E A and A F, each to his correſpondent, and the baſe D F equal to the baſe E F; being the ſides of an equilateral triangle; c the angle D A F ſhall be equal to the angle E A F: and therefore the angle B A C ſhall be divided into two equal parts: Therefore the right lined angle, &c. Which was to be done.

c) 8. 1.

PROP. 10. PROBL. 5.



To divide a terminated right line, given A B, *into two equal parts.*

Conſtruction ON A B a let there be deſcribed the equilateral triangle A B C, and then b let the angle A C B be divided in two equal parts by the right line C D: I ſay, that A B is divided into two equal parts in the point D.

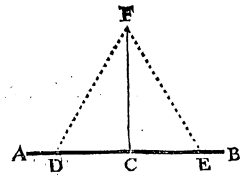
a) 1. 1.

b) 9. 1.

Demonſtration FOraſmuch as A C is equal to C B, (they being the ſides of an Equilateral triangle,) and C D is common, the two ſides A C and C D are equal to the two ſides B C and C D, each to his correſpondent, and the angle A C D equal to the angle B C D, by conſtruction; wherefore the baſe A D c ſhall be equal to the baſe B D: Therefore A B ſhall be divided in two equal parts. Which was to be done.

c) 4. 1.

PROP. 11. PROBL. 6.



On a given right line A B, *and from a given point therein* C, *to draw a right line* C F, *at right angles.*

Conſtruction IN the part A C let there be taken any point, as D, then a let C E be taken equal to C D, then on D E let there be deſcribed the equilateral triangle D F E; and let C F be drawn; b I ſay that the ſame C F is at right angles to the line A B.

a) 2, 3. 1.

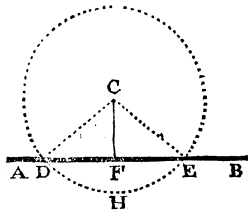
b) 5. 1.

De-

Demonſtration FOraſmuch as C D is equal to C E; and C F common, the triangles D C F and E C F ſhall have the two ſides D C and C F equal to the two ſides E C and C F, each to his correſpondent, and the baſes D F and E F equal, being the ſides of an equilateral triangle, the angles at the point C ſhall be equal; therefore both the one and the other, are right angles c: Therefore C F ſhall be at right angles to A B: Therefore on a right line, &c. Which was to be done.

c) 10. def.

PROP. 12. PROBL. 7.



On an infinite right line given A B, *and from a given point* C, *which is not in it, to draw a perpendicular line* C F.

Conſtruction LEt the contingent point H be taken on the other ſide of A B, and from the point C as a Center at the diſtance C H, let the circle D E be deſcribed, cutting A B in the points D and E; then a let D E be divided into two equal parts in the point F, and let C F, C D, and C E be drawn; I ſay that C F is a perpendicular to A B.

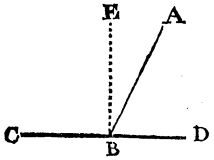
a) 10. 1.

Demonſtration FOraſmuch as D F is equal to E F, by Conſtruction, and F C common, the two ſides D F and F C of the triangle D F C, are equal to the two ſides E F and F C of the triangle E F C, each to his correſpondent, and the baſe C D equal to the baſe C E, they being both drawn from the center C to the circumference; b the angles on both ſides at the point F ſhall be equal; and c therefore both the one and the other, are right angles: Therefore C F is a perpendicular drawn to A B: Which was to be done.

b) 8. 1.

c) 10. def.

PROP. 13. THEOR. 6.



When a right line A B. *falling on a right line* C D, *maketh angles* A B D *and* A B C, *either it makes two right angles, or two angles equal to two right angles.*

Demonſtration FOr if the angle A B D be equal to A B C; they a ſhall be both right angles; if it be unequal, b let B E be drawn at right angles to C D; therefore E B D and E B C ſhall be right angles; Now foraſmuch as the two angles D B A and A B E c are equal to the right angle E B D, all the parts taken together, to their whole, if d the common angle E B C be added; then, the three angles D B A, A B E,

a) 10. def.

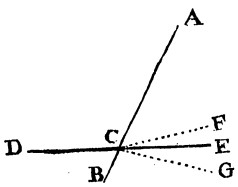
b) 11. 1.

c) 9. c. f.

d) 2. c. f.

and E B C shall be equal to the two right angles D B E and E B C: Again, forasmuch as the angle A B C is equal to the two angles A B E and E B C, the whole, to all his parts taken together, if you add the common D B A, the two angles D B A and A B C shall be equal to the three angles D B A, A B E, and E B C, but those three have been shewn to be equal to two right angles; therefore D B A and A B C shall be also equal to two right angles: Therefore when a right line, &c. Which was to be demonstrated.

PROP. 14. THEOR. 7.



If to a right line A B, *and at a point therein* C, *there be drawn two right lines* D C *and* E C, *not from the same side, making the angles* D C A *and* A C E, *on both sides, equal to two right angles, those right lines* D C *and* C E, *will directly meet with one another.*

Demonstration. For otherwise D C being prolonged towards E, would fall either above or below C E; Let it fall first of all above C E, (if possible,) as in the point F, in such sort as D C F may be a right line.

Forasmuch as the right line A C, falling on the right line D C F, makes the two angles D C A and A C F, [a] the same angles shall be right angles, or equal to two right angles. But the two angles D C A and A C E are equal to two right angles, by Supposition; and [b] all the right angles are equal to one another; therefore the two angles D C A and A C F are equal to the two angles D C A and A C E, taking away therefore the common angle D C A, [c] there will remain A C F equal to A C E, the part to its whole [d], which is impossible. The same absurdity will happen, if it shall be said that D C being prolonged, doth fall below C E, as in the point G; therefore D C and E C shall meet in a direct line. Therefore, &c. Which was to be demonstrated.

a) 13. 1.
b) 10. c. f.
c) 3. c. f.
d) 9. c. f.

COROLLARIE I.

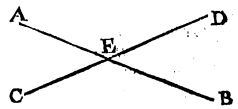
It follows from this Proposition, that two right lines indirectly meeting with one another, have not one and the same common segment; that is to say, one and the same part common as F C *and* E C, *which meet indirectly in the point* C, *the line* D C *cannot be said to be common to* C E *and* C F; *that is to say, that* D C *and* C E *making a right line* D C *and* C F *will not also make a right line, for otherwise, if as well* D C E, *as* D C F, *were a right line; (as we even now demonstrated,) the angle* H C F *should be equal to the angle* A C E, *the part to the whole, which is impossible.*

COROLLARIE II.

It follows yet that two right lines meeting with one another indirectly in a certain point

point, if they be both prolonged, they must of necessity cut one another, they being prolonged from the same part where they meet, as if the lines E C *and* F C, *which meet in the point* C, *be prolonged, to wit* E C, *it will fall directly in the point* D, *as hath been shewn; but if* F C *be also prolonged from the part of* C, *it will cut the line* E C, *prolonged at* D; *that is to say, it will fall under* C D: *for otherwise* F C *should fall on* C D; *and therefore* E C *and* F C *should have one and the same common segment, which is shewn to be impossible; and if it should be said that* F C *being prolonged, would fall above* C D, *the same absurdity as before would follow; to wit, that the part should be equal to its whole, which is impossible.*

PROP. 15. THEOR. 8.



If two right lines A B *and* C D, *do cut one another in* E, *they shall make the opposite angles at the head equal to one another,* A E D *to* C E B, *and* A E C *to* B E D.

Demonstration. Forasmuch as D E falls on A B, the two angles A E D and D E B shall be equal to two right angles; Again, forasmuch as B E falls on C D, the two angles C E B and B E D, [a] shall be equal to two right angles, and all the right angles are equal to one another: therefore the two angles A E D and D E B, [b] shall be equal to the two angles C E B and B E D; therefore if the common angle D E B be taken away; [c] there will remaine the angle A E D equal to C E B: in like manner it may be shewn that the angle A E C is equal to the angle D E B: Therefore, If two lines, &c. Which was to be demonstrated.

a) 13. 1.
b) 10. c. f.
c) 3. c. f.

COROLLARIE I.

EUCLIDE *gathers from the Demonstration of this Theorem, (according to the opinion of* Proclus) *that two right lines cutting one another, do make four angles equal to four right angles; for it hath been demonstrated, that as well the two angles* A E D *and* D E B, *as the two angles* A E C *and* C E B, *are equal to two right angles, by the* 13th. *Therefore the angles constituted at the point* E, *are equivalent to twice two right angles; and therefore are equal to four right angles.*

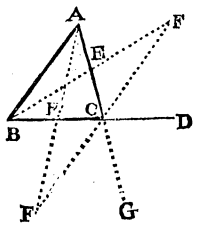
COROLLARIE II.

By the same reason we may gather that all the angles constituted at one and the same point, are only equal to four right angles: For if from the point E *in the aforegoing Figure, there be drawn as many right lines as you please, the four angles constituted in the point* E, *shall be divided into divers parts, the which are equal to their whole, [d] taken together; therefore those four being equal to four right angles, (by the first Corollary,) all their parts taken together, shall be likewise equal to four right angles; from whence it is manifest that every Space, constituted in a Plain, about some point, is equivalent to four right angles, as is manifest: forasmuch as all the angles which may be constituted about that point, are equal to four right angles: In like manner, it is manifest that any number of lines at pleasure, mutually cutting one another, will make the angles at the point of Section, equal to four right angles.*

d) Cor. 3. 1.

PROP.

PROP. 16. THEOR. 9.



Of every triangle, as A B G, *one side* B C *being prolonged, as to* D, *the exterior angle* A C D, *is greater than either of the interior and opposite angles* C A B *or* C B A.

Demonstration. FOr let the side A C be divided in two equal parts [a] in the point E, and by E let there be drawn B E F; and let [b] E F be made equal to E B, and F C joyned.

a) 10. 1.

b) 2. 1.

Forasmuch as A E is equal to E C, and E F to E B, by Construction, the two sides A E and E B of the triangle A E B, shall be equal to the two sides C E and E F of the triangle F E C, each to his correspondent, and the angles at the head E, conteined by those equal sides, [c] are equal; the base A B shall be equal to the base [d] F C, and the whole triangle to the whole triangle, and the other angles equal to the other angles, each to his correspondent, conteined by the same equal sides; to wit, E C F equal to E A B: But the whole [e] A C D is greater than E C F, it shall be therefore also greater than its equal E A B: In like manner, having divided B C into two equal parts in the point E, and drawn A E F, in such sort as E F may be equal to E A and C F joyned, and the side A C prolonged to the point G, it will appear that the exteriour angle B C G is greater than the interiour and opposite angle A B C: and therefore A C D which is [f] equal to B C G, shall be also greater than A B C: Therefore, Of every triangle one side, &c. Which was to be demonstrated.

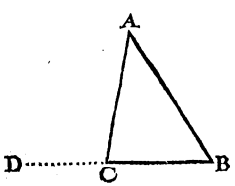
c) 15. 1.

d) 4. 1.

e) 9. c. f.

f) 15. pet.

PROP. 17. THEOR. 10.



Of every triangle as A B C, *two angles are lesse than two right angles, after what manner so ever they be taken.*

Demonstration. FOr first of all the side B C being prolonged to the point D, it will appear that the two angles A B C and A C B, are lesse than two right angles.

a) 16. pet.

Forasmuch as the exteriour angle A C D is greater [a] than the opposite and interiour angle B, if the common angle A C B be added, [b] the two angles A C D and A C B, will be greater than B and A C B; but A C B and A C D are equal [c] to two right angles: Therefore B and A C B are lesse than two right angles: In like manner, the side C A being prolonged to

b) 4. c. f.

c) 13. pet.

to the point E, it will appear that C A B and A B C are lesse than two right angles, &c. Therefore, Of every triangle, &c. Which was to be demonstrated.

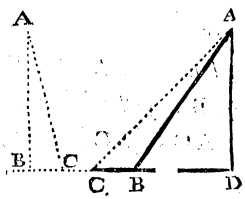
COROLLARIE I.

From these Demonstrations, it follows also that every triangle that hath one angle either right or obtuse, hath the other angles acute: for seeing it is apparent that any two angles of every triangle are lesse than two right angles; it is necessary that if one angle be right or obtuse, that all the other angles be acute, otherwise two angles of a triangle, should be right angles, or greater than two right angles.

COROLLARIE II.

It follows also from this Proposition, that if a right line make two unequal angles with another right line, the one obtuse, the other acute, that the perpendicular line drawn from any point thereof on the other line, will fall on that side where the acute angle is:

Let A B *with* C D *make two unequal angles, to wit,* A B D *acute, and* A B C *obtuse, and from any point, as from* A, *let* A D *be drawn perpendicular to* C B: *I say that* A D *will fall on the side of the acute angle* A B D; *Otherwise, Let it fall (if possible) on the side of the obtuse angle* A B C, *and let it be* A C, *therefore the two angles, to wit, the obtuse* A B C, *and the right angle* A C B *of the triangle* A B C, *shall be greater than two right angles, which is absurd, being they are [d] lesse than two right angles; therefore the perpendicular drawn from the point* A, *on* A B, *shall not fall on the side of the obtuse angle; therefore it shall fall on the side of the acute angle.*



d) 17. 1.

COROLLARIE III.

Likewise it is manifest from this Proposition, that all the angles of an Equilateral triangle, and the two angles on the base of an Isosceles triangle are acute: For seeing that [e] any two angles of an Equilateral triangle, and the two upper angles on the base of an Isosceles triangle are equal to one another; and as well the first two [f] as the last two, are lesse than two right angles; each of them shall be lesse than a right angle that is to say, it shall be acute, for if one of them were a right or obtuse angle, the two should be equal to two right angles, or greater than two right angles.

e) 5. 1.

f) 17. 1.

PROP. 18. THEOR. 11.

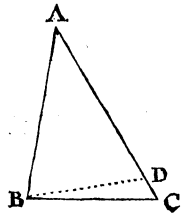
Of every triangle, as A B C, *the greatest side* A C, *subtendeth the greatest angle* A B C.

Demonstration. FOr seeing that A C is greater than A B, let [a] A D be cut off equal to A B, and let B D be joyned; forasmuch as A B and A D are equal, the two angles [b] A B D and A D B shall be equal; but the exterior angle A D B is greater [c] than the interior and opposite angle

a) 3. 1.

b) 5. 1.

c) 16. 1.

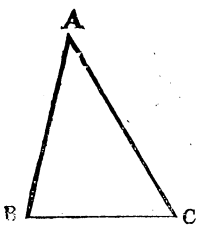


angle C: Therefore its equal ABD, ſhall be alſo greater than C: and therefore the whole ABC yet much greater than the angle C. By the ſame reaſon it will appear that the angle C is greater than the angle A: Therefore, Of every triangle, &c. Which was to be demonſtrated.

COROLLARIE.

It follows from this Propoſition, that the three angles of a Scalenum triangle are unequal: For let the triangle ABC *be a Scalenum, whoſe greateſt ſide let be* AC, *the leaſt* BC, *and the mean* AB; *I ſay, that all the angles of that triangle are unequal: For ſeeing that the ſide* AC *is put greater than the ſide* AB, *by this Propoſition the angle* B *ſhall be greater than the angle* C, *by the ſame reaſon, the angle* C *ſhall be greater than the angle* A; *foraſmuch as the ſide* AB, *is put greater than the ſide* BC: *Therefore the three angles are unequal:* B *the greateſt,* C *the mean, and* A *the leaſt.*

PROP. 19. THEOR. 12.



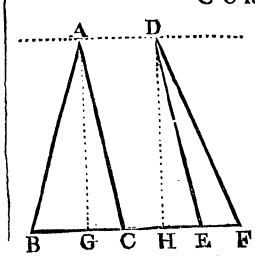
Of every triangle as ABC, *the greateſt angle* B *is ſubtended by the greateſt ſide* AC, (AC *is greater than* AB.)

Demonſtration FOr if it be denied, it ſhall be equal, or leſſe: if equal, the angle B [a] ſhall be equal to the angle C, which is abſurd; for B is put greater than C, if the ſide AC be ſaid to be leſſe than AB; the angle B ſubtended of the leaſt ſide AC, ſhall be leſſe than the angle C: ſubtended of the greateſt ſide AB [b], which is yet more abſurd: for the angle B is greater than the angle C, by ſuppoſition: Therefore the ſide AC ſhall be greater than AB: Likewiſe it will appear, that the ſide AC is greater than the ſide BC, putting the angle B greater than the angle A: Therefore, Of every triangle, &c. Which was to be demonſtrated.

a) 5. 1.

b) 18. 1.

COROLLARIE.



From this Propoſition it follows, that of all the right lines drawn from one and the ſame point, on a right line, that line which is perpendicular, is the leaſt.

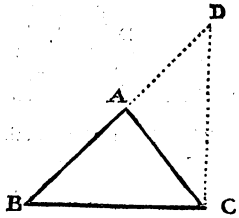
For, Let the lines AB, AG, *and* AC, *in the firſt triangle, be drawn from the point* A, *on the line* BC, *of which let* AG *be perpendicular to* BC: *I ſay, that* AG *is the leaſt: For ſeeing that in the triangle* ABG, *the two angles* AGB *and* ABG *are* [c] *leſſe than two right angles, and* AGB *is a right angles*

c) 17. 1.

ABG

A B G *shall be leſſe than a right angle*; *that is to ſay acute*; *therefore* d A B *ſhall be greater than* A G, *and in this manner, it might be demonſtrated that each of the lines in the other triangle*; *namely, the triangle* D E F, *is greater than the perpendicular* D H: *Therefore the perpendicular* A G *ſhall be the leaſt.* d) 19. 1.

PROP. 20. THEOR. 13.

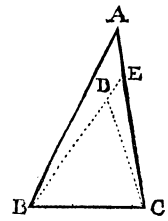


Of every triangle as A B C, *two ſides are greater than the other; after what manner ſoever they be taken* (*that is*, B A *and* A C *taken together*, *are greater than* B C; *likewiſe* A C *and* C B *are greater than* A B; *and laſtly*, A B *and* B C *are greater than* A C.)

Demonſtration LEt the ſide B A be prolonged towards D, a in ſuch ſort as A D may be put equal to A C, and let C D be drawn: Foraſmuch as the two ſides A D and A C are made equal, the two angles b on the baſe C D, to wit D, and A C D, ſhall be equal: but the angle B C D c is greater than the angle A C D, it ſhall be therefore alſo greater than D; therefore in the triangle B C D the ſide B D oppoſite d to the greateſt angle B C D, ſhall be greater than the ſide B C, oppoſite to the leaſt angle D; but the ſide B D is equal to the two ſides A C and A B: (for ſeeing that A D is equal to A C, if the common ſide e A B be added, the whole B A D ſhall be equal to A C and A B:) therefore the two ſides A B and A C together, ſhall be greater than the other ſide C B: In like manner, it may be demonſtrated, that two ſides (which you pleaſe,) of any triangle, are greater than the other ſide: Therefore, Of every triangle, &c. Which was to be demonſtrated.

a) 2. 1.
b) 5. 1.
c) 9. c. f.
d) 19. 1.
e) 2. c. f.

PROP. 21. THEOR. 14.



If on one of the ſides B C *of a triangle* A B C, *and from the extremities* B *and* C, *are drawn two right lines* B D *and* C D *within*, *meeting in a point* D, *thoſe lines* B D *and* C D *ſhall be leſſe than the two other ſides* B A *and* C A *of the triangle* B A C: *But they ſhall contein a greater angle*, (*the angle* B D C *is greater than the angle* B A C.

Demonſtration FOr let B D be prolonged to the point E of the ſide A C: foraſmuch as in the triangle B A E, the two ſides B A and A E, [a] are greater than the other B E, if the common ſide C E be added, [b] the ſides B A and A C ſhall be greater than the ſides B E and E C.

a) 20. 1. b) 4. c. ſ.

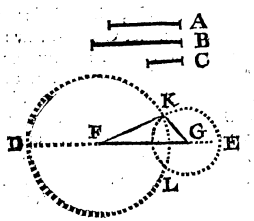
Again, foraſmuch as, in the triangle C E D, the two ſides C E and E D are [c] greater than the other C D, if the common ſide D B be added, the two ſides C E and E B, [d] ſhall be greater than C D and D B; but B A and A C have been ſhewn to be greater than B E and E C; therefore B A and A C ſhall be yet greater than B D and C D, which was in the firſt place propoſed.

c) 20. 1. d) 4. c. ſ.

Secondly, foraſmuch as the exterior angle B D C is [e] greater than the interior angle D E C, and the ſame D E C is greater than B A C, the angle B D C ſhall be much greater than the angle B A C: which was in the ſecond place propoſed: Therefore, If on one of the ſides, &c. Which was to be demonſtrated.

e) 16. 1.

PROP. 22. PROBL. 8.



Of three right lines F K, G K, *and* F G, *equal to three given right lines,* A, B, *and* C; *to conſtitute a triangle* F G K: *But two of them, after what manner ſoever they are taken, ought to be greater than the other; foraſmuch as of every triangle, two ſides after what manner ſoever they be taken, are greater than the other ſide.*

Conſtruction LEt there be taken ſome right line, as D E, prolonged infinitely towards E, and of the ſaid line D E. Let there be taken D F equal to [a] A and F G equal to B; and laſtly G E equal to C, and from the point F as a center at the ſpace F D. Let the circle D K be deſcribed againe from the center G, and with the ſpace or diſtance G E, let the circle E K be deſcribed; and let F K and G K be drawn: I ſay, that the triangle F G K is made of three lines equal to the three given lines A, B, and C.

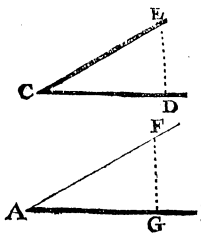
a) 3. 1.

Demonſtration FOraſmuch as the point F is the center of the circle D K, the line [b] F K ſhall be equal to F D, by the conſtruction; therefore F K and A ſhall be [c] equal to one another: Again, foraſmuch as G is the center of the circle E K, the line G K ſhall be equal to G E, by the ſame reaſon: But the line C is equal to G E by the conſtruction: therefore G K and C ſhall be equal, and F G is taken equal to B: Therefore the three right lines F K, F G, and G K, are equal to the three given right lines A, B, and C, &c. Which was to be done.

b) 15. def. 1. c) 1. c. ſ.

PROP.

PROP. 23. PROBL. 9.



To a given right line A B, *and at a point therein* A, *to conſtitute a right lined angle* F A G, *equal to a given right lined angle* D C E.

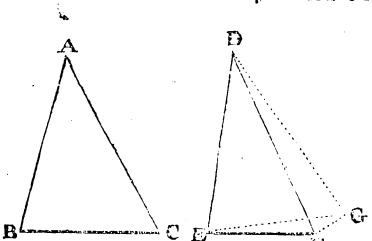
Conſtruction LEt the contingent points C and E be taken in both the one and the other lines D C and D E; and let C E be drawn; then of three lines equal to the right lines D C, C E, and E D, [a] let there be conſtituted the triangle A F G; in ſuch ſort, as A F be put equal to D C, F G to D E, and A G to C E.

a) 22. 1.

Demonſtration FOraſmuch then as the two ſides A F and A G of the triangle A F G, are equal to the two ſides C D and C E, of the triangle C D E, each to his correſpondent, and the baſe F G equal to the baſe E D; the angle F A G ſhall be [b] equal to the angle C D E: Therefore to a given right line, &c. Which was to be done.

b) 8. 1.

PROP. 24. THEOR. 15.



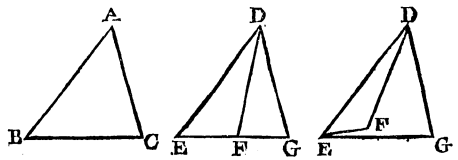
If two triangles A B C, *and* D E F, *have two ſides* A B *and* A C *equal to two ſides* D E *and* D F, *each to his correſpondent, and the angle* A *conteined of thoſe equal ſides* A B *and* A C, *greater than the angle* E D F, *they ſhall have the baſe* B C *greater than the baſe* E F.

Demonſtration FOr ſeeing that the angle A is greater than the angle E D F; at the line E D, and at a point therein D, let there be conſtituted the angle [a] E D G, equal to the angle A; and let D G be taken, equal to one of the two A C or D F; and let E G and F G be drawn: Foraſmuch then as the triangles A B C and E D G have the two ſides A B and A C equal to the two ſides D E and D G each to his correſpondent, and the angle A, equal to the angle E D G, conteined of the ſame equal ſides, [b] the baſe B C ſhall be equal to the baſe E G: Again,

a) 23. 1. b) 5. 1.

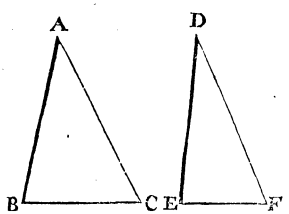
foraſmuch as D G is equal to D F, the triangle D G F is an Iſoſceles tri-
c) 9. c. ſ. angle: and therefore c the two angles D G F and D F G, on the baſe,
d) 19. 1. ſhall be equal: But the angle D G F is d greater than the angle E G F: therefore D F G equal thereto, ſhall be alſo greater than E G F, and therefore the whole E F G, yet greater than E G F: Therefore in the triangle E F G the ſide E G which doth ſubtend the greateſt angle E F G, ſhall be greater than E F, which ſubtendeth the leſſer angle E G F: But B C is ſhewn to be equal to E G: Therefore B C ſhall be alſo greater than E F.

e) 4. 1. Secondly, Let the baſe E G fall on E F, it e ſhall be equal to the baſe
f) 9. c. ſ. B C, as before; but E G is greater than F E, the whole than part thereof: Therefore B C f ſhall be alſo greater than E F.



g) 21. 1. Laſtly, let E G fall under E F; foraſmuch as the two ſides D G and
h) 5. c. ſ. E G are g greater than the two ſides D F and F E, and D F is made equal to D G; h taking away the equal ſides D G and D F, there will remaine E F leſſe than E G or B C, which we have ſhewn to be equal to E G: Therefore, If two triangles, &c. Which was to be demonſtrated.

PROP. 25. THEOR. 16.

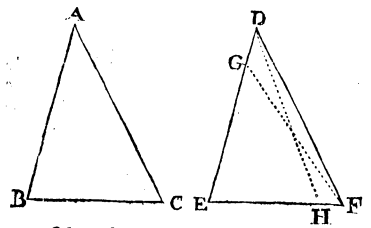


If two triangles A B C *and* D E F, *have two ſides* A B *and* A C, *equal to two ſides* D E *and* D F, *each to his correſpondent, and that they have the baſe* B C *greater than the baſe* E F, *they will have the angle* A, *conteined by the ſame equal right lines* A B *and* A C, *greater than the angle* D, *conteined by the equal right lines* D E *and* D F.

Demonſtration FOr if A were not greater, it ſhould be equal or leſſe, if equal, the ſides A B and A C being equal to the two ſides
a) 4. 1. D E and D F, each to his correſpondent; a the baſe B C ſhould be equal to the baſe E F, which is abſurd, for B C is put greater than E F: If leſſe, A B and A C being equal to D E and E F, each to his correſpondent;
b) 24. 1. B C ſhould be b leſſe than E F, which is more abſurd, for it is greater by

by ſuppoſition: Therefore ſeeing that it cannot be either equal, or leſſe, it ſhall be greater. Therefore, If two triangles, &c. Which was to be demonſtrated.

PROP. 26. THEOR. 17.



If two triangles A B C, *and* D E F, *have two angles* B *and* C, *equal to two angles* E, *and* D F E, *each to his correſpondent, and a ſide equal to a ſide; that is to ſay, either* B C *equal to* E F, *that which is adjacent to thoſe equal angles* B *and* C, *or elſe* A B *and* D E, *that which ſubtendeth one of thoſe equal angles* C *and* D F E, *they will have the other ſides* A C *and* C B, *equal to the other ſides* D F *and* F E, *each to his correſpondent, and the other angle* A, *equal to the other angle* D.

Demonſtration FOr if A B be not equal to D E, one of them ſhall be greater, which let be D E, (if poſſible,) from which let G E be cut off a equal to A B, and let G F be drawn; Foraſmuch as, the two a) 3. 1.
ſides A B and B C of the triangle A B C, are equal to the two ſides G E and E F of the triangle G E F, each to his correſpondent, and the angle B equal to the angle E, by ſuppoſition, the baſe b G F ſhall be equal to the b) 4. 1.
baſe A C, and all the triangle G E F, equal to all the triangle A B C, and the angle G F E equal to the angle C; but the angle D F E, is equal to the angle C by ſuppoſition; therefore the angle G F E ſhall be in like manner equal to the angle D F E, the part to its whole, which is abſurd: therefore the ſide A B is not equal to D E: Therefore ſeeing that the two ſides A B and B C are equal to the two ſides D E and E F, each to his correſpondent, and the angles B and E conteined of thoſe ſides, alſo equal, the baſe c A C ſhall be equal to the baſe D F, and the other angles A and c) 4. 1.
alſo equal, which was propoſed.

Secondly, Let the ſides A B and D E, ſubtending the equal angles C and D F E, be equal; I ſay again, that the other ſides A C and C B are equal to the other ſides D F and F E, each to his correſpondent, B C to E F, and A C to D F, and the other angle A equal to the other angle D; For if B C be not equal to E F: Let one of them, to wit E F, be the greater, from d which let E H be taken, equal to B C, and let D H be d) 3. 1.
drawn.

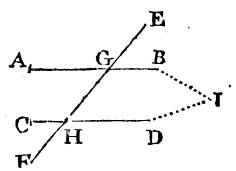
Foraſmuch then as the two ſides A B and B C, are equal to the two ſides D E and E H, and the angles B and E conteined of them, being put equal,

c) 4. 1. equal, the baſe is equal to the baſe, and the angle D H E [c] equal to the angle C: therefore the angle D H E ſhall be equal to the ſame angle D F E, the exterior angle to its interior and oppoſite angle, which is f) 16. 1. [f] abſurd. Therefore B C is not unequal to E F: Therefore by the fourth Propoſition of this Book it may be gathered as before, that the two other ſides, are equal to the two other, and the other angle to the other angle: Therefore, If two triangles, &c. Which was to be demonſtrated.

COROLLARIE.

It is eaſie to gather from this Propoſition, that the whole triangle is equal to the whole triangle, for by conſequence, as the two ſides A B *and* B C *are equal to the two* D E *and* E F, *each to his correſpondent, containing thoſe equal angles, the whole triangle is alſo equal to the whole triangle; which hath been already demonſtrated.*

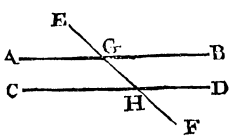
PROP. 27. THEOR. 18.



If a right line E F, *falling on two other right lines* A B *and* C D, *makes the alternate angles* A G H *and* G H D, *equal to one another; thoſe right lines* A B *and* C D, *ſhall be parallel to one another.*

Demonſtration. FOr if they be not parallel, they being infinitely prolonged on both parts, they will meet either on the part of A C or of B D, otherwiſe they ſhall be parallels; Let them meet then (if poſſible,) on the part of B D, as in the point I; and ſo G I H ſhall be a triangle, a) 16. 1. whoſe exterior angle [a] A G H ſhall be greater than the interior and oppoſite G H D, which is abſurd; for A G H is equal to G H D, by Suppoſition: Therefore A B and C D ſhall not meet, and [b] therefore they ſhall be b) 35. def. parallels: In like manner, it might be demonſtrated alſo that they can never meet on the part of A C: Therefore, If a right line, &c. Which was to be demonſtrated.

PROP. 28. THEOR. 19.



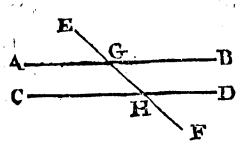
If a right line E F *falling on two right lines* A B *and* C D, *doth make the exterior angle* A G E, *equal to the interior and oppoſite angle* G H C, *and on the ſame part, or the interior angles on the ſame part,* A G H *and* C H G, *equal to two right angles; thoſe right lines* A B *and* C D *ſhall be parallel to one another.*

Demon-

Demonſtration. FOr aſmuch as the angle G H C is equal to the angle F G A by ſuppoſition; and that the angle B G H [a] is equal to the a) 15. 1. ſame angle E G A, being both oppoſites at the head G; the two angles B G H and G H C, which are alternate and oppoſite angles, [b] ſhall be b) 1. c. ſ. alſo equal to one another: Therefore [c] A B and C D ſhall be parallel to c) 27. 1. one another.

Secondly, If E F falling on A B and C D doe make the interior angles, and on the ſame part; to wit A G H and C H G, equal to two right angles: I ſay, that A B and C D are alſo parallel: For ſeeing that A G E and A G H [d] are equal to two right angles, and that the interior d) 13. 1. angles A G H and C H G are put equal to two right angles, the angles A G E and A G H ſhall be equal to the two angles A G H and C H G, taking away therefore the common angle A G H, there will remaine the 3. c. ſ. exterior angle, equal to the interior and oppoſite angle on the ſame part G H C: Therefore A B and C D ſhall be parallels by the firſt part of this Propoſition: Therefore, If a right line, &c. Which was to be demonſtrated.

PROP. 29. THEOR. 20.



If a right line E F *doth fall on two parallel right lines* A B *and* C D, *it will make the alternate angles* A G H *and* D H G *equal to one another, and the exterior angle* E G B, *equal to its interior and oppoſite angle* G H D, *and on the ſame part; and the two interior angles* B G H *and* G H D, *and on the ſame part, equal to two right angles.*

Demonſtration. FOr if they be not equal, let the one of them, to wit A G H, be the greater; foraſmuch then as the angle A G H is greater than the angle D H G; if the common angle B G H be added, [a] the two angles A G H and B G H, ſhall be greater than the two angles a) 4. c. ſ. B G H and D H G; but the two angles A G H and B G H [b] are equal to b) 13. 1. two right angles: Therefore B G H and D H G ſhall be leſſe than two right angles; therefore A B and C D ſhall not be parallels, [c] which is ab- c) 11. c. ſ. ſurd: for that they are put parallels: Therefore the angle A G H is not greater than the angle D H G, and therefore is equal thereto; if it were ſaid to be leſſe, the ſame abſurdity would happen.

Secondly, I ſay that the exterior angle E G B is equal to its interior and oppoſite angle, and on the ſame part G H D: Foraſmuch as the angle G H D is ſhewn to be equal to the angle A G H, and that the angle E G B is [d] equal to the ſame angle A G H, the angles E G B and G H D ſhall d) 15. 1. [e] be equal to one another; and ſo it may be ſhewn that E G A is equal e) 1. c. ſ. to G H C.

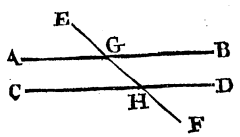
Thirdly, I ſay that the two interior angles, and on the ſame part, B G H and G H D, are equal to two right angles.

For

For ſeeing that the exterior angle E G B is ſhewn to be equal to G H D, if the common angle B G H be added; f the two angles B G H and G H D ſhall be equal to the two angles E G B and B G H; g but E G B and B G H are equal to two right angles: Therefore B G H and G H D ſhall be alſo equal to two right angles, by the ſame reaſon, the two angles A G H aud G H C will appear to be equal to two right angles: Therefore, &c. Which was to be demonſtrated.

f) 3. c. ſ.
g) 13. pet.

Otherwiſe, Let A B and C D be parallels: I ſay that the alternate angles A G H and H G D are equal to one another: If it be not ſo, the one of them ſhall be the greater: Let then A G H be the greater (if poſſible.) Wherefore adding the angle H G B, the two angles H G B and D H G ſhall be leſſe than the two angles H G B and A G H; but H G B and A G H are equivalent to two right angles; therefore H G B and D H G ſhall be leſſe than two right angles; therefore A B and C D ſhall not be parallel, which is contrary to Suppoſition: Therefore the angle A G H is equal to the angle G H D.

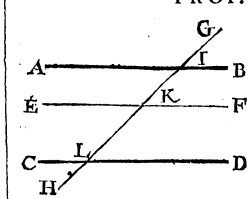


Secondly, The exterior angle E G B is equal to the interior and oppoſite angle, and of the ſame part G H D; for the angle E G B is equal alſo to the angle A G H, and the angle G H D is ſhewn to be equal to the angle A G H: Therefore the angle E G B ſhall be equal to the angle G H D, the exterior to the interior angle.

Laſtly, The two interior angles, and on the ſame part, B G H and G H D are equal to two right angles: For ſeeing that the angle E G B is ſhewn to be equal to the angle G H D, if the angle B G H be added to each of them, the two angles G H D and B G D ſhall be equal to the two angles E G B and B G H: But the angles E G B and B G H are equal to two right angles; therefore the angles B G H and D H G ſhall be alſo equal to two right angles: Which was to be demonſtrated.

PROP. 30. THEOR. 21.

The right lines A B *and* C D, *parallel to one and the ſame right line* E F, *are alſo parallel to one another.*



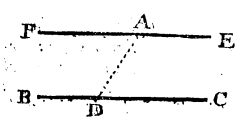
Demonſtration. LEt G H be drawn, cutting the ſame right lines in the points I, K, and L; foraſmuch as the right line G H doth fall on the parallels A B and E F, the angle A I K ſhall be equal a to the alternate angle F K I; Again, foraſmuch as G H falls on the parallels C D and E F, the angle D L K ſhall be equal to the angle I K F, the interior, to the exterior angle, or the exterior to the interior and oppoſite on the ſame part. But A I K is equal to the ſame I K F; therefore A I K b and D L K ſhall be equal to one another, which being alternate angles, c the lines A B and C D ſhall be parallel to one another: Therefore, The right lines, &c. Which was to be demonſtrated.

a) 29. 1.
b) 1. c. ſ.
c) 27. 1.

PROP.

PROP. 31. PROBL. 10.

From a given point A, *to draw a right line parallel to a given right line* B C.



Conſtruction. LEt the contingent point D be taken in the given line B C; and let A D be drawn, making any angle at pleaſure with B C, as the angle A D C, and in the point A of the given right line A D, let there be conſtituted the a right lined angle F A D, equal to the given right lined angle A D C, and let F A be prolonged directly towards E.

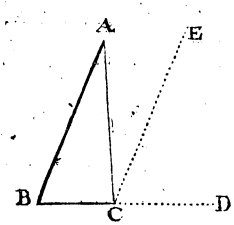
a) 23. 1.

Demonſtration. FOraſmuch as the two alternate angles F A D and C D A are equal by conſtruction, the line A E b ſhall be parallel to B C, and drawn from the given point A. Which was to be done.

b) 27. 1.

PROP. 32. THEOR. 22.

Of every triangle, as A B C, *one of the ſides* B C *being prolonged, the exterior angle* A C D *is equal to the two interior and oppoſite angles* A *and* B, *and the three interior angles* A, B, *and* A C B, *of any triangle, are equal to two right angles.*



Conſtruction. FOr from the point C let C E be drawn parallel to A B.

Demonſtration. FOraſmuch as A C doth fall on the parallels A B and C E, the alternate angles a A and A C E are equal; Again, foraſmuch as B D falls on the Parallels A B and C E, the angles B and E C D ſhall be b equal, the exterior angle to the interior and oppoſite angle on the ſame part: Therefore the two angles A C E and E C D; that is to ſay, the whole A C D ſhall be equal to the two angles A and B, the exterior angle to the two interior and oppoſite angles; Which was propoſed.

a) 29. 1.
b) 29. 1.

Secondly, I ſay that the three interior angles of the ſame triangle A B C; to wit, A, B, and A C B, are equal to two right angles.

Foraſmuch as A and B are equal to the angle A C D, as hath been ſhewn, if the common angle A C B be added, the three angles c A, B, and A C B, ſhall be equal to the two angles A C B and A C D; but the two angles A C B and A C D are equal to two d right angles; therefore

c) 2. c. ſ.
d) 13. 1.

the three angles A, B, and A C B are equal to two right angles: Therefore of every triangle, &c. Which was to be demonstrated.

COROLLARIE I.

From this 32 Proposition may be gathered that the three angles of any triangle taken together, are equal to three angles taken together, of any other triangle: forasmuch as the three angles of the one, as the three angles of the other, are equal c) 32. 1. *e to two right angles. Therefore if two angles of one triangle are equal to two angles of another triangle, the third angle of the one shall be also equal to the third angle of the other. Therefore if two angles of one triangle are equal to two angles of another triangle, each to his correspondent, the other angle of the one shall be equal to the other angle of the other, and those triangles shall be equiangled.*

COROLLARIE II.

It is also evident, that in every Isosceles triangle, whose angle conteined of the equal sides is a right angle, that each of the other angles which are on the base is the f) 32. 1. *half of a right angle, for both of them together do constitute a right angle: f Seeing the three angles are equal to two right angles, and that the third angle is put a right angle; therefore seeing that the two remaining angles are equal to one another, each of them shall be the half of a right angle: But if the angle conteined of the equal sides be obtuse, each of the other angles shall be lesse than half of a right angle, and the two angles together, lesse than a right angle; If lastly, the above mentioned angle be acute, each of the other angles shall be greater than half a right angle: forasmuch as the two together, are greater than a right angle.*

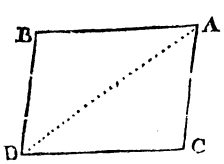
COROLLARIE. III.

It is likewise manifest, that each angle of an equilateral triangle, is the two g) 32. 1. *third parts of a right angle, for the two right angles, g to which the three angles of an equilateral triangle are equal, being divided into three angles, do make each angle the two third parts of a right angle.*

COROLLARIE IV.

It is also evident, that if a perpendicular line be drawn from one of the angles of an equilateral triangle on the opposite side there will be constituted two Scalene triangles, each of which shall have the angle adjacent to the perpendicular, a right angle, another shall be the two thirds of a right angle, to wit, that which is also an angle of the equilateral triangle: Lastly, the other angle shall be the third part of a right angle.

PROP. 33. THEOR. 23.



The right lines B D *and* A C, *which joyn together two right lines* B A *and* D C *equall and parallel, and on the same part, the same right lines* B D *and* A C *are in like manner equal and parallel.*

De-

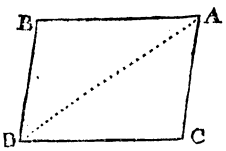
Demonstration FOr let A D be drawn, forasmuch as A D falls on the parallels B A and C D, a the alternate angles B A D and A D C shall be equal to one another, and forasmuch as B A is equal to D C by supposition, and A D common, the two sides B A and A D of the triangle A B D shall be equal to the two sides D C and D A of the triangle D C A, each to his correspondent, and the angle B A D equal to the angle C D A conteined of those equal sides, the base b B D shall be equal to the base A C, and the whole triangle A B D shall be equal to the whole triangle D C A, and the other angles equal to the other angles, each to its correspondent; to wit, C equal to B, and D A C equal to B D A, which being alternate angles, c the lines B D and A C shall be parallel: But they are also shewn to be equal: Therefore they shall be equal and parallel: Therefore, &c. Which was to be demonstrated.

a) 29. 1.

b) 4. 1.

c) 27. 1.

PROP. 34. THEOR. 24.



In Parallelograms the opposite sides A B *and* C D, *and the opposite angles* D B A *and* A C B, *are equal to one another, and their Diameter* A D *doth divide them into two equal parts.*

Demonstration FOr seeing that B A and D C are parallels, and that A D falleth on them, a the alternate angles B A D and A D C are equal to one another; Again, seeing that B D and A C are parallels, the alternate angles B D A and C A D are also equal: Therefore seeing that the two angles B A D and B D A of the triangle A B D, are equal to the two angles C A D and C D A of the triangle A C D, each to his correspondent, and the side adjacent to those equal angles, common to both the triangles, b the two sides B D and B A, shall be equal to the two other sides D C and A C, each to his correspondent: to wit, B A equal to its opposite D C, and B D equal to its opposite A C; and the angle B equal to the angle C: And forasmuch as the two angles at the point A are shewn to be equal to the two angles at the point D, the whole angle A shall be equal to its opposite and whole angle D: Therefore the sides and the angles opposite to one another, are equal.

a) 29. 1.

b) 26. 1.

But forasmuch as in the triangle A B D and D C A, the two sides B A and B D are equal to the two sides C D and C A, each to its correspondent, and the angles B and C, conteined of those sides, are equal, as hath been shewn, c the whole triangle B A D shall be equal to the whole triangle A C D: Therefore the Diameter doth divide the Parallelogram into two equal parts: Therefore the sides and the angles, &c. Which was to be demonstrated.

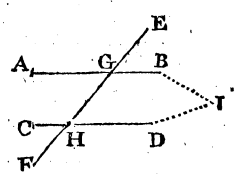
c) 4. 1.

c) 4. 1. equal, the baſe is equal to the baſe, and the angle D H E c equal to the angle C: therefore the angle D H E ſhall be equal to the ſame angle D F E, the exterior angle to its interior and oppoſite angle, which is f) 16. 1. f abſurd. Therefore B C is not unequal to E F: Therefore by the fourth Propoſition of this Book it may be gathered as before, that the two other ſides, are equal to the two other, and the other angle to the other angle: Therefore, If two triangles, &c. Which was to be demonſtrated.

COROLLARIE.

It is eaſie to gather from this Propoſition, that the whole triangle is equal to the whole triangle, for by conſequence, as the two ſides A B *and* B C *are equal to the two* D E *and* E F, *each to his correſpondent, containing thoſe equal angles, the whole triangle is alſo equal to the whole triangle; which hath been already demonſtrated.*

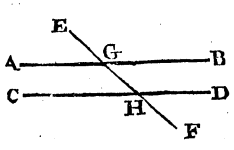
PROP. 27. THEOR. 18.



If a right line E F, *falling on two other right lines* A B *and* C D, *makes the alternate angles* A G H *and* G H D, *equal to one another; thoſe right lines* A B *and* C D, *ſhall be parallel to one another.*

Demonſtration IF or if they be not parallel, they being infinitely prolonged on both parts, they will meet either on the part of A C or of B D, otherwiſe they ſhall be parallels; Let them meet then (if poſſible,) on the part of B D, as in the point I; and ſo G I H ſhall be a triangle, a) 16. 1. whoſe exterior angle a A G H ſhall be greater than the interior and oppoſite G H D, which is abſurd; for A G H is equal to G H D, by Suppoſition: Therefore A B and C D ſhall not meet, and b) 35. def. b therefore they ſhall be parallels: In like manner, it might be demonſtrated alſo that they can never meet on the part of A C: Therefore, If a right line, &c. Which was to be demonſtrated.

PROP. 28. THEOR. 19.



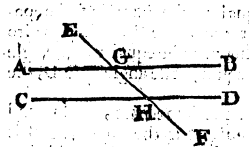
If a right line E F *falling on two right lines* A B *and* C D, *doth make the exterior angle* A G E, *equal to the interior and oppoſite angle* G H C, *and on the ſame part, or the interior angles on the ſame part,* A G H *and* C H G, *equal to two right angles; thoſe right lines* A B *and* C D *ſhall be parallel to one another.*

Demon-

Demonſtration FOr aſmuch as the angle G H C is equal to the angle F G A by ſuppoſition; and that the angle B G H a) 15. 1. is equal to the ſame angle E G A; being both oppoſites at the head G; the two angles B G H and G H C, which are alternate and oppoſite angles, b) 1. c. ſ. ſhall be alſo equal to one another: Therefore c) 27. 1. A B and C D ſhall be parallel to one another.

Secondly, If E F falling on A B and C D, doe make the interior angles, and on the ſame part; to wit A G H and C H G, equal to two right angles: I ſay, that A B and C D are alſo parallel: For ſeeing that A G E and A G H d) 13. 1. are equal to two right angles, and that the interior angles A G H and C H G are put equal to two right angles, the angles A G E and A G H ſhall be equal to the two angles A G H and C H G, taking away therefore the common angle A G H, 3. c. ſ. there will remaine the exterior angle, equal to the interior and oppoſite angle on the ſame part G H C: Therefore A B and C D ſhall be parallels by the firſt part of this Propoſition: Therefore, If a right line, &c. Which was to be demonſtrated.

PROP. 29. THEOR. 20.



If a right line E F *doth fall on two parallel right lines* A B *and* C D, *it will make the alternate angles* A G H *and* D H G *equal to one another, and the exterior angle* E G B, *equal to its interior and oppoſite angle* G H D, *and on the ſame part; and the two interior angles* B G H *and* G H D, *and on the ſame part, equal to two right angles.*

Demonſtration IF they be not equal, let the one of them, to wit A G H, be the greater, foraſmuch then as the angle A G H is greater than the angle D H G, if the common angle B G H be added, a) 4. c. ſ. a the two angles A G H and B G H, ſhall be greater than the two angles B G H and D H G; but the two angles A G H and B G H b) 13. 1. are equal to two right angles: Therefore B G H and D H G ſhall be leſſe than two right angles; therefore A B and C D ſhall not be parallels, c) 11. c. ſ. which is abſurd: for that they are put parallels: Therefore the angle A G H is not greater than the angle D H G, and therefore is equal thereto; if it were ſaid to be leſſe, the ſame abſurdity would happen.

Secondly, I ſay that the exterior angle E G B is equal to its interior and oppoſite angle, and on the ſame part G H D: Foraſmuch as the angle G H D is ſhewn to be equal to the angle A G H, and that the angle E G B is d) 15. 1. d equal to the ſame angle A G H, the angles E G B and G H D ſhall c) 1. c. ſ. e be equal to one another, and ſo it may be ſhewn that E G A is equal to G H C.

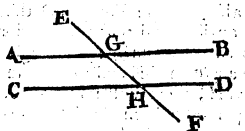
Thirdly, I ſay that the two interior angles, and on the ſame part, B G H and G H D, are equal to two right angles.

For

For seeing that the exterior angle EGB is shewn to be equal to GHD, if the common angle BGH be added; f the two angles BGH and GHD shall be equal to the two angles EGB and BGH; g but EGB and BGH are equal to two right angles: Therefore BGH and GHD shall be also equal to two right angles, by the same reason; the two angles AGH and GHC will appear to be equal to two right angles: Therefore, &c. Which was to be demonstrated.

f) 3. c. f.
g) 13. per.

Otherwise; Let AB and CD be parallels: I say that the alternate angles AGH and HGD are equal to one another: If it be not so, the one of them shall be the greater: Let then AGH be the greater (if possible.) Wherefore adding the angle HGB; the two angles HGB and DHG shall be lesse than the two angles HGB and AGH; but HGB and AGH are equivalent to two right angles; therefore HGB and DHG shall be lesse than two right angles; therefore AB and CD shall not be parallel, which is contrary to Supposition: Therefore the angle AGH is equal to the angle GHD.

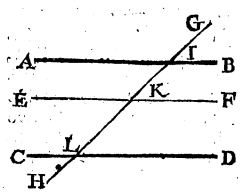


Secondly, The exterior angle EGB is equal to the interior and opposite angle, and of the same part GHD; for the angle EGB is equal to the angle AGH, and the angle GHD is shewn to be equal to the angle AGH: Therefore the angle EGB shall be equal to the angle GHD, the exterior to the interior angle.

Lastly, The two interior angles, and on the same part, BGH and GHD are equal to two right angles: For seeing that the angle EGB is shewn to be equal to the angle GHD, if the angle BGH be added to each of them, the two angles GHD and BGD shall be equal to the two angles EGB and BGH: But the angles EGB and BGH are equal to two right angles; therefore the angles BGH and DHG shall be also equal to two right angles: Which was to be demonstrated.

PROP. 30. THEOR. 21.

The right lines AB *and* CD, *parallel to one and the same right line* EF, *are also parallel to one another.*



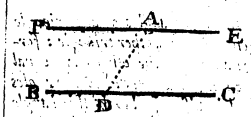
Demonstration LEt GH be drawn, cutting the same right lines in the points I, K, and L; forasmuch as the right line GH doth fall on the parallels AB and EF, the angle AIK shall be equal a to the alternate angle FKI; Again, forasmuch as GH falls on the paralleis CD and EF, the angle DLK shall be equal to the angle IKF, the interior, to the exterior angle, or the exterior to the interior and opposite on the same part But AIK is equal to the same IKF; therefore AIK b and DLK shall be equal to one another, which being alternate angles, c the lines AB and CD shall be parallel to one another: Therefore, The right lines, &c. Which was to be demonstrated.

a) 29. 1.
b) 1. c. f.
c) 27. 1.

PROP.

PROP. 31. PROBL. 10.

From a given point A, *to draw a right line parallel to a given right line* BC.



Construction LEt the contingent point D be taken in the given line BC, and let AD be drawn, making any angle at pleasure with BC; as the angle ADC, and in the point A of the given right line AD, let there be constituted the a right lined angle FAD, equal to the given right lined angle ADC, and let FA be prolonged directly towards E.

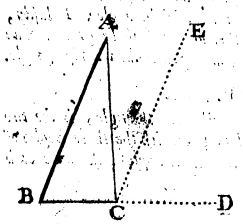
a) 23. 1.

Demonstration FOrasmuch as the two alternate angles FAD and CDA are equal by construction, the line AE b shall be parallel to BC, and drawn from the given point A. Which was to be done.

b) 27. 1.

PROP. 32. THEOR. 22.

Of every triangle, as ABC, *one of the sides* BC *being prolonged, the exterior angle* ACD *is equal to the two interior and opposite angles* A *and* B, *and the three interior angles* A, B, *and* ACB, *of any triangle, are equal to two right angles.*



Construction FOr from the point C let CE be drawn parallel to AB.

Demonstration FOrasmuch as AC doth fall on the parallels AB and CE, the alternate angles a A and ACE are equal; Again, forasmuch as BD falls on the Parallels AB and CE, the angles B and ECD shall be b equal, the exterior angle to the interior and opposite angle on the same part: Therefore the two angles ACE and ECD; that is to say, the whole ACD shall be equal to the two angles A and B, the exterior angle to the two interior and opposite angles; Which was proposed.

a) 29. 1.
b) 29. 1.

Secondly, I say that the three interior angles of the same triangle ABC; to wit, A, B, and ACB, are equal to two right angles.

Forasmuch as A and B are equal to the angle ACD, as hath been shewn, if the common angle ACB be added; the three angles c A, B, and ACB, shall be equal to the two angles ACB and ACD; but the two angles ACB and ACD are equal to two d right angles; therefore the

c) 2. c. f.
d) 13. 1.

the three angles A, B, and A C B are equal to two right angles: Therefore of every triangle, &c. Which was to be demonstrated.

COROLLARIE I.

From this 32 Proposition may be gathered that the three angles of any triangle taken together, are equal to three angles taken together, of any other triangle: forasmuch as the three angles of the one, as the three angles of the other, are equal c) *to two right angles. Therefore if two angles of one triangle are equal to two angles of another triangle, the third angle of the one shall be also equal to the third angle of the other. Therefore if two angles of one triangle are equal to two angles of another triangle, each to his correspondent, the other angle of the one shall be equal to the other angle of the other, and those triangles shall be equiangled.*

c) 32. 1.

COROLLARIE II.

It is also evident, that in every Isosceles triangle, whose angle conteined of the equal sides is a right angle, that each of the other angles which are on the base is the half of a right angle, for both of them together do constitute a right angle: f *Seeing the three angles are equal to two right angles, and that the third angle is put a right angle; therefore seeing that the two remaining angles are equal to one another, each of them shall be the half of a right angle: But if the angle conteined of the equal sides be obtuse, each of the other angles shall be lesse than half of a right angle, and the two angles together, lesse than a right angle; If lastly, the above mentioned angle be acute, each of the other angles shall be greater than half a right angle: forasmuch as the two together, are greater than a right angle.*

f) 32. 1.

COROLLARIE. III.

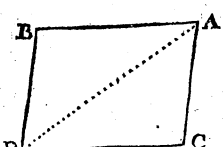
It is likewise manifest, that each angle of an equilateral triangle, is the two third parts of a right angle, for the two right angles, g *to which the three angles of an equilateral triangle are equal, being divided into three angles, do make each angle the two third parts of a right angle.*

g) 32. 1.

COROLLARIE IV.

It is also evident, that if a perpendicular line be drawn from one of the angles of an equilateral triangle on the opposite side there will be constituted two Scalene triangles, each of which shall have the angle adjacent to the perpendicular, a right angle, another shall be the two thirds of a right angle, to wit, that which is also an angle of the equilateral triangle: Lastly, the other angle shall be the third part of a right angle.

PROP. 33. THEOR. 23.



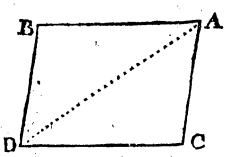
The right lines B D *and* A C, *which joyn together two right lines* B A *and* D C *equal and parallel, and on the same part, the same right lines* B D *and* A C *are in like manner equal and parallel.*

De-

Demonstration FOr let A D be drawn, forasmuch as A D falls on the parallels B A and C D, a the alternate angles B A D and A D C shall be equal to one another, and forasmuch as B A is equal to D C by supposition, and A D common, the two sides B A and A D of the triangle A B D shall be equal to the two sides D C and D A of the triangle D C A, each to his correspondent, and the angle B A D equal to the angle C D A conteined of those equal sides, the base b B D shall be equal to the base A C, and the whole triangle A B D shall be equal to the wholle triangle D C A, and the other angles equal to the other angles, each to its correspondent; to wit, C equal to B, and D A C equal to B D A, which being alternate angles, c the lines B D and A C shall be parallel: But they are also shewn to be equal: Therefore they shall be equal and parallel: Therefore, &c. Which was to be demonstrated.

a) 29. 1.
b) 4. 1.
c) 27. 1.

PROP. 34. THEOR. 24.



In Parallelograms the opposite sides A B *and* C D, *and the opposite angles* D B A *and* A C B, *are equal to one another, and their Diameter* A D *doth divide them into two equal parts.*

Demonstration FOr seeing that B A and D C are parallels, and that A D falleth on them, a the alternate angles B A D and A D C are equal to one another; Again, seeing that B D and A C are parallels, the alternate angles B D A and C A D are also equal: Therefore seeing that the two angles B A D and B D A of the triangle A B D, are equal to the two angles C A D and C D A of the triangle A C D, each to his correspondent, and the side adjacent to those equal angles, common to both the triangles, b the two sides B D and B A, shall be equal to the two other sides D C and A C, each to his correspondent: to wit, B A equal to its opposite D C, and B D equal to its opposite A C; and the angle B equal to the angle C: And forasmuch as the two angles at the point A are shewn to be equal to the two angles at the point D, the whole angle A shall be equal to its opposite and whole angle D: Therefore the sides and the angles opposite to one another, are equal.

But forasmuch as in the triangle A B D and D C A, the two sides B A and B D are equal to the two sides C D and C A, each to its correspondent, and the angles B and C, conteined of those sides, are equal, as hath been shewn, c the whole triangle B A D shall be equal to the whole triangle A C D: Therefore the Diameter doth divide the Parallelogram into two equal parts: Therefore the sides and the angles, &c. Which was to be demonstrated.

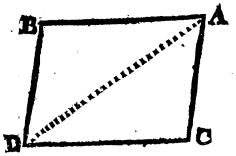
a) 29. 1.
b) 26. 1.
c) 4. 1.

COROLLARIE. I.

From this Propoſition may be in the firſt place gathered, that every quadrilateral or four ſided figure, which hath the oppoſite ſides equal, is a Parallelogram; for the two ſides B D *and* B A *being equal to* A C *and* C D, *each to his correſpondent, and the baſe* A D *common, the angles* B *and* C *ſhall be equal, and the two other angles alſo ſhall be equal to the two other angles, each to his correſpondent; ſo all the alternate angles which are at the points* A *and* D, *ſhall be equal to one another: Therefore the figure ſhall be a Parallelogram, all the ſides being Parallels.*

COROLLARIE II.

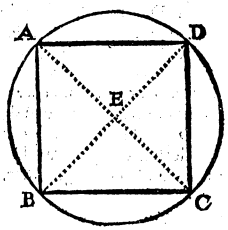
It follows alſo, that every four ſided figure which hath the oppoſite angles equal, is a Parallelogram, as in the former figure, the oppoſite angles being equal to wit, A *to* D, *and* B *to* C: *And by what hath been ſhewn in the* 32 *Propoſition, the four angles being equal to four right angles, the two angles* B *and* B A C *ſhall be equal to two right angles: In like manner* B *and* B D C *ſhall be alſo equal to two right angles: therefore as well* B A *and* C D *as* B D *and* A C *ſhall be parallels; and therefore the figure propoſed is a Parallelogram, and the ſame would happen if all the angles were right angles.*



We ſhall alſo ſay, that although the Diagonal line divideth every Parallelogram into two equal triangles, it doth not follow, that every figure of four ſides which divideth it ſelf into two equal triangles by the Diagonal, is a Parallelogram, as is eaſie to be underſtood in drawing from the point B *towards* A, *a line equal to* A C, *and from the point* D *a line equal to* B A, *on the ſame part, which will meet with one another at another point, as is manifeſt by what hath been heretofore demonſtrated; the facility whereof requireth no figure.*

COROLLARIE III.

It follows alſo that in the long Square and Rhomboides (for there are but four ſorts of Parallelograms, as we have already ſaid) the Diagonal line doth cut the angles unequally, becauſe of the inequality of the ſides in each triangle. See the former figure.

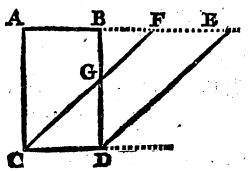


But in the Square and the Rhombe, the ſaid Diagonal line doth cut the angles into two equal parts; foraſmuch as the two ſides of one of the triangles are alſo equal to the two ſides of the other triangle, each to his correſpondent, and the angle conteined of them equal to the angle, and as appears by this ſquare A B C D, *in which the Diagonal lines* A C *and* B D *are alſo equal, and do cut one the other into two equal parts in the point* E, *becauſe of the equality of all the angles which are in the points* A, B, C, D, *and in the point* E.

But if the figure A B C D *were a Rhomb, the*

the two angles oppoſite ſhould be obtuſe, and the two others acute: Wherefore the Diagonal lines would be unequal; and yet nevertheleſſe would cut each other in two equal parts, as is eaſie to be underſtood by what hath already been demonſtrated, and alſo in the long Square and Rhomboides, and all figures of four ſides, in which the Diagonal lines do cut one another into two equal parts, is a Parallelogram; All which things are ſo eaſily demonſtrated, if the Propoſitions and Demonſtrations before mentioned be underſtood, as there was ſmall need of making any mention of them here.

PROP. 35. THEOR. 25.

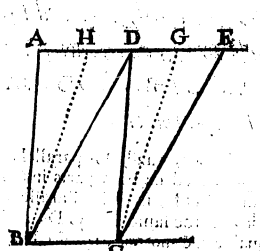


The Parallelograms A C D B *and* F C D E, *conſtituted on one and the ſame baſe* C D, *and between the ſame parallels* A B *and* C D *are equal to one another.*

Demonſtration Foraſmuch as A C D B is a parallelogram, A B is [a] equal to C D, by the ſame reaſon F E is equal to C D: [b] therefore A B and F E are equal to one another; therefore if the common line B F be added, [c] the whole A F ſhall be equal to the whole B E; [d] but A C is alſo equal to B D; therefore the two ſides F A and A C of the triangle F A C ſhall be equal to the two ſides E B and B D of the triangle B E D, each to his correſpondent, and the angles A and B conteined of thoſe ſides, [e] are equal, the exterior angle to the interior angle, and contrarily: therefore the baſe [f] F C ſhall be equal to the baſe E D, and the whole triangle F A C to the whole triangle E B D, from which the common triangle B G F being taken away, [g] the trapezium A B G C will remain equal to the Trapezium E F G D; to which Trapeziums if you again adde the common triangle C D G, the whole Parallelogram A C D B ſhall be equal to the whole Parallelogram E D C F: Therefore the Parallelograms conſtituted, &c. Which was to be demonſtrated.

a) 34. 1.
b) 1. c. f.
c) 2. c. f.
d) 34. 1.
e) 29. 1.
f) 4. 1.
g) 2. c. f.

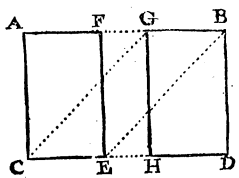
But if the Parallelograms conſtituted on one and the ſame baſe B C, were A B C D and D B C F, as in this ſecond figure; as well the ſide A D as D F, ſhould be equal to B C, as aforeſaid, and A B to D C, and the exterior angle F D C, to the interior and oppoſite angle A: Wherefore the two ſides F D and D C being equal to D A and A B, each to his correſpondent, and the angle F D C equal to the angle A, the whole triangle F D C ſhall be equal to the whole triangle D A B. Therefore if to both, there be added the common triangle B C D, the Parallelogram A B C D ſhall be equal to the Parallelogram D B C F.



And laſtly, if the Parallelograms conſtituted on the baſe B C, were A B C D and H B C G, as well A D as H G would yet be equal to B C; and

and therefore equal to one another: Therefore taking away the common line H D, there would remaine D G, equal to A H, and as before, the triangle G D C shall be equal to the triangle H A B: Therefore if to each of them there be added the common Trapezium H B C D, the whole Parallelogram A B C D shall be equal to the whole Parallelogram H B C G. Which was to be demonstrated.

PROP. 36. THEOR. 26.

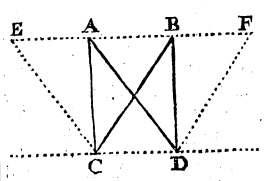


The Parallelograms A C E F *and* B G H D, *constituted on equal bases* C E *and* H D, *and between the same parallels* A B *and* C D *are equal to one another.*

Demonstration FOr let the right lines C G and E B be drawn; forasmuch as G B is [a] equal to H D, and C E is put equal to the same H D the two sides, [b] G B and C E shall be equal to one another, the which being parallels by supposition, the lines C G and E B which joyn them, [c] shall be also equal and parallel, and B G and C E shall be a Parallelogram: Therefore the Parallelogram A C E F shall be equal to the Parallelogram G C E B, being [d] on the same base C E, and between the same Parallels, by the same reasons, the Parallelogram G H D B shall be equal to the same Parallelogram G C E B, being on the same base G B, and between the same Parallels: Therefore the Parallelograms A C E F, and D B G H [e] are equal to one another: Therefore the Parallelograms, &c. Which was to be demonstrated.

a) 34. 1.
b) 1. c. 5.
c) 33. 1.
d) 35. 1.
c) 1. c. 5.

PROP. 37. THEOR. 27.



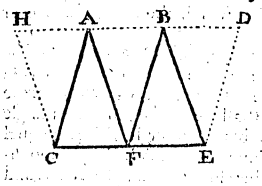
The triangles A C D *and* B C D, *constituted on one and the same base* C D, *and between the same parallels* A B *and* C D, *are equal to one another.*

Demonstration LEt C E be drawn parallel to A D, [a] and D F parallel to C B, meeting with A B, prolonged on both parts in the points E and F, the figures A D C E and B C D F, shall be Parallelograms, the which being constituted on one and the same base C D, and between the same Parallels, [b] shall be equal to one another; but the Diameters A C and B D doth divide them into [c] two equally: Therefore their halves (which are the triangles proposed A C D and B C D) [d] shall be also equal to one another: Therefore the triangles, &c. Which was to be demonstrated.

a) 13. 1.
b) 35. 1.
c) 34. 1.
d) 7. c. 5.

PROP.

PROP. 38. THEOR. 28.

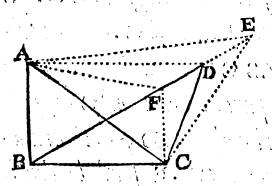


The triangles A C F *and* B E F, *constituted on equal bases* C F *and* F E, *and between the same Parallels* A B *and* C E *are equal to one another.*

Demonstration FOr let C H and E D be drawn [a] parallel to F A and F B, meeting A B prolonged on both parts in the points H and D, A F C H and B F E D shall be Parallelograms, and [b] equal to one another, of which the triangles proposed A C F and B F E are [c] the halves: Therefore those triangles shall [d] be also equal to one another: Therefore the triangles, &c. Which was to be demonstrated.

a) 31. 1.
b) 36. 1.
c) 34. 2.
d) 7. c. 5.

PROP. 39. THEOR. 29.



The equal triangles A B C *and* D B C, *constituted on one and the same base* B C, *and on the same part* A D, *are also between the same parallels; that is to say, that the line drawn from* A *to* D, *is parallel to* B C.

Demonstration FOr if A D were not parallel to B C, there might be drawn from the point A a parallel to the same B C, which would passe above A D, or under it: Let therefore [a] A F be drawn as a parallel to B C, if A D be not a parallel, and let C F be joyned: Forasmuch as A F and B C are parallels, the triangle F B C shall be equal to the triangle [b] A B C; but the triangle D C B is equal to the same triangle A B C by Supposition: therefore the triangles [c] F B C and D C B shall be equal to one another, the part to the whole, which is absurd.

If it be said that the line parallel to B C drawn from the point A towards D, shall passe above A D, the same absurdity will happen; therefore A D is parallel to B C: Therefore the equal triangles, &c. Which was to be demonstrated.

a) 31. 1.
b) 37. 1.
c) 1. c. 5.

PROP. 40. THEOR. 30.

The equal triangles A B C *and* D E F, *constituted on equal bases* B C *and* E F, *and on the same part* A D, *are also between the same parallels; that is to say, that the line drawn from the point* A *to* D, *is parallel to* B F.

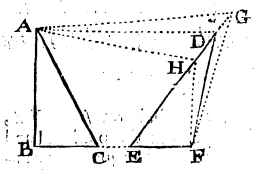
Demon-

Demonſtration FOr otherwiſe there might be drawn from the point A a line parallel to [a] B F; which ſhould fall above or under A D.

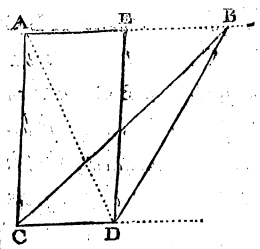
Let A G be drawn parallel to B F, (if poſſible,) and let G F be joyned: Foraſmuch as the lines A G and B F are parallels, the triangle G E F ſhall be equal to the triangle A B C, but the triangle D F E is equal to the ſame triangle A B C by Suppoſition: Therefore the triangles D F E and [b] G E F ſhall be equal to one another, the part to the whole, which is abſurd if the parallel be ſaid to paſſe below A D, the ſame abſurdity will happen: Therefore A D is parallel to B F: Therefore, The equal triangles, &c. Which was to be demonſtrated.

a) 31. 1.

b) 1. c. ſ.



PROP. 41. THEOR. 31.

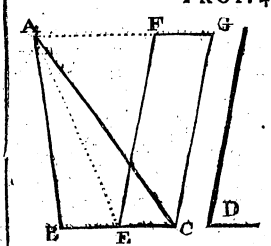


If a Parallelogram A C D E *hath the ſame baſe* C D, *as a triangle* B C D, *and is between the ſame parallels* A B *and* C D, *as is the triangle* B C D, *the Parallelogram* A C D E *ſhall be double to the triangle* B C D.

Demonſtration FOr having drawn the Diameter A D, the triangle A C D [a] ſhall be equal to the triangle B C D: But the Parallelogram A C D E is double to the triangle A C D: foraſmuch [b] as the triangles A C D and A E D are equal: Therefore the ſame Parallelogram A C D E is alſo double to the triangle B C D: Therefore, If a Parallelogram, &c. Which was to be demonſtrated.

a) 37. 1.
b) 34. 1.

PROP. 42. PROBL. 11.



To conſtitute a Parallelogram E F C G *equal to a given triangle* A B C, *in a given right lined angle* D.

THat is to ſay, that hath one angle equal to a given right lined angle.

Conſtruction LEt one of the ſides of the triangle A B C, (which you pleaſe) be divided; (to wit B C,) into two [a] equal parts in the point E,

a) 10. 1.

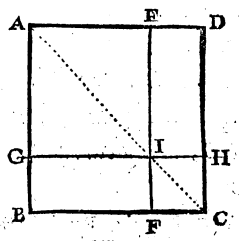
E, and let A E be joyned; and let the angle C E F be made equal to the given angle D; then let [b] A F be drawn parallel to B C, meeting E F in the point F; alſo let C G be drawn from the point C, parallel to E F, meeting A F prolonged in the point G, ſo ſhall the Parallelogram F E C G be conſtituted in the angle C E F, which is equal to the given angle D, which ſaid Parallelogram I ſay is equal to the given triangle A B C.

b) 31. 1.

Demonſtration FOraſmuch as the triangles A B E and A E C are equal; [c] being conſtituted on the equal baſes B E and E C, and between the ſame parallels A F and B C, the whole triangle A B C ſhall be double to the triangle A E C: But the Parallelogram F E C G is [d] alſo double to the ſame triangle A E C: Therefore the triangle A B C [e] and the Parallelogram F C ſhall be equal to one another, and the parallelogram F C ſhall have the angle C E F equal to the given angle D, by conſtruction: Therefore we have conſtituted, &c. Which was to be done.

c) 38. 1.
d) 41. 1.
e) 6. c. ſ.

PROP. 43. THEOR. 32.



Of every parallelogram, as A B C D, *the complements* D I *and* I B *of the parallelograms* G E *and* F H, *which are about the Diameter* A C, *are equal to one another.*

Demonſtration FOr ſeeing that the triangles A B C and A D C [a] are equal to one another (for the Diameter A C divideth the parallelogram into two equal parts, and that the two triangles A G I and A E I are alſo equal to one another, by the ſame reaſon, if thoſe two later be cut off from the two firſt, [b] the Trapeziums G B F C I and E D H C I, ſhall remain equal to one another; But the triangles F I C and H I C are [c] alſo equal to one another; therefore if theſe triangles be taken from the Trapeziums, the complements [d] D I and I B, will remain equal: Therefore, Of every, &c. Which was to be demonſtrated.

a) 34. 1.
b) 3. c. ſ.
c) 34. 1.
d) 3. c. ſ.

PROP. 44. PROBL. 12.

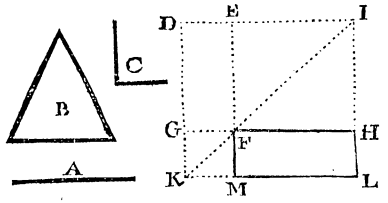
On a given right line A, *to apply a parallelogram,* L M F H, *equal to a given triangle* B, *in a given right lined angle* C.

Conſtruction LEt the parallelogram D E F G be conſtituted, [a] equal to the given triangle B, having the angle G F E, equal to C; then let G F be prolonged to the point H, in ſuch ſort as F H may be equal to A, and by H let there be drawn H I, [b] parallel to F E, meeting D E prolonged in the point I; after that let the Diameter I F be drawn, by the point F, meeting D G prolonged in K, and by K let K L be drawn, parallel

a) 42. 1.
b) 31. 1.

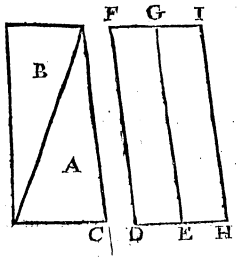
c) 31. 1. parallel c to G H, meeting I H prolonged in L, and let E F be prolonged to the point M: I say, that the Parallelogram L M F H is the Parallelogram required.

d) 15. 1. *Demonstration* FOr it hath the side F H equal to the given line A, by the construction, and the angle H F M is d equal to the angle E F G, the which is made equal to the angle C: Therefore H F M shall be also equal to C, and lastly, the parallelogram F L is equal to the parallelogram G E, the complement equal to the complement, which said parallelogram G E is made equal to the triangle B: Therefore F L shall be also equal to B: Therefore on a given, &c. Which was to be done.



PROP. 45. PROBL. 13.

To constitute a parallelogram D I, *equal to a given right lined figure* A B, *in a given right lined angle* C.



a) 42. 1. *Construction* LEt the Parallelogram D G be made a equal to the triangle A, having the angle F D E equal to the given right lined angle C; then to the line G E, let the parallelogram E I be applyed, equal b to the triangle B, having the angle G E H equal to the angle C; and so what was required is done.

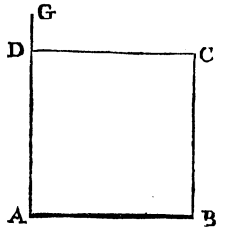
b) 44. 1.

c) 1. c. f. d) 2. c. f. *Demonstration* FOrasmuch as each of the angles F D E and G E H, is made equal to the angle C, c they shall be equal to one another; therefore if the angle G E D be added, d the two angles G E D and G E H shall be equal to the two angles G E D and F D E; but F D E and G E D are equal to two right angles; therefore G E D and G E H shall be also equal to two right angles; Therefore e D E H shall be a right line: By the same reasons it may be proved that F G and G I do make a right line. Now seeing that each of these lines D F and H I are equal and parallel f to E G, they shall be equal and parallel to one another, and the right lines F I and D H which joyn them together, g shall be also equal and parallel: Therefore D I shall be a parallelogram, made of the two parallelograms D G and E I, equal to the two triangles A and B: Therefore the parallelogram D I is equal to the right lined figure A B, having the angle D equal to the given right lined angle C: Which was to be done.

e) 14. 1. f) 34. 1. g) 30. 1.

PROP.

PROP. 46. PROBL. 14.



On a given right line A B, *to describe a Square* A B C D.

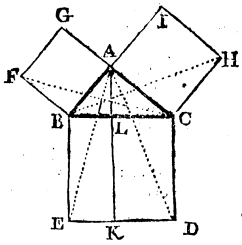
a) 11. 1. b) 31. 1. *Construction* LEt the right line A G be drawn a at right angles to A B, and from the point D let b D C be drawn parallel to A B, likewise let B C be drawn parallel to A D, meeting D C in the point C, the figure A B C D shall be a Parallelogram, and therefore the side D C c is equal to A B, and A D equal to B C: But A D is put equal to A B, therefore B C d shall be also equal to A B; therefore A B C D shall be equilateral: I say it is also a Rectangle.

c) 34. 1. d) 1. c. f.

e) 28. 1. *Demonstration* FOrasmuch as A D and B C are parallel, the e angles A and B are equal to two right angles, but A is made a right angle, therefore B shall be also a right angle, and the two opposite angles D and C f are equal to them, and therefore right angles: Therefore the Parallelogram A C is rectangled, and equilateral, and therefore shall be a Square: Therefore, On a right line, &c. Which was to be done.

f) 34. 1.

PROP. 47. THEOR. 33.



In right angled triangles as A B C, *the square* B D *of the side* B C, *which subtendeth the right angle* B A C, *is equal to the squares* A F *and* C I, *of the sides* A B *and* A C, *which do contein the same right angle* B A C.

a) 31. 1. *Demonstration* LEt A K be drawn a parallel to B E or to C D, cutting B C in the point L, and let the right lines A D, A E, C F, and B H be joyned: Forasmuch as the angles B A C and B A G are right angles, b G A and A C shall make one only right line, by the same reason B A and A I shall also make one right line.

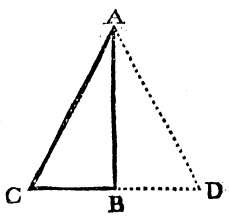
b) 14. 1. Again, Forasmuch as the angles A B F and C B E are equal (as being right angles, if the common angle A B C be added, the whole A B E shall be c equal to the whole C B F; likewise the whole A C D shall be equal to the whole B C H: Forasmuch therefore as the two sides A B and B E of the triangle A B E, are equal to the two sides C B and B F of the triangle C B F, each to his correspondent, (as appears by the Definition of Squares,) and the angles A B E and C B F conteined by those sides are shewn to be equal: d Those triangles A B E and C B F shall be equal; but the Parallelogram A F is e double to the triangle C B F,

c) 2. c. f. d) 4. 1. e) 41. 1.

CBF, being on the same base, and between the same parallels: It shall be therefore also double to the triangle ABC, and the Parallelogram BK is double to the same triangle ABC, it being on the same base, and between the same parallels: Therefore the parallelogram BK shall be equal to the square AF.

By the same discourse it might be shewn that the Parallelogram LD is equal to the square CI: Therefore the square EC shall be equal to the two squares AF and CI: Therefore, In triangles, &c. Which was to be demonstrated.

PROP. 48. THEOR. 34.



If the square described of one of the sides AC *of a triangle* ABC *be equal to the squares of the two other sides* AB *and* BC *of the same triangle* ABC, *the angle* ABC *is a right angle.*

Demonstration LEt BD be drawn equal to BC, and at [a] right angles to AB, and let AD be joyned: Seeing that ABD is a right angle, the square of AD [b] is equal to the two squares of AB and BD: but the square of BD is equal to the square of BC, seeing that the lines BD and BC are made equal, wherefore the square of AD shall be equal to the squares of BA and BC: Therefore seeing that the square of AC is equal to the same squares of BA and BC, [c] the same squares of AC and AD shall be equal; and therefore the lines AD and AC are equal: Forasmuch then, as in the triangles ABD and ABC, the two sides BD and BA are equal to the two sides BA and BC, each to his correspondent, and the bases AD and AC equal, the angles at the point B conteined of those [d] sides shall be equal to one another: But ABD is made a right angle: Therefore ABC shall be also a right angle: Therefore the square, &c. Which was to be demonstrated.

a) 11. 1.
b) 47. 1.
c) 1. c. f.
d) 8. 1.

The End of the First Element of EUCLIDE.

THE

THE

SECOND ELEMENT

OF

EUCLIDE.

THE ARGUMENT.

EUCLIDE treats in this second Book of the power of right lines, seeking the quantity of the squares of the parts of every right line divided, and of rectangled Parallelograms conteined under the parts of the same divided line, compared as well to one another, as with the square of the whole, &c. and for that cause he here unfoldeth in the first place, by these two Definitions two things which are necessary for the well understanding of what is to be demonstrated hereafter.

DEFINITIONS.

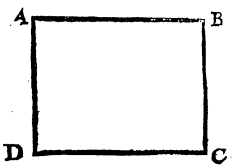
1 *Every rectangled Parallelogram is said to be conteined under two righe lines, which do contein the right angle.*

IN this first he declares under what lines the rectangled parallelogram is said to be conteined, and what is to be understood by a parallelogram to be conteined under two right lines; which for the more easie understanding, you must first know that every rectangled parallelogram is that which hath all his angles right angles, and of those there are two sorts, to wit, the square, and the oblong or long square, longer on one part then on the other: for in those all the angles are right angles, as we have

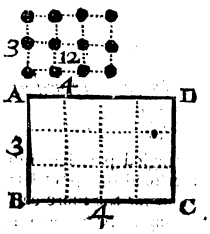
have already said in the Definitions: And in every parallelogram if one angle be given a right angle, the three others will of necessity be right angles; For example, in the parallelogram A B C D, Let the angle A be a right angle, I say that the three others B, C, and D, are also right angles, for seeing that A B and D C are parallels, [a] the two interior angles A and B, are equal to two right angles: But A is a right angle by supposition; therefore B shall be also a right angle: But [b] forasmuch as each one of them is equal to his opposite, as the angle A to the angle C, and the angle B to the angle D, the angles D and C shall be also right angles: Therefore *EUCLIDE* saith that every parallelogram right angled is said to be conteined under two right lines, which contein a right angle, as the Rectangled parallelogram A B C D, is said to be conteined under A B and A D, or under A D and D C, or lastly, under A B and B C: forasmuch as each two of those lines so taken, do expresse the magnitude or space of the whole parallelogram, to wit, one of them, as A B or D C its length, and the other as A D or B C its breadth: Therefore these two lines of the rectangled parallelogram, which contein the right angle being expressed, we conceive immediately the whole quantity thereof, and also its length and breadth is understood: It happens also that by the imaginary motion of one of the lines, according to the other, the whole parallelogram is constituted; for if it be conceived in the understanding, that the right line A B be moved transversly according to A D, in such sort as that it may alwayes constitute a right angle with A D, until the point A arrive to the point D, and the point B to the point C, the whole parallelogram A D C B shall be described: the same will happen if A D be proposed to move transversly according to A B; therefore by good reason, a right angled parallelogram is said to be conteined under two such right lines.

a) 29. 1.

b) 34. 1.



Now this hath a great affinity and neerness with the multiplication of one number by another, for even so, as by the multiplication of 3 by 4, 12 is produced, which is constituted in form of a parallelogram, from whence it is said to be conteined under 3 and 4: So the parallelogram A B C D, conteined under the two right lines A B and B C, (of which A B conteins 3 Palmes, or other measures, and B C 4) conteins 12 square Palmes, or other measures, which is made by the imaginary passing or motion of the line A B, of 3 Palmes in the line, or according to the line B C of 4 Palmes, as is represented by the figure; and is well known to Arithmeticians and Geometricians, and is demonstrated by *Regiomontanus* in his first Book of triangles, Prop. 16. Whence it comes, that some say, that the rectangled parallelogram is

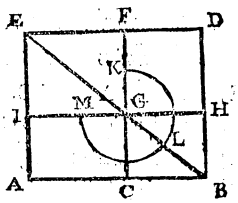


is constituted, or made of the product of two lines which are about the right angle, the one being multiplyed by the other; as the fore-mentioned Parallelogram is produced by the multiplication of the line A B, in the line B C, or (which is the same thing,) of the line B C in the line A B, for the same parallelogram is produced, whether the lesser line be applyed to the greater, or the greater to the lesser. In like manner, also the same number is produced, if the lesser number be multiplyed by the greater, or the greater by the lesser, as it is demonstrated by *EUCLIDE* in the Seventh Book, Proposition 16. For that 12 is as well produced by 3 multiplyed in 4, as by 4 multiplyed in 3.

¶ The Reader is here advertised that in this second and the other following Books, the rectangled parallelogram is termed by *EUCLIDE* simply Rectangle, which other Geometricians do also observe, so that by Rectangle is to be understood alwayes Rectangled Parallelogram: Again, to the end the same letters be not so often repeated, Geometricians are accustomed to expresse the Parallelogram as well Rectangled, as not Rectangled, by two letters only; to wit, those which are diametrically opposite, as to denote the Parallelogram before mentioned, we say the Parallelogram A C, or else B D.

2 *Of every Parallelogram, one of the Parallelograms (which you please,) described about the Diameter, together with the two complements, is called* Gnomon.

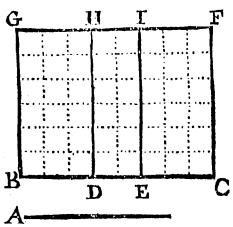
IN the Parallelogram A B D E, be it rectangled, or not: Let it be divided into four Parallelograms, as was shewn in the 37th. Definition of the First Book, of which C H and I F, are said to be about the Diameter, and the two others A G and G D are said to be complements; the figure compounded of which you please of the Parallelograms which are about the Diameter, as of C H, with the two complements A G and G D, such as is the figure M L K is called Gnomon: by the same reason, the figure H D E I C G, compounded of the Parallelogram I F, which is about the Diameter, and the two complements A G and G D shall be called Gnomon.



PRO-

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.



If there be two right lines A *and* BC, *and that one of them* BC, *be divided into as many parts as you please* BD, DE, *and* EC, *the Rectangle conteined under those two right lines* A *and* BC, *is equal to the Rectangles conteined under the undivided line* A, *and under each of the parts of the divided line* BC, *to wit, under* A *and* BD, *under* A *and* DE, *and under* A *and* EC.

Demonstration LEt the Rectangle be BF, compriſed under A and BC; that is to ſay, that BG be put equal to A, which may thus be done, draw BG and GF [a] at right angles, to the right line BC, and make each equal to A, and joyn the right line GF, ſo BG and CF ſhall be [b] parallels, becauſe of the right angles B and C, and alſo equal to one another; ſeeing [c] that each of them is put equal to A: therefore the right lines GF and BC which joyn them, [d] ſhall be alſo equal and parallel, therefore BF ſhall be a Rectangle, and conteined under BC and A; that is to ſay, his equal BG, according to the firſt Definition of this Book: Then from the points D and E, draw DH and EI, [e] parallel to BG, and therefore parallel [f] to one another: And ſeeing that by the ſame conſtruction BH and BI are Parallelograms, the right lines DH and EI ſhall [g] be equal to BG; that is to ſay to A: Seeing then that BG is equal to A, the Rectangle BH ſhall be conteined under the undivided line A, and the ſegment BD; and the Rectangle DI, ſhall be conteined under the undivided line A, and the ſegment DE: In like manner EF ſhall be

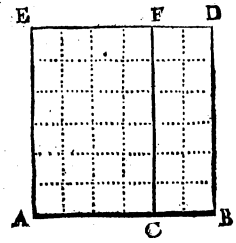
a) 11. 1.
b) 28. 1.
c) 1. c. ſ.
d) 33. 1.
e) 31. 1.
f) 30. 1.
g) 34. 1.

conteined under A, and the ſegment EC: Now ſeeing that all thoſe parts taken together, are equal to their whole BF; it is evident that the Rectangle compriſed of the two lines A and BC, ſhall be equal to each of the Rectangles compriſed of A, and each of the ſegments of BC: Therefore if there be two lines, &c. Which was to be demonſtrated.

SCHOLIUM.

We have divided theſe figures into equal parts, to render the Demonſtrations more eaſie, and to fit them to numbers, (See the precedent figure:) Let the line BG *equal to* A, *be* 6 *feet, or other meaſures, and the line* BC 8, *by the firſt Definition of this Book, the Rectangle* BF *ſhall be* 48 *Superficial feet, and by the ſame reaſon, the Rectangle* BH *ſhall be* 18, DI 12, *and* EF *alſo* 18, *which three Rectangles together do make alſo* 48, *to wit, all the parts taken together, equal to their whole.*

PROP. 2. THEOR. 2.



If a right line AB *be divided at pleaſure in* C, *the Rectangles* CD *and* AF, *conteined under the whole* AB, *and each of the parts* AC *and* CB, *are equal to the ſquare of the whole* AB.

Demonſtration DEſcribe [a] AD the ſquare of the whole line AB, and by C [b] draw CF parallel to AE or BD, it ſhall be [c] equal to AE; that is to ſay, to AB, to which AE is equal, by the Definition of a ſquare: Foraſmuch then as AE is equal to AB, the Rectangle AF ſhall be conteined under the whole AB, and the ſegment AC: In like manner, foraſmuch as CF is equal to AB, the Rectangle CD is conteined under the whole AB, and the other ſegment CB: Therefore ſeeing the Rectangles CD and AF are equal to the ſquare AD, it is manifeſt that the Rectangles conteined under the whole AB, and each of the ſegments AC and CB are equal to the ſquare of the whole AB: Therefore, If a line, &c. Which was to be demonſtrated.

a) 46. 1.
b) 31. 1.
c) 34. 1.

SCHOLIUM.

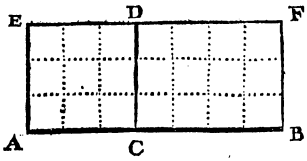
This may alſo be ſeen by numbers, by dividing 6 *in* 4 *and* 2, *as appeares by the figure, applying it to numbers,* AF *ſhall be* 24, *and* CD 12, *according to the firſt Definition, which make* 36, *and ſo much is the ſquare of* AD 6.

PROP. 3. THEOR. 3.

If a right line AB *be divided at pleaſure in* C, *the Rectangle* AF, *conteined under the whole* AB, *and one of the*

parts A C, *is equal to the Rectangle* C F, *conteined under thoſe parts, and* A D *the ſquare of the part* A C *firſt taken.*

Demonſtration DEſcribe AD the ſquare of the part A C, and by the point B draw B F [a] parallel to A E, meeting E D prolonged at F. a) 31. 1.

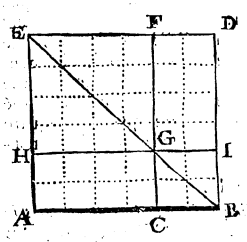


Foraſmuch then, as the right line A E is equal to the ſegment A C, by the Definition of a ſquare, the Rectangle A F ſhall be conteined under the whole A B and the part A C: Again, foraſmuch as C D is equal to C A; by the ſame reaſon, the Rectangle C F ſhall be conteined under the parts A C and C B; therefore ſeeing that the Rectangle A F is equal to the ſquare A D, and to the Rectangle C F: It is manifeſt that the Rectangle conteined under the whole A B, and the ſegment A C is equal to the Rectangle conteined under the part A C and C B, and to the ſquare of A C firſt taken: Therefore, If a right line, &c. Which was to be demonſtrated.

SCHOLIUM.

To fit this Theorem to numbers: Let A B 7 *be divided into* 4 *and* 3: *I ſay that the number* 21, *the product of* 3 *by* 7, *is equal to* 12, *the product of the two ſegments* 4 *and* 3 *multiplyed by one another, and to the number* 9 *the ſquare of the firſt part* 3: *In like manner* 23 *the product of* 7 *by* 4, *is equal to* 12 *the product of* 4 *in* 3, *and to the number* 16, *the ſquare of* 4 *the number firſt taken.*

PROP. 4. THEOR. 4.



If a right line A B *be divided at pleaſure in* C, *the ſquare* A D *of the whole* A B, *is equal to the ſquares* H F *and* C I, *of the parts* A C *and* C B, *and to twice the Rectangle* A G *or* G D, *conteined under the ſame parts* A C *and* C B.

Demonſtration DEſcribe the ſquare of the line A B, to wit, A D, and having drawn the Diameter B E, draw through the point C the right line C F, parallel to A E or B D, cutting the Diameter in the point G, and by G draw I G H parallel to A B, or to D E, and ſo the ſquare of A B ſhall be divided into four Parallelograms.

Foraſmuch as in the triangle A B E, the ſides A B and A E are equal, the two angles A B E and A E B [a] on the baſe E B are equal: But the angle a) 5. 1.

angle in the ſquare, and the three angles A, A E B, and A B E [b] are equal to two right angles, and therefore the two angles A B E and A E B, are each half a right angle, being equal to one another, and foraſmuch as H I and A B are parallels, the exterior angle G H E [c] ſhall be equal to its interior and oppoſite angle, and therefore a right angle: But the three angles of the triangle [d] E G H are equal to two right angles, and G E H is ſhewn to be half of a right angle, and E H G a right angle: Therefore the other H G E ſhall be alſo half a right angle; therefore in the triangle E G H, the two angles G E H and E G A ſhall be equal, being each of them half a right angle; therefore the two ſides H E and [e] H G ſhall be equal: But the two oppoſite ſides E F and F G [f] are equal unto them, each to its correſpondent: Therefore the Parallelogram H F ſhall be a ſquare, having all the ſides equal, and the angles right angles: For one of the angles, to wit E H G, being a right angle, all the four ſhall be right angles, as is ſhewn by the firſt Definition of this Book, and by the ſame reaſons it may be ſhewn that C I is a ſquare: Therefore F H and C I, [g] are the ſquares of the parts A C and C B; ſeeing that H G is equal to A C. b) 32. 1. c) 29. 1. d) 32. 1. e) 6. 1. f) 34. 1. g) 34. 1.

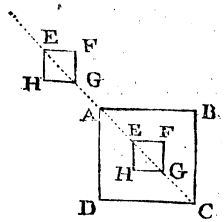
The Rectangles A G and G D ſhall be in like manner conteined under the ſegments A C and C B: foraſmuch as C G and G I are equal to C B, becauſe of the ſquare C I, and E G is equal to G H, by reaſon of the ſquare F H; that is to ſay [h] to A C: Therefore ſeeing the ſquare A D is equal to the two ſquares H E and C I, and to the two Rectangles A G and G D, it is manifeſt that A D the ſquare of the whole A B, is equal to the ſquares of the parts A C and C D, and to twice the Rectangle conteined under the ſame part A C and C B: Therefore, If a right line, &c. Which was to be demonſtrated. h) 34. 1.

COROLLARIE I.

By this Demonſtration it is manifeſt that the Parallelograms deſcribed about the Diameter of a ſquare are ſquare.

Which is apparent by the Demonſtration of this Theorem, where it is demonſtrated that the Rectangles C I *and* F H, *which are about the Diameter* B E *are ſquares: for the ſame Demonſtration ſerves in all other ſquares.*

Nevertheleſſe, this Corollary ought to be underſtood of Parallelograms, which are about the Diameteter of the ſquare, and which have ſome angle common with the whole ſquare, as are the Parallelograms C I *and* F H, *for the one hath the angle* A B D, *and the other the angle* A E D, *common with the ſquare.*

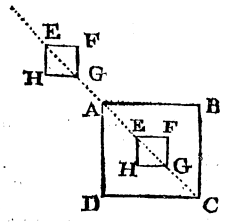


It is alſo true of any Parallelograms which are about the Diameter of the ſquare, being prolonged, although it have no angle common with the ſquare, provided their ſides be parallel to the ſides of the ſquare.

For about the Diameter A C *of the ſquare* B D, *deſcribe the Parallelogram* F H, *be it without or within the ſquare, which notwithſtanding muſt have the ſides parallel to the ſides of the ſquare: I ſay that* F H *is a ſquare.*

For ſeeing that A B *and* E F *are parallels* [i] *the angles* B A C *and* F E G *ſhall be equal, the exterior to the interior, and by the ſame reaſon the angles* B C A *and* F G E *ſhall be equal alſo: But the angles* B A C *and* i) 29. 1.

BCA *are the halves of right angles, as hath been already demonstrated: Therefore the angles* FEG *and* FGE *shall be also the halves of right angles,* k *and therefore the sides* EF *and* FG *opposite to them, shall be equal, and* l *the angle* F *shall be a right angle: Therefore seeing that the sides* EF *and* FG m *are equal to their opposite sides* GH *and* HE, *the Parallelogram* FH *shall be equilateral; but it is also rectangled, as hath been shewn by the first Definition of this Book: Forasmuch as one of the angles* F *is demonstrated to be a right angle: Therefore* FH *shall be a square: Which was proposed.*



k) 6. 1.
l) 32. 1.
m) 34. 1.

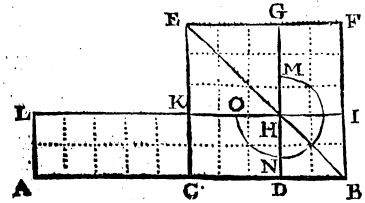
COROLLARIE. II.

It follows also from the Demonstrateon of this Proposition that the Diameter of any square doth divide its angle into two equal parts, for it hath been demonstrated that the angles AEB *and* DEB *in the fore-going Proposition, are the halves of right angles, which is also demonstrated by the Eighth, and Thirty fourth Propositon of the first Book.*

SCHOLIUM.

This fourth Theorem shall be also shewn by numbers; (See the figure belonging to this Proposition) let the line AB *of 6 feet be divided into* AC *and* CB, 4 *and* 2, *the square of* AB *shall be 36 Superficial feet, and the squares of* AC *and* CB, *to wit,* HF *and* CI *shall be 16 and 4, and the two rectangles under* AC *and* CB, *which are* AG *and* GD, *are each 8, all which parts together make 36; to wit, as much as the square of the line* AB, *which is* ABDE, *as is manifest.*

PROP. 5. THEOR. 5.



If a right line AB *be divided into two equal parts* AC *and* CB, *and into two unequal parts* AD *and* DB, *the Rectangle* AH, *conteined of the the unequal parts* AD *and* DB, *of the whole* AB, *with the square* KG, *of the intermediate part* CD, *is equal to the square* CF, *described of* CB, *the half of the whole* AB.

Demonstration DEscribe CF the square a of CB, and draw the Diameter BE, and by the point D draw b DG, parallel to BF, or CE, cutting the Diameter in the point H, and by the point H draw KI, parallel to AB, and from the point A draw AL, parallel to CE, meeting

a) 46. 1.
b) 33. 1.

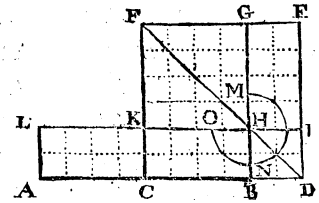
meeting with IK prolonged in the point L; The Parallelogramis KG and DI, about the Diameter BE shall be c squares, and therefore DH equal to DB and KH to CD: therefore the Rectangle AH shall be conteined under AD and DB, and KG shall be the square of CD, the which AH with KG, is equal to the square CF: For seeing that d the complements CH and HF are equal, if the common square DI be added, the Parallelogram DF shall be equal to the Parallelogram CI: But the Parallelogram AK is e equal to the Parallelogram CI, being constituted on equal bases AC and CB, and between the same parallels AB and LI: Therefore AK shall be equal to DF, to which if you adde the common CH, the Gnomon MNO shall be equal to the Rectangle AH: Wherefore seeing that the Gnomon MNO and the square KG, are equal to the square CF, the Rectangle AH with the square KG, shall be equal to the same square CF: Therefore, If a line, &c. Which was to be demonstrated.

c) Cor. 4, 2.
d) 43. 1.
e) 36. 1.

SCHOLIUM.

The same is also manifest in applying it to numbers: Let the line AB *be* 10; *then as well* AC *as* CB *shall be* 5, *and let* CD *be* 3, AD *shall be* 8, *and* DB 2: *Therefore the Rectangle* AH *shall be* 16, *and the square* KG 9, *and the Gnomon also* 16, *which with the square* KG, *is* 25, *which is the square* CF, *equivalent to the Rectangle* AH 16, *and* KG 9, *which together make* 25, *as is manifest.*

PROP. 6. THEOR. 6.



If a right line AB, *be divided into two equal parts in* C, *and there be added to it another right line* BD, *directly, the Rectangle* AI, *conteined under the whole* AB, *with the added line* BD, *to wit* AD, *and the added line* BD, *with the square* KG, *of the half* CB, *is equal to the square* CE, *described of the line* CD, *compounded of the half* CB, *and the added line* BD, *as of one line.*

Demonstration ON CD describe the square CE, and having drawn the Diameter DF by B, draw BG parallel to DE or CF, dividing the Diameter in the point H; and by H draw IK, parallel to CD, and by A draw AL parallel to CF, meeting IK prolonged in the point L: Therefore BI and KH shall be squares, and DI shall be equal to BD, a and KH is equal to CB: Wherefore the Rectangle AI shall be conteined under AD and DB, and KG shall be the square of CB, which Rectangle AI and square KG, are equal to the square CE: for seeing that

a) 34. 1.

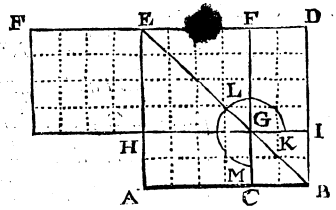
b) 36. 1. that the Parallelogram A K is equal to the Parallelogram C H; [b] Seeing that the bases A C and C B are equal, and the Parallelogram H E is equal to the same C H, the complement to the complement, A K and H E shall be equal to one another: Therefore if you add to them the common C I, the Gnomon M N O shall be equal to the Rectangle A I: Therefore seeing the Gnomon M N O, with the square K G, is equal to the square C E, the Rectangle A I with the square K G, shall be also equal to the square C E: Therefore, If, &c. Which was to be demonstrated.

This is also easie to be understood by the figure, where the Rectangle A K and H E are each 8 equal parts, to each of which if you add the Rectangle C I 12, the Rectangle A I will be equivalent to the Gnomon, to wit 20: Therefore A I with K G shall be equal to the square C E.

SCHOLIUM.

To apply this Theorem to Numbers: Let 8 be divided into two equal parts, to wit into 4 and 4; and let 2 be added, it will be seen how the number 20, the product of the whole compound number 10, multiplied by the added 2 with 16, the square of the half 4, is equal to 36, the square of 6 compounded of the half 4, and of the added number 2, as may be also understood by the figure it self.

PROP. 7. THEOR. 7.



If a right line A B, *be divided at pleasure in* C, *the two squares together, to wit,* A D, *that of the whole* A B, *and* F H *that of one of the segments* A C, *are equal to twice the Rectangle* A F, *conteined under the whole* A B, *and the said segment* A C, *and to* C I *the square of the other segment* C B.

Demonstration DEscribe the square of the line A B, to wit A D, and having drawn the Diameter B E by C, draw C F parallel to A E or B D, cutting the Diameter in the point G, by which point draw H I parallel to AB, by the first Corollary of the fourth Proposition, C I and H F shall be squares: And [a] forasmuch as G H is equal to A C, and the figure H F shall be the square of the segment A C: Again, forasmuch as A E is equal to A B, the Rectangle A F shall be conteined under the whole AB, & the segment AC, by the same reason, the Rectangle H D shall be conteined under the same A B and A C: Forasmuch as D E and E H are equal to A B and A C, by reason of the squares A D and H F.

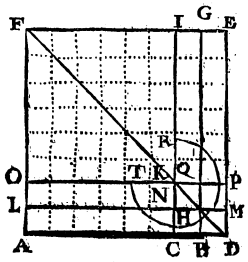
a) 34. 1.

Forasmuch then as the square A D is equal to the Rectangles A F and F B; that is to say, to the Gnomon K L M, and to the square C I, if you add the common square H F, the squares A D and H F shall be equal to the Rectangles A F and D H, (each of which is conteined under the whole A B and the segment A C,) and to C I the square of the other part C B: Therefore, If a right line, &c. Which was to be demonstrated.

SCHOLIUM.

This is manifest also if it be applyed to numbers, for A B *being divided into 6 parts,* A C *shall be 4, and* C B *2, and* A F *shall be 24, and* G D *with* H F *also 24, and* C I *4, which parts together, make 52 equal parts: And the two squares together* A D *and* H F *do also make 52 such parts.*

PROP. 8. THEOR. 8.



If a right line A B, *be divided at pleasure in* C, *four times the Rectangle* A H, *conteined under the whole* A B, *and one of the segments* C B, *with the square* O I *of the other segment* A C, *is equal to the square* A E, *described of the whole* A E, *and of the said segment* C B, *as of one line* C D.

Demonstration PRolong A B towards D, and put B D equal to B C, and on the whole A D describe the square A E, and having drawn the Diameter D F, draw C I and B G parallels to D E, cutting the Diameter in the points H and K, through which points draw L M and O P, parallel to A D, which doth divide the first Parallels in the points N and Q.

In the first place the Parallelograms O I, N Q, B M, L G, and C P, which are about the Diameter D F shall be squares, by the first Corollary of the fourth Proposition: And forasmuch as [a] O K is equal to A C, O I shall be the square of the segment A C: Again, seeing that N H is equal [b] to C B, N Q shall be the square of the segment C B: and therefore equal to the square B M; seeing that C B and B D are equal, therefore each of the lines B H and H Q is equal to the segment C B, and so the two Rectangles A H and L Q, shall be comprised under the whole A B, and the segment C B: [c] Seeing that L H is equal to A B, by the same reason, N G and H E shall be comprised under A B and C B: Seeing that the right lines N H and H M are equal to C B and B D; and G H and E M equal to F L, that is to say to L H, or to his equal A B: And forasmuch as the squares N Q and B M are equal; if you add the common rectangle K G, B M and K G together, shall be equal to the Rectangle N G: Wherefore the five rectangles A H, L Q, H E, B M, and K G, which constitute the Gnomon R H T, are equal to four times the Rectangle conteined under the whole A B, and the segment C B:

a) 34. 1.

b) 34. 1.

c) 34. 1.

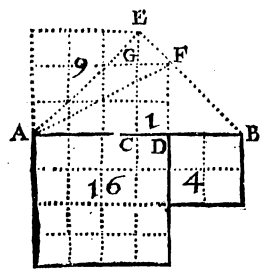
Now

Now seeing that the Gnomon R S T, and the square O I, are equal to the square A E; four times the Rectangle conteined under the whole A B, and the segment C B, with the square of the other segment A C, is equal to the square of the line A D, compounded of A B and of the same segment C B: Therefore, If a line, &c. Which was to be demonstrated: And this may be also understood by the figure divided into equal parts.

SCHOLIUM.

Now divide 10 *at pleasure, into* 6 *and* 4, *the number* 240, *which is made of the whole* 10, *multiplyed by the part* 6, 4 *times, with the number* 16 *the square of the other part* 4; *that is to say,* 256 *is equal to the square number of* 16, *which is compounded of the given number* 10, *and of the said part* 6, *as is manifest: In the same manner, the number* 160, *which is made of the whole* 10, 4 *times multiplyed by the part* 4 *with* 36, *the square of the other part* 6; *that is to say* 196, *is equal to the square of the number* 14, *compounded of* 10 *and* 4 *as is evident.*

PROP. 9. THEOR. 9.



If a right line A B, *be divided into two equal parts in* C, *and into two unequal parts in* D, *the squares* 16 *and* 4, *of the unequal segments* A D *and* D B, *of the whole* A B, *are double to the square* A E, *of the half* A C, *and the square* 1 *of the intermediate Section* C D.

Demonstration FRom the point C draw C E at right angles to A B, and equal to A C, and joyn A E and E B together; and from D draw also D F at right angles to A B, cutting E B in the point F, by which point draw F G, parallel to A B, dividing C E in the point G; and lastly joyn A F; Forasmuch as in the triangle A C E, the two sides A C and C E are equal, the two angles C A E and C E A shall be
a) 32. 1. equal: but A C E is a right angle; therefore the two [a] others are equivalent to a right angle: and therefore the angle A E C shall be the half of a right angle: By the same reason, the angle B E C shall be shewn to be half a right angle, and [b] by consequence the whole A E B shall be a angle.

b) 29. 1. Again, forasmuch as in the triangle F G E, the angle E G F [b] is equal to the right angle E C B, the exterior angle to the interior, the two
c) 32. 1. others [c] shall be equal to a right angle: But it is demonstrated that the angle F E G is half a right angle; Therefore F E G shall be also half a
d) 6. 1. right angle, and therefore are both equal: Therefore [d] the sides E G and G F shall be equal to one another: In the same manner it shall be shewn that the two lines F D and D B are equal to one another.

Now forasmuch as in the triangle A C E, the angle C is a right angle, the

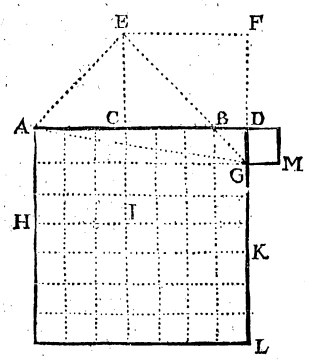
[e] the square of the side A E shall be equal to the two squares of A C and C E, the which are equal to one another, being the lines A C and C E are equal: Therefore the square of A E shall be double to the square of A C: Again, forasmuch as in the triangle E G F, the angle G is a right angle, and the two sides G E and G F equal [f] the square of E F shall be double to the square of G F; that is to say, of the square of C D, which is [g] equal to G F: Therefore the two squares of A E and E F, are double to the two squares of A C and C D: But these said squares of A E and E F [h] are equal to the square of A F: Therefore the square of A F shall be double to the two squares of A C and C D: and the square of A F is equal to the two squares of A D and D F: Therefore the two squares of A D and D F are double to the two squares of A C and C D: But the square of D F is equal to the square of D B: for those lines are shewn to be equal: In like manner, therefore the squares of the segments A D and D B shall be also double to the squares of A C and C D, the half of the line A B, and of the middle Section: Therefore, If a line, &c. Which was to be demonstrated.

e) 47. 1.
f) 47. 1.
g) 34. 1.
h) 34. 1.

SCHOLIUM.

Let 10 *be divided equally into* 5 *and* 5, *and unequally into* 7 *and* 3, *the middle Section shall be* 2, *as by the fifth Proposition, the squares of the unequal parts* 7 *and* 3, *which are* 49 *and* 9, *are double to* 25 *and* 4, *the squares of the half and of the middle Section, as is manifest.*

PROP. 10. THEOR. 10.

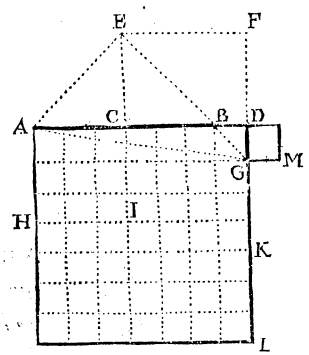


If a right line A B, *be divided into two equal parts in* C, *and there be added to it directly a right line* B D, *the two squares* A L *and* B G *together, of the whole* A B, *with the added line* B D, *are double to the square* H C, *described of the half* A C, *and to the square* C K, *compounded of the half* C B, *and of the added line* B D, *as of one line.*

Demonstration DRaw C E at right angles to the line A B, and make it equal to the half A C, and joyn the right lines A E and E B, and by D draw F D G, parallel to

CE, meeting with EB prolonged in the point G, and by E draw EF parallel to CD, meeting DF in the point F, and let the right line AG be joyned, we ſhall now ſhew that the angle AEB is a right angle as in the foregoing Propoſition, and CEB half a right angle;
a) 29. 1. a and therefore its alternate angle EGF in like manner half a right
b) 34. 1. angle: But the angle b F is a right angle being oppoſite to the right
c) 34. 1. angle C in the Parallelogram CF; c therefore the other angle FEG ſhall
be alſo half a right angle, and therefore equal to EGF: Wherefore EF
d) 6. 1. and FG d oppoſite to the angles FEG and EGF; are in like manner equal,
by the ſame diſcourſe it ſhall be ſhewn that BD and DG are equal;
foraſmuch as the angle BDG is a right angle, and BGD half a
right angle, &c.

c) 47. 1. Foraſmuch therefore, as the ſquare of AE is c equal to the equal ſquares of the equal right lines AC and CE, the ſame ſquare of AE ſhall be double to the ſquare of AC: Again, foraſmuch as the ſquare of EG



f) 47. 1. f is equal to the equal ſquares of the equal right lines EF and FG, the ſame ſquare of EG ſhall be in like manner double to the ſquare of EF, or of CD its equal; for g CD is
g) 34. 1. equal to EF: therefore the two ſquares of AE and EG are double to the ſquares of AC and CD; But the ſquare of AG is equal to the ſquares h of
h) 47. 1. AE and EG, and therefore double to the ſquares of AC and CD, and the ſquares of AD and DG are equal to the ſquare of AG: Therefore the ſquares of AD and DG are double to the ſquares deſcribed of AC and CD: Therefore ſeeing that the ſquare of DG is equal to the ſquare of its equal BD, the ſquares of AD and DB ſhall be in like manner double to the ſquares deſcribed of AC and CD: Therefore, If a right line, &c. Which was to be demonſtrated.

SCHOLIUM.

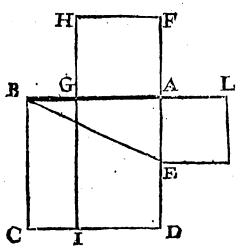
Divide the number 10 *into two equal parts* 5 *and* 5, *to which add ſome other number as* 3, *in ſuch ſort as the whole compound number may make* 13, *whoſe ſquares* 169 *and* 9, *of the numbers* 13 *and* 3, *are double to the ſquares* 25 *and* 64, *which are produced of the numbers* 5 *and* 8, *as is manifeſt.*

PROP. 11. PROBL. 1.

To divide a given right line AB, *in ſuch ſort as that the Rectangle* CG *conteined under the whole* AB, *and one of the ſegments* BG, *may be equal to the ſquare* AH *of the other ſegment* AG.

Demon-

Conſtruction DEſcribe the ſquare A C of the right line A B, and having divided the ſide D A into two equal parts in the point E, draw E B, and prolong D A to the point F, in ſuch ſort as E F be put equal to E B, and from A B cut off A G equal to A F; for A B is greater than A F: Seeing [a] that E A and A B are greater than E B, or E F its equal: Therefore taking away the common part E A, there will reſt A B greater than A F: I ſay that A B is divided in the point G, in ſuch ſort as the Rectangle conteined under A B and B G, is equal to the ſquare of A G, in ſuch ſort as that B G is the leſſer ſegment, and A G the greater. a) 26. 1.



Demonſtration FOr deſcribe A H the ſquare of A G, and prolong the ſide H G towards I, in ſuch ſort as that H I may divide D C in the point I, which [b] ſhall be parallel to F D and B C; therefore C G and D H ſhall be rectangled Parallelograms, and C G ſhall be conteined under the whole A B, and the ſegment B G; for C B is equal to A B, which ſaid Rectangle is equal to the ſquare G F of the other ſegment A G. b) 34. 1.

For ſeeing that D A being divided into two equal parts in E, and A F added thereto, [c] the Rectangle conteined under D F and F A, that is to ſay, the Rectangle D H, (for F H is equal to F A,) with the ſquare of the half A E, is equal to the ſquare of E F, that is to ſay, to the ſquare of E B, which is equal to E F: But the ſquare of E B is equal [d] to the ſquares of E A and A B: Wherefore the Rectangle D H, with the ſquare of A E ſhall be alſo equal to the ſquares of A E and A B, taking away therefore the common ſquare of A E, there will remain the Rectangle D H, equal to the ſquare of A B, which is the ſquare A C: Therefore if the common Rectangle D G be taken away, there will remain the Rectangle G C, equal to the ſquare G F, which was propoſed: Therefore we have divided, &c. Which was to be done. c) 6. 2. d) 47. 1.

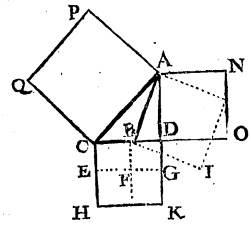
SCHOLIUM.

This Problem cannot in any kind be demonſtrated by numbers: For no number can be divided in ſuch ſort into two parts, as the number produced of the whole multiplyed by one of the parts, may be equal to the ſquare number of the number remaining, as ſhall be demonſtrated in the fourteenth Propoſition of the Ninth Book.

¶ The Demonſtration of the ten foregoing Theorems in numbers, according to *Barlaam*, ſhall follow at the End of this Second Book.

PROP. 12. THEOR. 14.

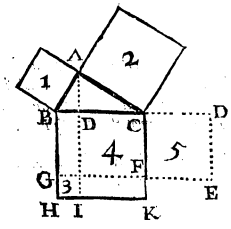
In Ambligonium *triangles* A B C, *the ſquare* A Q, *of the ſide* A C, *which ſubtendeth the obtuſe angle*



ABC, *is greater than the ſquares* AI *and* HF, *of the ſides* AB *and* BC, *which do contein the obtuſe angle* ABC, *by twice the Rectangle* CF, *conteined under one of the ſides which are about the obtuſe angle, to wit,* CB, *on which being prolonged, there doth fall the perpendicular* AD, *and the line* BD, *taken without, between the perpendicular* AD, *and the obtuſe angle* ABC.

a) 4. 2. *Demonſtration* FOr ſeeing that CD is divided at adventure in B, [a] the ſquare of CD, to wit, CK, is equal to the two ſquares of CB and BD, and to twice the Rectangle conteined under CB and BD, adding therefore the common ſquare of AD, to wit, DN, the two ſquares of CD and DA ſhall be equal to the three ſquares of the lines CB, BD, and DA, and to twice the Rectangle conteined under CB and BD: But
b) 47. 1. the ſquare of AC [b] is equal to the two ſquares of CD and DA, the ſame ſquare of AC ſhall be therefore alſo equal to the three ſquares of CB, BD, & DA, and to twice the Rectangle conteined under CB and BD;
c) 47. 1. therefore ſeeing that the ſquare of BA is [c] equal to the ſquares of BD and DA, the ſquare of AC ſhall be equal to the ſquares of CB and BA, and to twice the Rectangle conteined under CB and BD, which is propoſed: Therefore, In, &c. Which was to be demonſtrated.

PROP. 13. THEOR. 12.



In Oxigonium *triangles* ABC, *the ſquare* I *of the ſide* AB, *which ſubtendeth the acute angle* ACB, *is leſſe than the ſquares* 2, *and* BK, *of the ſides* AC *and* CB, *which contein the accute angle* ACB, *by twice the Rectangle* BF, *conteined under one of the ſides* BC, *which are about the accute angle* ACB, *to wit, that on which the*

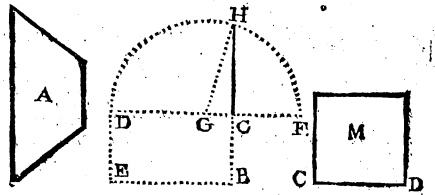
perpendicular AD *doth fall, and the line* DC *taken from within, between the perpendicular* AD, *and the accute angle* ACB.

Demonſtration FOr ſeeing that the right line BC is divided at adventure in the point D, the ſquares of [a] BC and of CD, are equal to twice the Rectangle conteined under BC and CD, and to the ſquare BD: Therefore if you add the common ſquare of DA, the three ſquares of BC, CD, and DA, ſhall be equal to twice the Rectangle conteined under BC and CD, and to the two ſquares of BD and DA: But [b] the ſquare of CA is equal to the two ſquares of CD and DA; therefore the two ſquares of BC and CA are equal to twice the Rectangle conteined under BC and CD, and to twice the ſquares of BD and DA; Therefore ſeeing that the ſquare of AB is equal to the two ſquares of BD and DA, the two ſquares of BC and CA ſhall be equal to twice the Rectangle conteined under BC and CD, and to the ſquare of AB, which was propoſed. a) 7. 2. b) 47. 1.

In the ſame manner, we ſhall demonſtrate that the ſquares of AB and BC are equal to twice the Rectangle conteined under CB and BD, and to the ſquare of AC, that is to ſay, that the ſquare of the ſide AC is leſſe than the ſquares of the ſides AB and BC, by twice the Rectangle conteined under CB and BD: Therefore, In Oxigonal triangles, &c. Which was to be demonſtrated.

PROP. 14. PROBL. 2.

To conſtitute a ſquare M, *equal to a given right lined figure* A.



Conſtruction MAke [a] the rectangled Parallelogram BCDE, equal to the right lined figure A, and prolong one of its ſides, as DC to F, and make CF equal to GB, and divide DF into two equal parts in the point G, which ſhall be the ſame as the point C, or not; if it fall on the point C, the right line BC, (ſeeing it is made equal to CF,) ſhall be equal to CD; therefore the Rectangle BD ſhall be a ſquare, and ſeeing [b] that the ſides BE and ED are equal to the oppoſite ſides BC and CD, you have what was ſought: But if the point G doth not fall on the point C, from the point G as a center, and at the diſtance GF or GD, deſcribe the ſemicircle FHD, and prolong BC a) 45. 1. b) 34. 1.

BC to the Circumference H, and draw alſo GH: I ſay that the ſquare of the right line CH is equal to the right lined figure A.

Demonſtration FOraſmuch as DF is divided into two equal parts in G, and in two unequal parts in C, the Rectangle conteined
c) 5. 2. under DC and CF, to wit the Rectangle BD, c with the ſquare of GC is equal to the ſquare of GF, or of GH equal thereto: But the ſquare
d) 34. 1. of GH is d equal to the two ſquares GC and CH: Therefore the Rectangle BD, with the ſquare of GC, is equal to the two ſquares of GC and CH: Therefore if you take away the common ſquare GC, there will remain the Rectangle BD; that is to ſay, the right lined figure A equal to the ſquare of CH: Therefore, We have conſtituted, &c. Which was to be done.

The End of the Second Element of EUCLIDE.

The Demonſtration of the ten firſt Theorems of this Second Book of EUCLIDE in Numbers, according to *Barlaam.*

¶ PRINCIPLES.

1 *A number is ſaid to multiply another number: when the number multiplyed is ſo oftentimes added to it ſelf, as there be unities in the number which multiplyeth: whereby is produced a certain number, which the number multiplyed, meaſureth by the unities which are in the number which multiplieth.*

2 *And the number produced of that multiplication is called a plain or ſuperficial number.*

3 *A ſquare number is that which is produced of the multiplication of any number into its ſelf.*

4 *Every leſſe number compared to a greater, is ſaid to be a part of the greater, whether the leſſe meaſure the greater, or meaſure it not.*

5 *Numbers, whom one and the ſelf ſame number meaſureth equally, that is, by one and the ſelf ſame number are equal the one to the other.*

6 Num-

6 Numbers that are equimultiplices to one and the self same number, that is, which contein one and the same number equally and alike, are equal the one to the other.

PROPOSITION I.

C....4
A..2 D..2 E..2 B
F.....24
G.....8 H.....8 I.....8 K

Two numbers being given, if one of them be divided into any numbers, how many soever: the plain or superficial number which is produced of the multiplication of the two numbers first given, the one into the other, shall be equal to the superficial numbers which are produced of the multiplication of the number not divided into every part of the number divided.

Demonstration Suppose that there be two numbers A B and C. And divide the number A B into certain other numbers, how many soever, as into A D, D E, and E B. Then I say that the superficial number which is produced of the multiplication of the number C into the number A B, is equal to the superficial numbers which are produced of the multiplication of the number C into the number A D, and of C into D E, and of C into E B. For let F be the superficial number produced of the multiplication of the number C into the number A B, and let G H be the superficial number produced of the multiplication of C into A D: And let H I be produced of the multiplication of C into D E: and finally, of the multiplication of C into F B, let there be produced the number I K: Now forasmuch as A B multiplying the number C produced the number F: Therefore the number C measureth the number F by the unities which are in the number A B. And by the same reason may be proved that the number C doth also measure the number G H, by the unities which are in the number A D, and then it doth measure the number H I, by the unities which are in the number D F; and finally, that it measureth the number I K by the unities which are in the number E B. Wherefore the number C measureth the whole number G K by the unities which are in the number A B. But it before measured the number F by the unities which are in the number A B, wherefore either of these numbers F and G K is equimultiplex to the number C: But numbers which are equimultiplices to one and the self same numbers, are equal the one to the other [a]: Wherefore the number F is equal to the number G K: But the number F is the superficial number produced of the multiplication of the number C into the number A B: and the number G K is composed of the superficial numbers produced of the multiplication of the number C not divided into every one of the numbers A D, D E, and E B: If therefore there be two numbers given, and the one of them be divided, &c. Which was to be demonstrated.

a) Def. 6.

PROPOSITION II.

If a number given, be divided into two other numbers: the su-

ſuperficial numbers, which are produced of the multiplication of the whole into either part, added together, are equal to the ſquare number of the whole number given.

A....4 C..2 B
D.....36
B.....24 F.....12

Demonſtration Suppoſe that the number given be A B: and let it be divided into two other numbers A C and C B. Then I ſay that the two ſuperficial numbers, which are produced of A B into A C, and of A B into B C, thoſe two ſuperficial numbers (I ſay) being added together, ſhall be equal to the ſquare number produced of the multiplication of the number A B into it ſelf. For let the number A B multiplying it ſelf produce the number D. Let the number A C alſo multiplying the number A B, produce the number E F: Again, let the number C B multiplying the ſelf ſame number A B, produce the number F G: Now foraſmuch as the number A C multiplying the number A B, produced the number E F: therefore the number A B meaſureth the number E F by the unites which are in A C. Again, foraſmuch as the number C B multiplyed the number A B, and produced the number F G; therefore the number A B meaſureth the number F G, by the unites which are in the number C B: But the ſame number A B before meaſured the number E F by the unites which are in the number A C: Wherefore the number A B meaſureth the whole number E G, by the unites which are in A B. Farther, foraſmuch as the number A B multiplying it ſelf, produced the number D: therefore the number A B meaſureth the number D by the unites which are in himſelf. Wherefore it meaſureth either of theſe numbers; namely, the number D, and the number E G, by the unities which are in himſelf. Wherefore how Multiplex the number D, is to the number A B, ſo Multiplex is the number E G to the ſame number A B. But numbers which are equimuliplices to one and the ſelf ſame number, are equal the one to the other. Wherefore the number D is equal to the number E G. And the number D is the ſquare number made of the number A B, and the number E G is compoſed of the two ſuperficial numbers produced of A B into B C, and of B A into A C. Wherefore the ſquare number produced of the number A B, is equal to the ſuperficial numbers produced of the number A B into the number B C, and of A B into A C, added together: If therefore a number be divided into two other numbers, &c. Which was to be demonſtrated.

PROPOSITION III.

A....4 C..2 B
D.....12
E.....8 F....4 G

If a number given, be divided into two numbers: the ſuperficial number which is produced of the multiplication of the whole into one of the parts, is equal to the ſuperficial number which is produced of the parts the one into the other, and to the ſquare number produced of the aforeſaid part.

Demon-

Demonſtration Suppoſe that the number given be A B, which let be divided into two numbers A C and C B. Then I ſay that the ſuperficial number which is produced of the multiplication of the number A B into the number B C, is equal to the ſuperficial number which is produced of the multiplication of the number A C into the number C B, and to the ſquare number produced of the number C B. For let the number A B multiplying the number C B, produce the number D. And let the number A C multiplying the number C B, produce the number E F: and finally, let the number C B multiplying himſelf produce the number F G. Now foraſmuch as the number A B multiplying the number C B, produced the number D. Therefore the number C B meaſureth the number D, by the unites which are in the number A B. Again, foraſmuch as the number A C multiplying the number C B, and produced the number E F, therefore the number C B meaſureth the number E F, by the unites which are in A C. Again, foraſmuch as the number C B multiplied it ſelf and produced the number F G: therefore the number C B meaſureth the number F G by the unites which are in it ſelf. But as we have before proved, the ſelf ſame number C B meaſureth alſo the number E F by the unites which are in the number A C, wherefore the number C B meaſureth the whole number E G, by the unites which are in the number A B. And it alſo meaſureth the number D by the unites which are in the number A B. Wherefore the number C B equally meaſureth either number, namely, the number D, and the number E G. But thoſe numbers whom one and the ſelf ſame number meaſureth equally, are equal the one to the other. Wherefore the number D is equal to the number E G. But the number D is a ſuperficial number produced of the multiplication of the number A B into the number B C, and the number E G is the ſuperficial number produced of the multiplication of the number A C into the number C B, and of the ſquare of the number C B. Wherefore the ſuperficial number produced of the multitiplication of the number A B into the number C B, is equal to the ſuperficial number produced of the number A C into the number C B, and to the ſquare of the number C B. If therefore a number be divided into two numbers, the ſuperficial number, &c. Which was required to be proved.

A....4 C..2 B
D.....12
E.....8 F....4 G

PROPOSITION IV.

If a number given be divided into two numbers, the ſquare number of the whole, is equal to the ſquare numbers of the parts, and to the ſuperficial number which is produced of the multiplication of the parts the one into the other twice.

Demonſtration Suppoſe that the number given be A B; which let be divided into two numbers A C and C B. Then I ſay, that the ſquare number of the whole number A B, is equal to the ſquares of the parts; that is, to the ſquares of the numbers A C and C B, and to the ſuperficial number produced of the multiplication of the numbers A C

and C B, one into the other twice. Let the ſquare number produced of the multiplication of the whole number A B, into himſelf be D. And let C A multiplyed into himſelf produce the number E F: And C B multiplyed into it ſelf let it produce G H: and finally, of the multiplication of the numbers A C and C B, the one into the other twice, let there be produced either of theſe ſuperficial numbers F G and H K. Now foraſmuch as the number A C multiplying it ſelf produced the number E F: therefore the number A C meaſureth the number E F by the unites which are in it ſelf. And foraſmuch as the number C B multiplyed the number C A,

A.....6 C..2 B
D.....64
E.....36 F.....12 G....4 H.....12 K

and produced the number F G: therefore the number A C meaſureth the number F G by the unites which are in the number C B. But it before alſo meaſured the number E F by the unites which are in it ſelf. Wherefore the number A B multiplying the number A C, produceth the number E G. And therefore the number E G is the ſuperficial number produced of the multiplication of the number B A into the number A C. And by the ſame reaſon may we prove that the number G K is the ſuperficial number produced of the multiplication of the number A B into the number B C. Farther the number D is the ſquare of the number A B. But if a number be divided into two numbers, the ſquare of the whole number is equal to the two ſuperficial numbers which are produced of the multiplication of the whole into either the parts, (by the ſecond Theorem.) Wherefore the ſquare number D is equal to the ſuperficial number E K. But the number E K is compoſed of the ſquares of the numbers A C and C B, and of the ſuperficial number which is produced of the multiplication of the number A C & C B the one into the other twice: and the number D is the ſquare of the whole number A B. Wherefore the ſquare number produced of the multiplicatiõ of the number A B into its ſelf is equal to the ſquare numbers of the parts, that is, to the ſquare numbers of the numbers A C and C B, and to the ſuperficial number produced of the multiplication of the numbers A C and C B, the one into the other twice: If therefore a number given be divided into two numbers, &c. Which was required to be proved.

PROPOSITION V.

A....4 C..2 D..2 B
E.....16
F.....12 G....4 H
K....4 L....4 M....4 N....4 X

If an even number be divided into two equal parts, and again, alſo into two unequal parts, the ſuperficial number which is produced of the multiplication of the unequal parts the one into the other, together with the ſquare of the number ſet between the parts, is equal to the ſquare of half the number.

Demonſtration Suppoſe that A B be an even number, which let be divided into two equal numbers A C and C B, and into two unequal numbers A D and D B. Then I ſay, that the ſquare number which is produced of the multiplication of the half number C B into it ſelf, is equal to the ſuperficial number produced of the multiplication of the unequal

unequal numbers A D and D B, the one into the other, and to the ſquare number produced of the number C D, which is ſet between the ſaid unequal parts. Let the ſquare number produced of the multiplication of the half number C B into it ſelf be E. And let the ſuperficial number produced of the multiplication of the unequal numbers A D and D B, the one into the other, be the number F G; and let the ſquare of the number D C which is ſet between the parts be G H. Now foraſmuch as the number B C is divided into the numbers B D and D C; therefore the ſquare of the number B C, that is, the number E, is equal to the ſquares of the numbers B D and D C, and to the ſuperficial number which is compoſed of the multiplication of the numbers B D and D C, the one into the other twice, (by the fourth Propoſition of this Book:) Let the ſquare of the number B D be the number K L; and let N X be the ſquare of the number D C: and finally, of the multiplication of the numbers B D and D C, the one into the other twice, let be produced either of theſe numbers L M and M N. Wherefore the whole number K X is equal to the number E. And foraſmuch as the number B D multiplying it ſelf produced the number K L; therefore it meaſureth it by the unites which are in it ſelf. Moreover, foraſmuch as the number C D multiplying the number B D, produced the number L M; therefore alſo D B meaſureth L M by the unites which are in the number C D: but it before meaſured the number K L by the unites which are in it ſelf. Wherefore the number D B meaſureth the whole number K M by the unites which are in C B. But the number C B is equal to the number C A. Wherefore the number D B meaſureth the number K M by the unites which are in C A. Again, foraſmuch as the number C D multiplying the number D B produced the number M N: therefore the number D B meaſureth the number M N, by the unites which are in the number C D: but it before meaſured the number K M by the unites which are in the number A C. Wherefore the number B D meaſureth the whole number K N by the unites which are in the number A D. Wherefore the number F G is equal to the number K N. For numbers which are equimultiplices to one and the ſelf-ſame number, are equal the one to the other. But the number G H is equal to the number N X: for either of them is ſuppoſed to be the ſquare of the number C D. Wherefore the whole number K X is equal to the whole number F H: But the number K X is equal to the number E. Wherefore alſo the number F H is equal to the number E. And the number F H is the ſuperficial number produced of the multiplication of the numbers A D and D B the one into the other, together with the ſquare of the number D C. And the number E is the ſquare of the number C B. Wherefore the ſuperficial number produced of the multiplication of the unequal parts A D and D B, the one into the other, together with the ſquare of the number D C, which is ſet between thoſe unequal parts, is equal to the ſquare of the number C B, which is the half of the whole number A B: If therefore an even number be divided into two equal parts, &c. Which was required to be proved.

PROPOSITION VI.

If an even number be divided into two equal numbers, and unto it be added ſome other numbers, the ſuperficial number which is made of the multiplication of the number compo-

ed of the whole number and the number added, into the number added, together with the square of the half number, is equal to the square of the number compoſed of the half and the number added.

A...3 C...3 B....4 D
E.....49
F.....40 G.....9 H
K.....16 L.....12 M.....12 N.....9 X

Demonſtration SUppoſe that A B be an even number, and let it be divided into two equal numbers A C and C B: and unto it let there be added an other number B D. Then I ſay, that the ſuperficial number produced of the multiplication of the number A D into the number D B, is equal to the ſquare of the number C D. For let the ſquare number of the number C D be the number E, and let the ſuperficial number produced of the multiplication of the number A D into the number D B, be the number F G; and finally, let the ſquare number of C B be the number G H: And foraſmuch as the ſquare of the number C D is (by the 4th. Propoſition) equal to the ſquares of the numbers D B and B C together, with the ſuperficial number which is produced of the multiplication of the numbers D B and B C, the one into the other twice. Let the ſquare of the number B D be the number K L: and let the ſuperficial numbers produced of the multiplication of the numbers D B and B C, the one into the other twice be either of theſe numbers L M and M N: and finally, let the ſquare of the number B C be the number N X. Wherefore the whole number K X ſhall be equal to the ſquare of the number C D: But the ſquare of the number C D is the number E. Wherefore the number K X is equal to the number E. And foraſmuch as the number B D multiplying it ſelf, produced the number K L: therefore the number B D meaſureth the number K L, by the unites which are in it ſelf, but it alſo meaſureth the number L M by the unites which are in the number C B. Wherefore the number D B meaſureth the whole number K M by the unites which are in the number C D. The number D B alſo meaſureth the number M N by the unites which are in the number C B: and the number C B is equal to the number C A by ſuppoſition. Wherefore the number D B meaſureth the whole number K N by the unites which are in the number A D. But the number D B doth alſo meaſure the number F G, by the unites which are in the number A D: for by ſuppoſition, the number F G is the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other. Wherefore the number F G is equal to the number K N. But the number H G is equal to the number N X: for either of them is the ſquare number of the number C B. Wherefore the whole number F H is equal to the number K X, and the number K X is proved to be equal to the number E. Wherefore the number F H ſhall alſo be equal to the number E. And the number F H is the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other, together with the ſquare of the number C B, and the number E is the ſquare of the number C D: Wherefore the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other, together with the ſquare of the number C B, is equal to the ſquare of the number C D: If therefore an even number, &c. Which was to be demonſtrated. PRO-

PROPOSITION VII.

If a number be divided into two numbers: the ſquare of the whole number together with the ſquare of one of the parts, is equal to the ſuperficial number produced of the multiplication of the whole number into the foreſaid part twice, together with the ſquare of the other part.

A.....5 C...3 B

Demonſtration SUppoſe that the number A B be divided into the numbers A C and C B. Then I ſay, that the ſquare numbers of the numbers B A and A C are equal to the ſuperficial number produced of the multiplication of the number B A into the number A C twice, together with the ſquare of the number B C. For foraſmuch as (by the 4th. Prop. of this Book,) the ſquare of the number A B is equal to the ſquares of the numbers B C and C A, and to the ſuperficial number produced of the multiplication of the numbers B C and C A, the one into the other twice: add the ſquare of the number A C common to them both. Wherefore the ſquare of the number A B, together with the ſquare of the number A C is equal to two ſquares of the number A C, and to one ſquare of the number C B, and alſo to the ſuperficial number produced of the multiplication of the numbers B C and C A, the one into the other twice. And foraſmuch as the ſuperficial number produced of the multiplication of the numbers B A and C A, the one into the other once, is equal to the ſuperficial number produced of the multiplication of B C into C A once, and to the ſquare of the number C A (by the third Propoſition of this Book): therefore the number produced of the multiplication of B A into A C twice, is equal to the number produced of the multiplication of B C into C A twice, and alſo to two ſquares of the number C A. Add the ſquare number of B C common to them both. Wherefore two ſquares of the number A C, and one ſquare of the number C B, together with the ſuperficial number produced of the multiplication of B C into C A twice, are equal to the ſuperficial number produced of the multiplication of the number B A into the number A C twice together, with the ſquare of the number C B. Wherefore the ſquare of the number A B, together with the ſquare of the number A C is equal to the ſuperficial number produced of the multiplication of the number B A into the number A C twice, together with the ſquare of the number C B. If therefore a number be divided ihto two numbers, &c. Which was required to be demonſtrated.

8		64	the ſquare of the whole A B.
8		25	the ſquare of the part A C.
64	the ſquare of the whole	89	
5		80	the ſuperficial number.
5		9	the ſquare of the other part B C
25	the ſquare of the part	89	

8 8
5
40 40
80 the superficial number produced of the multiplication of the whole into the part twice.

3 3
9 the square of the other part.

PROPOSITION VIII.

If a number be divided into two numbers, the superficial number produced of the multiplication of the whole into one of the parts four times, together with the square of the other parts, is equal to the square of the number composed of the whole number and the foresaid part.

A.....6 C..2 B..2 D

Demonstration Suppose that the number A B be divided into two numbers A C and C B. Then I say, that the superficial number produced of the multiplication of the number A B into the number C B four times, together with the square of the number A C, is equal to the square of the number composed of the numbers A B and C B. For unto the number B C let the number B D be equal. Now forasmuch as the square of the number A D is equal to the squares of the numbers A B and B D, and to the superficial number produced of the multiplication of the numbers A B and B D, the one into the other twice, (by the 4th. Prop. of this Book.) And the number B D is equal to the number B C: therefore the square of the number A D is equal to the squares of the numbers A B and B C, and to the superficial number produced of the multiplication of the numbers A B and B C, the one into the other twice. But the squares of the numbers A B and B C are equal unto the superficial number produced of the multiplication of the numbers A B and B C, the one into the other twice, and to the square of A C (by the eighth Proposition.) Wherefore the square of the number A D is equal to the superficial number produced of the multiplication of the numbers A B and B C, the one into the other four times, and to the square of the number A C. But the square of the number A D is the square of the number composed of the numbers A B and B C: for the number B D is equal to the number B C. Wherefore the square of the number composed of the numbers A B and B C, is equal to the superficial number produced of the multiplication of the numbers A B and B C, the one into the other four times, and to the square of the number A C. If therefore a number be divided into two numbers, &c. Which was to be demonstrated.

8 8 8 8
2 2 2 2
16 16 16 16
64 the superficial number produced of the multiplication of the numbers A B and B C, the one into the other four times.

6
6
36 the square of A C.

10
10
100 the square of the number composed of A B and B C.

64 the superficial number produced of the multipl. made 4 times.
36 the square number of A C.
100

PROPOSITION IX.

If a number be divided into two equal numbers, and again be divided into two unequal parts: the square numbers of the unequal numbers, are double to the square which is made of the multiplication of the half number into it self, together with the square which is made of the number set between them.

A.....5 C...3 D..2 B.

Demonstration FOr let the number A B being an even number be divided into two equal numbers A C and C B: and into two unequal numbers A D and D B. Then I say that the square numbers of A D and D B are double to the squares which are made of the multiplication of the numbers A C and C D into themselves. For, forasmuch as the number A B is an even number, and is divided also into two equal numbers A C and C B, and afterward into two unequal numbers A D and D B: therefore the superficial number produced of the multiplication of the numbers A D and D B, the one into the other, together with the square of the number D C, is equal to the square of the number A C, (by the fifth proposition.) Wherefore the superficial number produced of the multiplication of the numbers A D and D B, the one into the other twice, together with two squares of the number C D, is double to the square of the number A C. Forasmuch as also the number A B is divided into two equal numbers A C and C B, therefore the square number of A B is quadruple to the square number produced of the multiplication of the number A C into it self (by the 4th. Proposition) Moreover forasmuch as the superficial number produced of the multiplication of the numbers A D and D B, the one into the other twice together, with two squares of the number D C, is double to the square number of C A: and forasmuch

as

as there are two numbers, of which the one is quadruple to one and the self-same number, and the other is double to the same number: therefore that number which is quadruple, shall be double to that number which is double. Wherefore the square of the number A B is double to the number produced of the multiplication of the numbers A D and D B, the one into the other twice together, with the two squares of the number D C. Wherefore the number which is produced of the multiplication of the numbers A D and D B, the one into the other twice, is lesse than half of the square of the number A B, by the two squares of the numbers D C. And forasmuch as the number produced of the multiplication of the numbers A D and D B, the one into the other twice, together with the number composed of the squares of the numbers A D and D B is (by the fourth Proposition) equal to the square of the number A B: therefore the number composed of the squares of the numbers A D and D B is greater than the half of the square number of A B, by the two squares of the number D C. And the square of the number A B is quadruple to the square of the number A C. Wherefore the number composed of the squares of the numbers A D and D B is greater than the double of the square of the number A C by two squares of the number D C. Wherefore the said number is double to the squares of the numbers A C and C D: If therefore a number be divided, &c. Which was to be demonstrated.

8 2
8 2
64 the square of the unequal part A D. 4 the square of the unequal part B D.

5
5
25 the square of the half A C.

3
3
9 the square of C D, namely, of the number set between.

25
9
34 the squares of the half, and of the number set between.

64 68 | 2
4 34
68 the squares of the unequal parts.

PROPOSITION X.

If an even number be divided into two equal numbers, and unto it be added any other number: the square number of the whole number composed of the number and of that which is added, and the square number of the number added: these two square

square numbers (I say,) added together, are double to these square numbers; namely, to the square of the half number, and to the square of the number composed of the half number, and of the number added.

A ...3 C ...3 B ..2 D

Demonstration Suppose that the number A B being an even number, be divided into two equal numbers A C and C B: and unto it let be added an other number B D. Then I say, that the square numbers of the numbers A D and D B, are double to the square numbers of A C and C D. For, forasmuch as the number A D is divided into the numbers A B and B D: therefore the square numbers of the numbers A D and D B are equal to the superficial number produced of the multiplication of the numbers A D and D B, the one into the other twice, together with the square of the number A B, (by the seventh Proposition.) But the square of the number A B is equal to four squares of either of the numbers A C or C B (for AC is equal to the number C B): wherefore also the squares of the numbers A D and D B are equal to the superficial number produced of the multiplication of the numbers A D and D B, the one into the other twice, and to four squares of the number B C or C A. And forasmuch as the superficial number produced of the multiplication of the numbers A D and D B the one into the other, together with the square of the number C B, is equal to square of the number C D, (by the sixth Proposition): therefore the number produced of the multiplication of the numbers A D and D B, the one into the other twice together, with two squares of the number C B, is equal to two squares of the number C D. Wherefore the squares of the numbers A D and D B are equal to two squares of the number C D, and to two squares of the numbers A C. Wherefore they are double to the squares of the numbers A C and C D. And the square of the number A D is the square of the whole and of the number added: And the square of D B is the square of the number added; the square also of the number C D is the square of the number composed of the half and of the number added: If therefore an even number be divided, &c. Which was required to be proved.

8 2
8 2
64 the square of A D. 4 the square of D B

5
5
25 the square of C D, namely, of the number composed of the half and of the number added.

3 25 64
3 9 4
9 the square of the half A C. 34 68 | 2.
68 34

The End of the ten first Propositions according to Barlaam.

THE THIRD ELEMENT OF EUCLIDE.

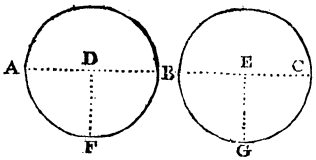
THE ARGUMENT.

THis Third Book of *EUCLIDE* treateth of the circle, which of all figures is the moſt perfect, the properties and paſſions whereof are herein ſet forth and demonſtrated Here alſo is declared the nature and property of lines applyed to circles, either touching or cutting one another, either within or without the circle, as alſo of right lined angles drawn in the ſegments of circles, &c. With diverſe uſeful Propoſitions applicable to diverſe neceſſary and profitable uſes.

DEFINITIONS.

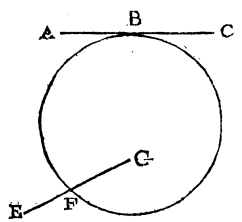
1 *Equal Circles are ſuch whoſe diameters are equal, or whoſe right lines drawn from the centers to the circumferences, are equal.*

FOraſmuch as *EUCLIDE* demonſtrateth in this Third Book divers properties of Circles, he unfoldeth in the firſt place certain terms; whoſe uſe is frequent therein: he ſhewes therefore firſt of all, that thoſe circles are equal whoſe diameters or ſemidiameters are equal; for ſeeing the circle is deſcribed by the revolution of the ſemidiameter about one



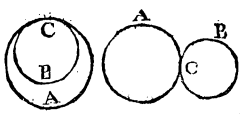
one of its extremities fixed and unmoveable, as is ſhewn in the Firſt Book: It is manifeſt that thoſe circles are equal, whoſe ſemidiameters or right lines, drawn from the centers to the circumferences are equal: as if the diameters A B and B C are equal, or the right lines D F and E G drawn from the centers D and E, the circles A F B and B G C ſhall be equal; and contrarily, if the circles be equal, the diameters or right lines drawn from the centers to the circumferences, ſhall be equal.

Whence it appeares, that the circles (whoſe diameters or right lines drawn from the centers to the circumferences are unequal,) are unequal; and therefore that circle that hath the greater Diameter, is alſo the greateſt circle; and contrarily of unequal circles, the diameters are unequal.

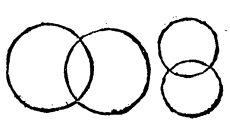


2 *A right line is ſaid to touch a circle, which touching it, if it be prolonged, doth not cut it.*

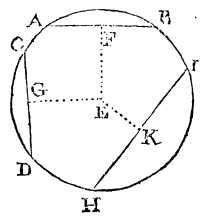
AS A B ſhall be ſaid to touch the circle, if it ſo touch it at B, as that being prolonged to C, doth not cut it; But foraſmuch as E F doth ſo touch touch the circle in the point F, that being prolonged to the point G, it cuteth it, and falls within, it ſhall not be ſaid to touch the circle, but to cut it.



3 *Thoſe circles are ſaid to touch one another, which in touching one another, do not interſect or cut one another.*

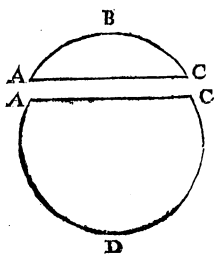


AS the two circles A C and B C are ſaid to touch at C, if they touch in ſuch ſort as that they cut not one another, be the one within the other or without it: But if two circles touch one another otherwiſe, they ſhall be ſaid to cut one another.



4 *In a circle right lines are ſaid to be equally diſtant from the center, when the perpendiculars which are drawn on them from the center, are equal; but that is ſaid to be fartheſt diſtant from the center, on which the greateſt perpendicular doth fall.*

AS the two right lines A B and C D in the circle A B C D ſhall be ſaid to be equally diſtant from the center, if the perpendiculars E F and E G be equal: But the line C D ſhall be ſaid to be more diſtant from the center than the line H I, if the perpendicular E G be greater than the perpendicular E K.



5 *A Segment, or Section of a circle is a figure compriſed under a right line, and the circumference of the circle.*

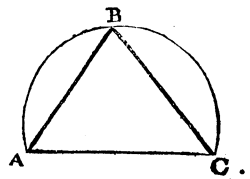
IF in the circle A B C D, let there be drawn a right line by chance as A C, as well the figure A B C conteined under the right line A C, and the circumference A D C ſhall be ſaid to be a Segment, or Section of a circle.

From whence may be gathered, that there are three ſorts of ſegments; to wit, the ſemicircle, when the right line which with the circumference, conſtituteth the ſegment, doth paſſe through the center: The ſegment greater than the ſemicircle, when the center of the circle is within it, as A D C: and the leaſt ſegment is when the figure of the ſaid ſegment is leſſe than the ſemicircle, as A B C. The right line A C ſhall be alſo called a Chord, and the circumference A B C or A D C an Arch.

6 *The angle of the ſegment, or of the ſection, is that which is compriſed under a right line, and the circumference of a circle.*

EUCLIDE now defineth three kinds of angles which are conſidered in circles. In the firſt place the angle of the ſegment, as is the mixt angle B A C or B C A, conteined under the right line A C, and the circumference A B C, and if the ſegment be a ſemicircle, it ſhall be called the angle of the Semicircle, if the ſegment be greater, it ſhall be termed

termed the angle of the greater ſegment; if leſſe than the ſemicircle, it ſhall be termed the angle of the leſſer ſegment.

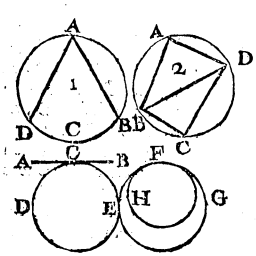


7 *But an angle is in a ſegment or in a ſection, when there is taken ſome point in the circumference of the ſegment, and from it are drawn two right lines, on the extremities of the right line, which is the baſe of the ſegment, and is that (I ſay,) which is conteined under thoſe drawn right lines.*

LET A B C be a ſegment of a circle, and let the right line A C be the baſe, and from any point as B taken in the circumference, draw the two right lines A B and B C, to the points A and C, which are the extremities of the baſe A C, the right lined angle A B C conteined of them, is ſaid to be an angle in a ſegment.

8 *But when the right lines which do contein the angle, do take or receive ſome circumference, the angle is ſaid to inſiſt upon it.*

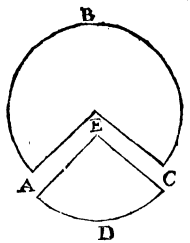
AS the angle D A B conteined of the two right lines A B and A D, which do receive the circumference B C D, ſhall be ſaid to be, or to inſiſt thereon, and differeth not from the afore-going Definition, but only in the term; For if you draw a right line from B to D for the baſe, it ſhall be ſaid to be in the ſegment, according to the precedent Definition, and by this, to reſt or inſiſt on the circumference B C D: Nevertheleſſe, it refers to divers things, for the angle in the ſegment refers to the ſegment in which it is; and the angle inſiſting on the circumference refers it ſelf to the circumference which is its baſe: Therefore if you take ſome ſegment of a circle, as B C D, of the circle A B C D, the angle which is in this ſegment, ſhall not be the ſame with that which inſiſteth or reſteth upon it; for the angle which is therein ſhall be B C D: But the angle which inſiſteth or reſteth on the ſame circumference, ſhall be the angle B A D which is different from it.



Beſides, theſe three ſorts of angles aforeſaid, the Geometricians do conſider alſo an angle of contingence, which is conteined of a right line, touching the circle, and of the circumference of a

circle

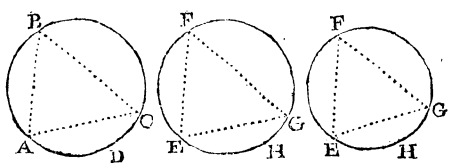
circle, or of two circumferences, touching one another either without or within, as for example, if the right line AB touch the circle CED in the point C, the mixt angle ACD or BCE shall be termed the angle of contingence: In like manner, the curvilined angle GFH or EFH, made of the circumferences of two circles touching one another, shall be also termed the angle of contingence.



9 *A Sector of a circle, is a figure conteined under two right lines, which constitute an angle at the center; and the circumference comprised between the same lines.*

AS the figure AECD, conteined under the right lines AE and EC, which make the angle at the center E, and under the circumference ADC, comprised between A and C, the extremities of the same lines, is termed a Sector of a circle.

10 *Like Segments or Sections of Circles, are those which receive equal angles, or in which the angles are equal to one another.*



AS if in the circles ABCD and EFGH, the right lined angles B and F in the Sections ABC and EFG, are equal, those Sections which do receive those said equal angles, or in which those equal angles are, shall be said to be like Sections, and the circumferences ABC and EFG in like manner shall be alike, that is to say, that the Section ABC shall be such part of the whole circle ABCD, as the Section EFG is of the circle EFGH, and the circumference ABC shall be the same part of the whole circumference ABCD, as the circumference EFG is of the whole circumference EFGH.

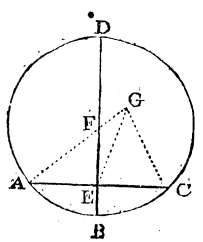
By the same reason, the Arches or Circumferences on which equal angles insist, are also alike, as are the Arches or Circumferences ADC and EHG, on which the equal angles B and F insist.

From these things may be gathered that like Sections of one and the same circle, or of equal circles, are also equal to one another, being the same parts of one and the same thing, or of equal things.

PRO-

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. PROBLEM 1.



A circle ABCD *being given, to find the center* F.

Construction DRaw any line therein at adventure, as AC, [a] which may be divided in two equal parts in the point E, and by E draw BD at right angles, terminating on both sides of the circumference in the points B and D, and let it be divided into two equal parts in the point F: I say, that F is the center of the proposed Circle. a) 10. 1.

Demonstration FOr no other point therein can be the center, considering that every other point doth divide it unequally: Therefore if F be not the center of the circle, let G be the center thereof without the line BD, from which draw the right lines GA, GE, and GC.

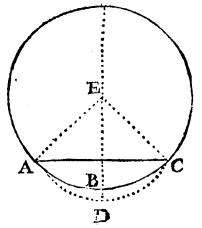
Forasmuch as, in the triangle AEG, the two sides AE and EG are equal to the two sides CE and EG of the triangle CEG, and the base AG equal to the base CG, (for that they are said to be drawn from the center,) [b] the angles AEG and CEG shall be equal, and therefore right angles. But the angle AEF is a right angle by construction; therefore the right angles AEF and AEG shall be equal, the part to the whole, which is absurd: Therefore G is not the center of the circle; nor by the same reason, is any other point out of the line DB: Therefore F shall be the center: Therefore we have, &c. Which was to be done. b) 8. 1.

COROLLARIE.

From the Proposition it is evident that if in a circle a right line be divided into two equal parts, and at right angles, by another right line, the center of the circle shall be in that dividing line, provided it end and terminate on both sides of the circumference, or at least; that it be not lesse than the semidiameter of the circle, or that the divided line doth passe by the center, considering that it is demonstrated that

that the center of the circle A B C D, *cannot be other-where then in the point* F *of the line* B D, *which divideth* A C *into two equal parts, and at right angles.*

PROP. 2. THEOR. 1.



If in the circumference of a circle A B C, *there be taken two points at pleasure* A *and* C, *the right line* A C *joyning those points* A *and* C, *will fall within the circle* A B C.

Demonstration FOr if it fall not within the circle, let it (if possible,) fall without such as is the right line A D C, (as the contradictor will have it,) and having [a] found the center thereof E, draw the right lines E A, E C, and E D, to some point as at D, of the line A D C, in such sort as that it cut the circumference of the circle A B C in the point B.

a) 1. 3.

Forasmuch as the two sides E A and E C of the triangle E A C, to which the line A D C is put for base, are equal, (being drawn from the center,) [b] the angles E A D and E C D shall be equal. But the exterior [c] angle E D A is greater than its interior and opposite angle E C D: seeing that the side C D is prolonged to the point A. Therefore the same angle E D A [d] shall be also greater than the angle E A D, which is equal to E C D: Therefore E A opposite to the greatest angle E D A, shall be greater than E D, opposite to the least E A D: Therefore E B equal to E A should be also greater than E D, the part than the whole, which is absurd: Therefore the right line drawn from A to B, will not fall without the circle, but within it: In like manner, we might shew that it will not fall on the circumference: Therefore, If in a circle, &c. Which was to be demonstrated.

b) 5. 1.
c) 16. 1.
d) 19. 1.

COROLLARIE.

From the Demonstration of this second Propesition it is manifest, that the right line which toucheth the circle, in such sort as that it divide not the said circle, toucheth it only in one point, for if it touch it in two points, [e] *the part of the line between those points would fall within the circle; and therefore would divide it, which is contrary to Suppostion.*

e) 2. 3.

PROP. 3. THEOR. 2.

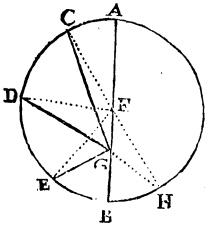
If in a circle B C D E, *some right line* E C *passing by the center* A, *doth cut some other right line* B D, *which doth not passe by the center* A, *into two equal parts in* F, *it will also cut it at right angles, and if it divide it at right angles, it will also divide it into two equal parts.*

Demonstration FOr having drawn the two lines A B and A D, the two sides A F and F B of the triangle A F B, are equal to the two sides A F and F D of the triangle A F D, and the bases A B and A D are

equal to D B. Again, forasmuch as D is also the center of the circle A B, the line D A shall be equal to D B, But D G is demonstrated also to be equal to D B: therefore [a] D A and D C shall be equal, the part to the whole, which is impossible: Therefore the point D is not the center of both the circles A B and C B: Therefore, If two circles, &c. Which was to be demonstrated.

a) 1. c. 5.

PROP. 7. THEOR. 6.



If in the diameter A B *of the circle* A C D E B, *some point* G *be taken, which is not the center of the circle, and from this point* G *to the circumference* A C D E B, *there fall right lines* G C, G D, G E, *the greatest line shall be* G A, *that line wherein the center* F *is; but the least shall be* G B, *that which remaines (to wit, of the diameter) of the others* G C *and* G A, *alwayes the neerest* G C, *to that* G A *which passeth by the center* F, *is greater than* G D, *that which is more remote; and from this same point* G *there will only fall two equal right lines* G E *and* G H, *to the circumference on both sides of the least line* G B.

Demonstration DRaw from the center F to the points C, D, and E, the right lines F C, F D, and F E, forasmuch as the two sides G F and F C of the triangle G F C, are greater than the other G C: But G F and F C [a] are greater than the other side G C, but G F and F C are equal to G F and F A; that is to say, to the whole G A, the same G A shall be also greater than G C, and by the same reason greater than G D, and so of the others: Therefore G A shall be the greatest of all those which fall from the point G to the circumference.

a) 20. 1.

Secondly, Forasmuch as in the triangle E F G, the side E F is [b] lesse than the two sides E G and G F: but E F is equal to F B; therefore F B shall be lesse than the two sides E G and G E. Therefore the common part F G being taken away, there will yet remain G B, lesse than G E, and by the same reason G B shall be shewn to be lesse than any of the others G D and G C: Therefore G B is the least of all those which are drawn from G to the circumference.

b) 20. 1.

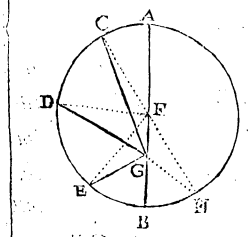
Thirdly, Forasmuch as the two sides G F and F C of the triangle G F C, are equal to the two sides G F and F D of the triangle G F D, and the angle G F C is greater than the angle G F D, the whole than the part, [c] the base G C shall be greater than the base G D; by the same

c) 24. 1.

reaſon G C ſhall be greater than G E; therefore the line which is neereſt to that which paſſeth by the center, is greater than that which is more remote.

Laſtly, make the angle B F H on the other ſide of the line F B, equal to the angle B F E, and draw the right line G H: foraſmuch as the two ſides E F and F G of the triangle E F G are equal to the two ſides H F and F G of the triangle H F G, each to his correſpondent ſide, and that the angles E F G and H F G conteined of thoſe ſides are equal by the conſtruction, d the right lines G E and G H, on both ſides of the leſſer line G B, ſhall be equal to one another. Now that no other line can be equal to either of them is manifeſt; for if from the point G you draw another line on that ſide whereon H is, it will fall below H, and therefore is is leſſe than G H, as being more remote from that which paſſeth by the center, as hath been demonſtrated: and therefore alſo is leſſe than G E, equal to G H: If it paſſe above G H, it ſhall be greater by the ſame reaſon: Therefore there cannot be drawn two right lines equal to one another, but only on each ſide of the leaſt line G B: Therefore, If in the diameter, &c. Which was to be demonſtrated.

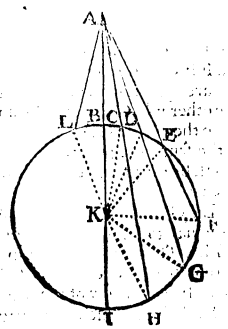
d) 4. 1.



COROLLARIE.

It follows that from every point taken in the Circle, there can be drawn only two right lines to the circumference equal to one another: foraſmuch as that from every point taken in a circle, there can be drawn but one diameter.

PROP. 8. THEOR. 7.



If without the circle B C D E *there be taken ſome point* A, *and from that point there be drawn right lines to the circle, one of which* A I *paſſeth by the center* K, *and the others* A H, A G, A F *at pleaſure; of all the right lines which do fall on the concave circumference, the greateſt* A I, *is that which paſſeth by the center* K, *but of the others alwayes the neereſt to that which paſſeth by the center, ſhall be greater than that which is more remote: But of thoſe which*

which fall on the convex circumference, the leaſt is that which is put between the point A *and the diameter* B I, *and of the others, that which is the neareſt to the leaſt* A B, *is alwayes leſſe than that which is more remote, and from this point* A *can be drawn only two right lines equal to one another, to the circle on both ſides of the leaſt line.*

Demonſtration 1. FOr from the center K draw the right lines K C, K D, K E, K F, K G, K H: foraſmuch as the two ſides A K and K H of the triangle A K H a are greater than A H. But A K and K H are equal to A K and K I, that is to ſay, to the whole A K I, the right line A I ſhall be alſo greater than A H, by the ſame reaſon, A I ſhall be alſo greater than each of the others A G and A F: Therefore A I is the greateſt of all thoſe which are drawn from the point A into the circle. a) 20. 1.

Secondly, foraſmuch as the ſides A K and K H of the triangle A K H, are equal to the two ſides A K and K G of the triangle A K G, and that the whole angle A K H is greater than the angle A K G its part, b the baſe A H ſhall be greater than the baſe A G, by the ſame reaſon, A H ſhall be greater than A F: In like manner, A G ſhall be greater than A F: Therefore the line next to that which paſſeth by the center is greater than that which is more remote from the center. b) 24. 1.

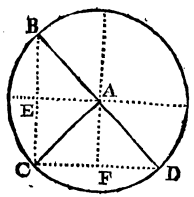
Thirdly, foraſmuch as in the triangle A C K the right line A K c is leſſe than the two others A C and C K: if the equal lines B K and C K be cut off, there will yet remain A B leſſe than A C: by like reaſon, A B ſhall be leſſe than A D, and leſſe alſo than A E: Wherefore A B is the leaſt of all the lines drawn from the point A without the circle, to the convex circumference. c) 20. 1.

Fourthly, foraſmuch as in the triangle A D K there do fall the two right lines A C and C K, comming from the extremities of the ſide A K, d the ſame lines A C and C K ſhall be leſſe than A D and D K: Therefore having taken away the equal lines C K and D K, there will yet remaine A C, leſſe than A D, and in like manner, A C ſhall be leſſe than A E, and A D alſo leſſe than A E: Therefore the line neereſt to the leaſt line A B, is leſſe than that which is more remote. d) 21. 1.

In the laſt place, Make the angle A K L equal to the angle A K D, and draw A L: Foraſmuch as the two ſides A K and K D of the triangle A K D, are equal to the two ſides A K and K L of the triangle A K L: But the angles A K D and A K L conteined of thoſe ſides are equal; e the right lines A D and A L on both ſides of the leaſt line A B ſhall be equal to one another, and no other line drawn from the point A to the ſaid circumference, ſhall be equal to any of them, for it would fall above A L towards B, and therefore leſſe than A L; that is to ſay, than A D, equal to A L; or on the other ſide of A L, and therefore greater, as hath been demonſtrated: Therefore there can be only drawn two equal right lines to the circumference on both ſides of the leaſt line A B: Therefore, If you take &c. Which was to be demonſtrated. e) 4. 1.

PROP.

PROP. 9. THEOR. 8.

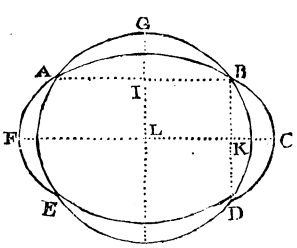


If within a circle BCD, *there be taken a certain point* A, *and from that point* A *there do fall more than two equal right lines* AB, AC, *and* AD, *to the circumference: the point taken* A, *is the center of the circle.*

Demonstration JOyn the right light lines BC and CD, which lines shall be divided in two equal parts in the points E and F, and by the point A draw the right lines AE and AF, and prolong them on both sides to the circumference: Forasmuch then a BE is equal to EC, and EA common, the two sides AE and EB of the triangle AEB shall be equal to the two sides AE and EC of the triangle AEC, and the base AB is equal to the base AC, by supposition; a the angle AEB shall be equal to the angle AEC; b and therefore are both right angles, by the same reason it shall be shewn that the angles at the point F are right angles: Therefore seeing that the right lines AE and AF do divide the right lines BC and CD in two equal parts, and at right angles; in each of the same dividing lines AE and AF, shall be found the center of the circle, being prolonged to the circumference: Therefore the point A in which they intersect shall be the center of the c circle, it being not to be found any other-where than in the common point where they intersect and divide one another, it being both in the one and the other: Therefore, If within a circle, &c. Which was to be demonstrated.

a) 8. 1.
b) 10. def.
c) Cor. 1. 3.

PROP. 10. THEOR. 9.



A circle ABCDEF, *doth not divide another circle* AGBDHE *in more than two points.*

Demonstration LEt the circle ABCDEF divide the circle AGBDHE in more than two points, A, B, and D, (if possible,) and having joyned the right lines AB and BD, divide them in two equal parts in the points I and K, and by those points a draw IL and KL at right angles to the lines AB and BD, and let them be prolonged on both sides to the circumference.

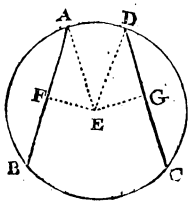
For-

a) 11. 1.

are equally distant from the center E, *and those that are equally distant from the center are equal to one another.*

Demonstration FOr from the center E draw EF and EG perpendicular to AB and CD, and joyn EA and ED: the lines EF and EG shall divide a AB and CD in two equal parts, which being put equal, their halves AF and DG shall be equal.

a) 3. 3.



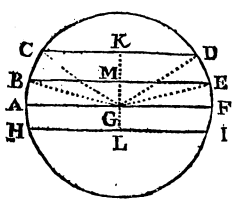
Demonstration FOrasmuch as the squares of the equal lines EA and ED are equal: and b that the square of EA is equal to the squares of AF and FE: In like manner, the square of ED is equal to the squares of DG and GE, the squares of AF and FE shall be likewise equal to the squares of DG and GE; therefore having taken away the equal squares of the equal right lines AF and DG, the square of FE and GE will remain equal: Therefore EF and EG shall be equal: Therefore c AB and CD shall be equally distant from the center E.

b) 47. 1.
c) 4. def. 3.

Again, Let AB and CB be equally distant from the center E: I say that they are equal to one another; for (the construction made as before,) we shall shew that EF and EG d are equal, and that they divide c AB and CD in two equal parts, and the squares of the equal right lines EA and ED are equal to one another: But the square of AE is f equal to the two squares of AF and FE, and that of ED also equal to the two squares of DG and GE: therefore the squares of AF and FE are equal to the squares of DG and GE; having then taken away the equal squares of the equal lines EF and EG: there will remain the squares of AF and DG equal, and therefore their doubles AB and CD are also equal: Therefore, in a circle, &c. Which was to be demonstrated.

d) 4. def. 3.
e) 3. 3.
f) 47. 1.

PROP. 15. THEOR. 14.



In a circle ABCD, *the greatest line is the diameter* AF, *but of the others* CD, *and* HI, *always that* HI *which is nearest to the center* G, *is greater than* CD, *that which is more remote.*

Demonstration 1. FOr draw from the center G the perpeudiculars GK and GL to CD and HI: and forasmuch as CD is more remote from the center than HI, the perpendicular GK shall be greater a than GL: Therefore let GM be cut off from the line GK equal to GL, and by M draw BME, perpendicular to GK, and joyn the right lines GB, GC, GD, and GE.

a) 4. def. 3.

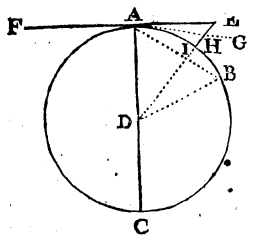
Forasmuch as the perpendiculars G M and G L are equal, B E and H I shall be equally distant from the center, and therefore equal to one another; Again, forasmuch as G B and G E are b greater than B E, but are equal to the diameter A F, the diameter A F shall be also greater than B E; by the same reason it shall be shewn that A F is the greatest of all the other lines.

b) 10. 1.

Lastly, forasmuch as the sides G B and G E of the triangles B G E are equal to the sides G C and G D of the triangle C G D; and the angle B G E is greater than the angle C G D, c the base D E shall be greater than the base C D. And therefore H I which hath been shewn to be equal to B E, shall be in like manner greater than C D: Therefore, In a circle, &c. Which was to be demonstrated.

c) 24. 1.

PROP. 16. THEOR. 15.



The right line E F *drawn from the extremitie* A, *of the diameter* A C *of the circle* A B C, *at right angles to the same diameter, shall fall without the circle* A B C, *in the space comprised between the same right line* E F, *and the circumference* A B C, *and there shall not fall another right line* G A, *and the angle of the semicircle* D A B *is greater than any acute right lined angle: But the rest* E A H *is lesse.*

Demonstration IF it be said that E F doth not fall without the circle: Let it (if possible,) fall within, as the line A B, and draw D B;
a) 5. 1. a the angles D A B and D B A shall be equal; but D A B is a right angle:
b) 17. 1. therefore D B A shall be also a right angle, which is absurd; b seeing that the two angles of a triangle are lesse than two right angles: In like manner we shall shew that it will not also fall on the circumference; therefore it shall fall without at A E.

I say now, that between A E and the circumference B A, there cannot fall another right line drawn from the point A: For let A G fall there (if possible,) and so then the angle D A G shall be acute, and on A G draw D H perpendicular to A G, cutting the circumference in I, D H shall only
c) Co. 17. 1. fall on the side of the acute angle D A G. Forasmuch c as in the triangle
d) 17. 1. D A H d the two angles D H A and D A H are lesse than two right angles, and D H A is a right angle, by the construction, D A H shall be lesse than a right angle; and therefore D A, that is to say, D I equal thereto; shall
e) 19. 1. be e greater than D H, the part than the whole, which is impossible: Therefore, not any right line shall fall between the circumference A B, and the right line perpendicular A E, but will cut the circle.

I say, Lastly, that the angle of the semicircle conteined of the diameter A C, and the circumfereece B A is greater than any right lined acute angle,

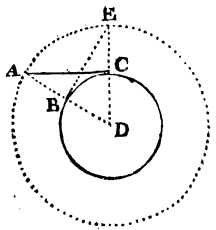
angle: for seeing it hath been shewn that every right line drawn from the point A, below the perdendicular A E, doth fall within the circle, it will of necessity make with the line A C a right lined acute angle, lesse than the angle of the semicircle: But with A E it will constitute a right lined acute angle, greater than the angle of contingence, considering that the angle of contingence is a part of the angle of the semicircle; but this acute angle is some whole, in respect of the angle of contingence, which is manifest, the line A B being drawn at adventure, below A E: for seeing that the same A B doth fall within the circle, as hath been demonstrated, the right lined acute angle C A B is lesse than the angle of the semicircle, conteined under the diameter A C, and the circumference A B C: Seeing that the acute angle is part of the angle of the semicircle: But the angle of contingence conteined under the tangent A E, and the circumference A B C is lesse than the right lined angle B A E, seeing that it is a part thereof: Therefore the angle of the semicircle is greater than any right lined acute angle, and the remainder the angle of contingence lesse: Therefore, the line, &c. Which was to be demonstrated.

COROLLARIE.

It is manifest from hence, that the right line drawn from the extremity of the diameter at right angles, doth touch the circle; for it hath been demonstrated that it falls without the circle: Wherefore it reacheth the circle in this point (the extream of the diameter) only.

As if from the point A *in the circumference of the circle* A B C, *there were to be drawn a line which should touch the circle in the point* A, *we shall draw a line from* A *to the center* D, *and to that draw another;* A E *at right angles, and that line shall touch the circle in the point* A, *as hath been demonstrated.*

PROP. 17. PROBL 2.



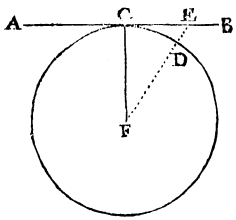
From a given point A, *to draw a right line* A C, *which toucheth a given circle* B C.

Construction FInd the center D, and joyne the line A D, which divideth the circle in the point B, and on the center D, with the distance D A, describe the circle A E, and from the point B draw B E at right angles to A D, dividing the circle A E in the point E; and lastly, draw D E, dividing the circle B C in the point C, and joyn C A: I say, that A C doth touch the given circle in the point C.

Demonstration FOr seeing that the two sides B D and D E of the triangle B D E are equal to the two sides C D and D A of the triangle C D A, each to his correspondent, as appears by the Definition of a circle, and the angle D conteined of those sides is common, the a) 4. 1.
a bases B E and C A, and the angles D B E and D C A shall be equal. But D B E is a right angle by Construction: Therefore D C A shall be also a right angle: Therefore seeing that C A is drawn from the extremity

b) Co. 16. 3. C, of the ſemidiameter D C at right angles, it will b touch the circle: Therefore from the given point A, &c. Which was to be done.

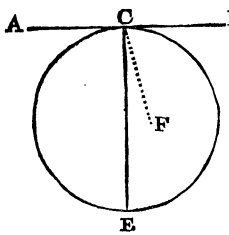
PROP. 18. THEOR. 16.



If ſome right line A B, *doth touch a circle* C D, *and from the center* F *to the point of touching* C, *there be joyned ſome right line* F C, *that joyned line* F C *ſhall be perpendicular to the touch line* A B.

Demonſtration FOr if it be denyed, Draw F E perpendicular to A B, cutting the circumference in the point D: a) 17. 1. Foraſmuch as in the triangle C F E, a the two angles F C E and E F C b) 18. 1. are leſſe than two right angles, and E F C is a right angle, by conſtruction, F C E ſhall be leſſe than a right angle: Therefore b C F, that is to ſay, F D its equal, ſhall be greater than F E the part than the whole, which is impoſſible: Therefore F C is perpendicular to A B: Therefore if a right line, &c. Which was to be demonſtrated.

PROP. 19. THEOR. 17.



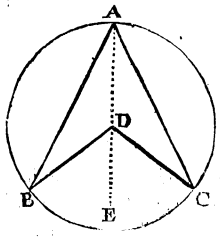
If ſome right line A B, *doth touch a circle* C E, *and from the point of touching* C, *there be drawn a right line* C E *at right angles to the line touching in that ſame line* C E, *ſhall be the center of the circle.*

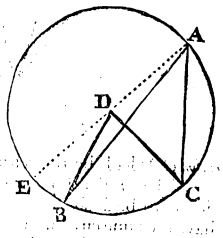
Demonſtration FOr if it be ſaid that the center is not in the line C E, but without; Let it (if poſſible,) be in the point F, from a) 18. 1. which point F draw the right line F C, a which ſhall be perpendicular to A B. Therefore the right angle F C B ſhall be equal to the right angle E C B, the part to the whole, which is impoſſible: Therefore the center of the circle ſhall not be out of the line C E: Therefore, If a right, &c. Which was to be demonſtrated.

PROP. 20. THEOR. 18.

In a circle A B C, *the angle* B D C, *which is at the center* D,

D, *is double to the angle* B A C, *which is at the circumference, when they have for their baſe one and the ſame circumference* B C.





Demonſtration JOyn A D together, and prolong it to E; foraſmuch as the two lines D A and D B are equal, D A D and D B A ſhall a) 5. 1. be equal: But the exterior angle B D E is equal b to the two interior angles D A B and D B A, and therefore double to each of them, as of b) 32. 1. D A B: In like manner, we ſhall ſhew that C D E is double to the angle D A C: Wherefore the whole angle B D C ſhall be double to the whole angle B A C which was propoſed: Again, let the angle C A B not encloſe the angle C D B, and joyn A D, prolonged to E; foraſmuch as the two ſides D A and D C are equal, c the angles D A C and D C A ſhall be c) 5. 1. equal, but the exterior angle C D E is equal d to them: Therefore it d) 32. 1. ſhall be double to D A C, but the angle E D B at the center, is double to E A B, as is demonſtrated; Therefore the remainder B D C ſhall be double to the remainder B A C; e for when the whole is double to the e) 10. 1. whole, and the part cut off to the part cut off, the remainder ſhall be double to the remainder, as followeth.

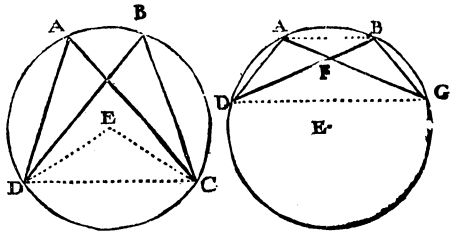
LEMMA.

Let the whole A B *be double to the whole* C D, *and* A E *the part cut off, double to* C F *the part cut off, the reſt* E B *ſhall be double to the reſt* F D; *for ſeeing that* A B *is double to* C D, *being divided in two equal parts at* E, *as well* A E *as* E B, *ſhall be equal to* C D: *Therefore* A E *being alſo double to* C F, C D *equal to* A E, *ſhall be alſo double to* C F, *and therefore double to its other half* F D: *Therefore the reſt* E B *equal to* C D, *ſhall be alſo double to the reſt* F D: *Which was to be demonſtrated.*

PROP. 21. THEOR. 19.

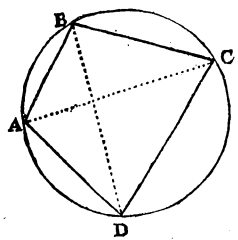
In a circle A B C D, *the angles* D A C *and* C B D, *which are in one and the ſame ſegment* C B A D, *are equal to one another.*

De-



Demonſtration. FOr let E be the center, to which joyn the lines E D and E C: Foraſmuch as the angle D E C is conſtituted in the center, it [a] ſhall be double to D A C, the angle at the circumference: a) 20. 3. for they have the ſame circumference D C for their baſe, by the ſame reaſon, it ſhall be double to the angle C B D: Therefore each of the two angles A and B, ſhall be the half of D E C, and therefore equal to one another: Therefore, In a circle, &c. Which was to be demonſtrated.

PROP. 22. THEOR. 20.



The oppoſite angles B A D *and* B C D *of quadrilateral figures* A C, *inſcribed in a circle* A B C D, *are equal to two right angles.*

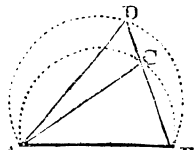
Demonſtration. FOr, joyn the two diagonal lines A C and B D; foraſmuch as the two angles A C D and A B D, are in one and the ſame ſegment A B C D, they ſhall be equal to one another, by the ſame reaſon, the two angles B C A and B D A, in the ſegment B C D A, ſhall be alſo equal to one another; therefore the whole angle B C D ſhall be equal to the two angles A B D and A D B; therefore having added the common angle B A D, the two angles B C D and B A D ſhall be equal to the three angles B D A, D B A, and B A D, of the triangle A B D, but the three angles of the triangle A B D [a] are equal to two a) 32. 1. right angles, therefore the two oppoſite angles B A D and B C D ſhall be alſo equal to two right angles: In like manner, it might be ſhewn that the two oppoſite angles at the points D and B, are equal to two right angles: Therefore the angles, &c. Which was to be demonſtrated.

PROP. 23. THEOR. 21.

On one and the ſame right line A B, *there cannot be conſtituted two ſegments of circles* A D B *and* A C B, *alike and unequal, and on the ſame ſide.*

De-

Demonſtration FOr if it be poſſible, Let two ſegments of circles be conſtituted alike and unequal: On the line A B, and on the ſame part or ſide A C B and A D B, thoſe ſegments will only cut one another in the points A and B; Seeing [a] that a circle cuts not another circle in more than two points: Therefore the circumference of one of the ſegments ſhall be without the circumference of the other. Draw then the right line B C D, cutting the circumference in the points C and D, and joyn A D and A C: Foraſmuch as the ſegments A C B and A D B are put alike ſegments by the tenth Definition of this Book, the angle A C B ſhall be equal to the angle A D B, which is abſurd, [b] for the exterior angle A C B is greater than the interior A D B, they are not then like ſegments: Therefore on one and the ſame, &c. Which was to be demonſtrated.

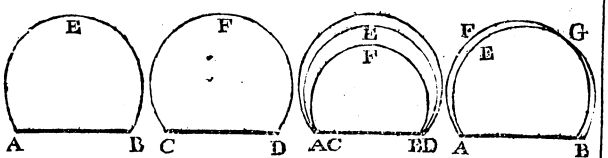


a) 10. 3.

b) 16. 1.

PROP. 24. THEOR. 22.

Like ſegments of circles A E B *and* C F D, *conſtituted on equal right lines* A B *and* C D *are equal to one another.*



Demonſtration FOr the lines A B and C D being equal, they will agree to one another, if it be underſtood that they be placed the one on the other, to wit, A B on C D, and the ſegment A E B will agree with the ſegment C F D: otherwiſe the ſegment A E B ſhall fall without the ſegment C F D, or elſe within it, or a part without, and a part within; and therefore on one and the ſame right line are conſtituted like ſegments of circles, [a] which is abſurd: For if part fall without, and part within, they ſhould cut one another in more than two points, as at B and G, which is impoſſible: For a circle doth not cut another circle in more than two points: Therefore they ſhall agree, and therefore ſhall be equal to one another: Therefore like ſegments, &c. Which was to be demonſtrated.

a) 23. 3.

PROP. 25. PROBL. 3.

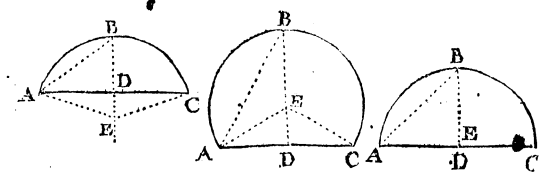
The ſegment A B C, *of a circle, being given, to deſcribe the circle of which it is a ſegment.*

Conſtruction JOyn A C, and divide it into two equal parts in the point D, by which draw D B at right angles to A C, and joyn A B: The angle D B A ſhall be greater than the angle D A B, or equal or leſſe.

Demon-

Demonstration FIrst of all, Let it be greater, (which will happen when the segment shall be lesse than the semicircle,) for then B D prolonged, will passe [a] by the center, which is without the segment; seeing that it is lesse than the semicircle, and [b] D A shall be greater than D B, for D B is the rest of the Diameter: Therefore [c] D B A shall be greater than D A B: Let the angle B A E be made equal to D B A, and let the right line A E cut B D prolonged, at E: I say, that E is the center of the circle, of which A B C is the segment.

a) Co. 1. 3.
b) 7. 3.
c) 18. 1.



For having drawn E C, the two sides A D and D E of the triangle A D E shall be equal to the two sides C D and D E of the triangle C D E, and the angles conteined of them are right angles; [d] therefore A E and E C are equal, but E A is also equal to E B: Seeing [e] that the angles E A B and E B A are equal: Therefore the three sides E A, E B, and E C are equal: [f] Therefore F shall be the center of the circle A B C; seeing that from that point there falls more than two equal right lines, to the circumference.

d) 4. 1.
e) 16. 1.
f) 9. 3.

Secondly, Let the two angles D B A and D A B be equal: [g] D A shall be equal to D B: But D C is also equal to D B. Seeing that A C is divided into two equal parts in the point D, all the three lines D C, D B, and D A shall be equal; therefore D shall be the center of the circle. and the segment proposed shall be a semicircle.

g) 6. 1.

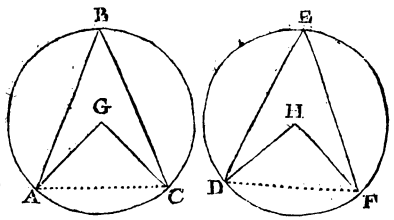
Lastly, if the angle D B A be lesse than the angle D A B: Let the angle B A E be made equal to the angle A B E; and let E C be joyned: I say that E is the center of the circle: For it will be shewn by the same reasons as above, that the three lines E A, E B, and E C, are equal; and therefore also the segment proposed will exceed the semicircle: Therefore a segment, &c. Which was to be done.

PROP. 26. THEOR. 23.

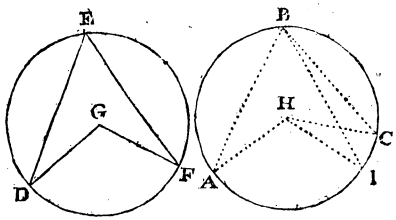
In equal circles A B C *and* D E F, *the equal angles* D H F *and* A G C, *insist on equal circumferences* A C *and* D F, *whether they be constituted at the centers* G *and* H, *or on the circumferences* A C *and* D F.

Demonstration FOr joyn the right lines A C and D F; forasmuch as the two sides G A and G C of the triangle A G C, are equal to the sides H D and H F of the triangle D H F; because of the equality of the circles, and the angles G and H equal by supposition, [a] the bases A C and D F shall be equal: but the angles B and E constituted at the circumferences, are equal by supposition, and insist on the same bases: Therefore the segments A B C and D E F shall be alike [b], and therefore equal; which being subtracted from the equal circles, there will remaine the segments A C and D F equal; and therefore the circumferences A C and D F equal. Which was to be demonstrated.

a) 4. 1.
b) 24. 3.



PROP. 27. THEOR 24.



In equal circles A B C *and* D E F, *the angles* A H C *and* D G F *that insist on the equal circumferences* A C *and* D F, *are equal to one another, whether they be constituted at the centers* H *and* G, *or else on the circumferences* A C *and* D F.

Demonstration FOr if they be not equal: Let the angle H be the greater, and make the angle A H I equal to the angle D H F [a]: therefore the circumference A I shall be equal to the circumference D F: But the circumference A C is equal to the circumference D F, by supposition; therefore A I and A C shall be equal to one another, the part to the whole, which is impossible; therefore the angles A H C and D G F are equal.

a) 26. 3.

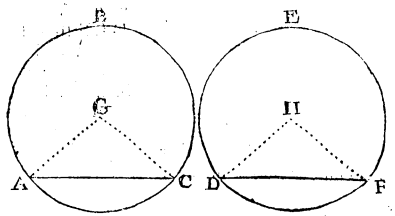
Secondly, Constitute the angles B and E at the circumferences, having equal circumferences, A C and D F for bases; by the same reason it might be shewn that they are equal to one another: Which was to be demonstrated.

PROP. 28. THEOR. 25.

In equal circles A B C *and* D E F, *equal right lines* A C *and* D F *do take away equal circumferences* A B C *and* D E F, *to wit, the greatest from the greatest, and the least from the least.*

O Demon-

Demonstration FOr let the centers G and H be taken, and the right lines A G, G C, D H, and H F, be drawn; the two sides G A and G C of the triangle A G C are equal to the two sides H D and H F, of the triangle D H F, and the bases A C and D F are equal by supposition, therefore the angles [a] G and H are equal: Therefore [b] the circumferences A C and D F, on which they insist are equal, which being taken from the equal circles, there will remain the circumferences A B C and D E F, also equal: Therefore, In circles, &c. Which was to be demonstrated.



a) 8. 1.
b) 26. 3.

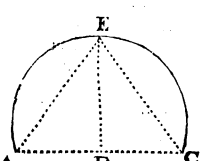
PROP. 29. THEOR. 26.

In equal circles A B C *and* D E F, *equal circumferences* A B C *and* D E F, *subtend equal right lines* A C *and* D F.

Demonstration FOr, Let the centers G and H be taken, and let the right lines A G, G C, D H, and H F, be drawn: Forasmuch as the two sides A G and G C of the triangle A G C, are equal to the two sides D H and H F of the triangle D H F; [a] and the angles G and H equal; seeing they insist on equal circumferences, [b] the bases which are the right lines A C and D F, are equal: Therefore in circles, &c. Which was to be demonstrated. As by the fore-going figure is evident.

a) 27. 1.
b) 4. 1.

PROP. 30. PROBL 4.



To divide a given circumferense A E C, *into two equal parts.*

Construction DRaw the right line A C, which divide in two equal parts, [a] at the point D, by which point draw the right line D E, at [b] right angles to A C, and joyn the right lines A E and C E, the circumference shall be divided in two equal parts, in the point E.

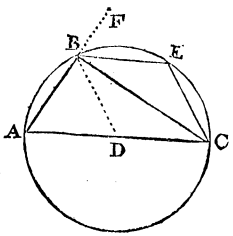
a) 10. 1.
b) 11. 1.

Demonstration FOr seeing that A D and C D are equal, and D E common, the two sides A D and D E of the triangle A D E shall be equal to the two sides C D and D E of the triangle C D E; and the angles at the point D are equal, to wit, right angles; [c] the bases A E and C E shall be equal; [d] and therefore the circumference A E and C E also equal: Therefore we have divided, &c. Which was to be done.

c) 4. 1.
d) 28. 3.

PROP.

PROP. 31. THEOR. 27.



In a circle A B C, *the angle* A B C, *which is in the semicircle is a right angle: But that which is in the greatest segment* B C A, *is lesse than a right angle, and that which is in the least segment* B E C, *is greater than a right angle; And moreover, the angle* A B C *of the greatest segment is greater than a right angle: But the angle* E B C *of the least segment is lesse than a right angle.*

Demonstration FOr draw B D to the center D, and prolong A B directly towards F: Forasmuch as D A and D B are equal, the angles [a] D A B and D B A shall be equal to one another; by the same reason, the two angles D B C and D C B shall be also equal; for D B and D C are also equal to one another: Therefore the two angles D B A and D B C, that is to say, the whole A B C, is equal to the two angles B A C and B C A: But the exterior angle C B F [b] is equal to the same two interior and opposite angles B A C and B C A: Therefore the two angles A B C and C B F shall be equal to one another; and therefore right angles: Therefore the angle A B C in the semicircle is a right angle; which was in the first place proposed.

a) 5. 1.
b) 32. 1.

I say, secondly, that the angle B A C in the greatest segment B A C, is lesse than a right angle: Forasmuch as in the triangle A B C, the two angles A B C and B A C, are lesse than two right angles, and A B C is demonstrated to be a right angle, the angle B A C shall be lesse than a right angle; which is in the second place proposed.

Thirdly, I say, that the angle B E C in the least segment B E C, is greater than a right angle: Forasmuch as in the quadrilateral figure A B E C, described in the circle, the two opposite angles [c] E and A are equal to two right angles, and the angles B A C is shewn to be lesse than a right angle; therefore the other angle B E C shall be greater than a right angle; which was in the third place proposed.

c) 22. 3.

Fourthly, I say, that the angle of the greatest segment B A C, which is the angle A B C, conteined of the circumference A B, and of the right line B C, is greater than a right angle: for seeing that the right angle A B C is conteined of the right lines A B and B C, and is a part thereof, the whole angle A B C, conteined of the circumference A B, and the right line B C, shall be greater than a right angle; which was in the fourth place proposed.

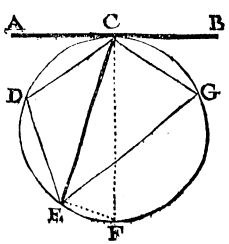
Lastly, I say, that the angle of the least segment, which is the angle C B E, conteined of the right line C B, and of the circumference B E C, is lesse than a right angle; for that it is lesse than the right angle F B C,

the part than the whole, which was in the laſt place propoſed, &c. Which was to be demonſtrated.

COROLLARIE.

Hence it is manifeſt that the angle of a triangle which is equal to two others, is a right angle; foraſmuch as the angle contiguous thereto, (which is made without the triangle, the ſide being prolonged,) is equal to the ſame; which appears by the firſt Demonſtration; it is the half of the three angles of a triangle, which are equivalent to two right angles.

PROP. 32. THEOR. 28.



If ſome right line AB *do touch a circle* CDE, *and from the point of touching* C, *there be drawn ſome right line* CE, *to the circle dividing it, the angles* ACE *and* BCE, *that it makes with the touch line* ACB, *ſhall be equal to the angles which are in the alternate ſegments* CDE, *and* CGFE, (viz. ACE *equal to* G, *and* BCE *equal to* D.

Demonſtration FOr, Draw the right line CF by the center, and joyn EF, the line [a] CF ſhall be perpendicular to AB, and the angle CEF ſhall be a right angle; and therefore the two remaining angles EFC and ECF ſhall be equal to a right angle, as to the right angle ACF: Therefore if the common angle ECF be taken away, there will remain the angle ACE, equal to the angle EFC, which is equal to the angle G: Foraſmuch as they are in one and the ſame ſegment: Therefore the angle ACE ſhall be alſo equal to the angle G. a) 18. 1.

And foraſmuch as in the quadrilateral figure DEGC, inſcribed in the circle, the two oppoſite angles D and G are equal [b] to two right angles, and the two angles ACE and BCE are alſo equal to two right angles; if the two equal angles ACE and CGE be taken away, there will remain the angle BCE, equal to the angle D, in the alternate ſegment CDE: Therefore, if a right line, &c. Which was to be demonſtrated. b) 21. 3.

PROP. 33. PROBL. 5.

On a right line given AB, *to deſcribe a ſegment of a circle, which may receive an angle equal to a given right lined angle* C.

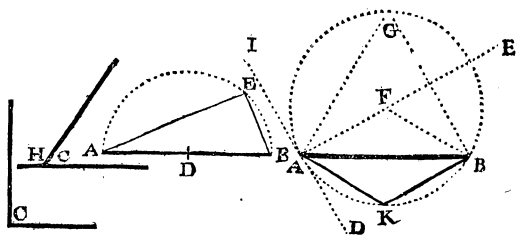
Firſt, Let the angle given C, be a right angle.

Conſtruction DIvide the right line AB into two equal parts in the point D, and from the point D as a center, and with the diſtance DA, deſcribe the ſemicircle, in which make the angle AEB which is a right angle; and therefore [a] equal to the given right angle C. a) 31. 3.

Secondly, Let the given angle C, be acute.

Conſtruction THen from the point A, of the given right line AB, make the angle DAB equal to the acute angle C, and from the point A draw AE at right angles to AD, which ſhall fall above AB; then make the angle ABF equal to the angle FAB, and let BF cut the right line AE in the point F, [b] the lines FA and FB ſhall be equal. Therefore if from the center F, with the diſtance FA, the circle AGB be deſcribed, it will paſſe by the point B: I ſay then that the angle which ſhall be made in the ſegment AGB, deſcribed on the line AB, ſhall be equal to the angle C. Therefore let the angle AGB, be made in the ſaid ſegment. b) 6. 1.

Demonſtration FOraſmuch as AE doth paſſe by the center, and that from the extremity A, there is drawn AD at right angles thereto, DA will [c] touch the circle in the point A: Wherefore the angle DAB is to ſay, the given angle C ſhall be equal to the angle G, in the alternate ſegment AGB. c) C. 16. 3.



Thirdly, Let the given angle H be obtuſe.

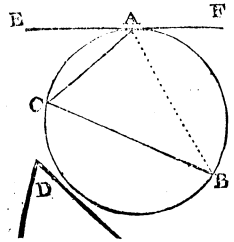
Conſtruction MAke again the angle IAB equal to the angle H, and from the point A, draw AE, at right angles to AI, which will fall above AB: finiſh the reſt as is before taught, and ſo there ſhall be deſcribed on the line AB the ſegment AKB, in which the angle K is equal to the obtuſe angle given H.

Demonſtration FOr the angle IAB, that is to ſay, the given angle H, is equal to the angle K, in the alternate ſegment AKB, for that it is the ſame Demonſtration: Therefore, On a given, &c. Which was to be done.

PROP. 34. PROBL. 6.

From a given circle ABC, *to cut off a ſegment, which may receive an angle, equal to a given right lined angle* D.

Con-



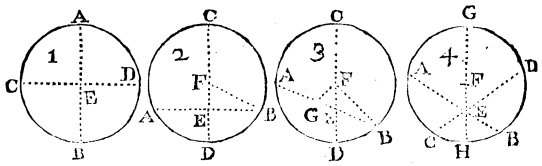
Construction DRaw the right line E F, touching the circle in the point A, then make the angle F A B equal to the given angle D, and draw the right lines A C and B C: I say, that the angle A C B in the segment A C B is equal to the given angle D.

Demonstration THe angle F A B, which is made equal to the given angle D, is [a] equal to the angle C, in the alternate segment A C B: Therefore the same angle C, shall be in like manner equal to D: We have therefore cut off from the given circle, the segment A C B, &c. Which was to be done.

a) 32. 3.

PROP. 35. THEOR. 29.

If in a circle A B C, *two right lines* A B *and* C D, *do divide one another* (*in the point* E,) *the Rectangle conteined under the two parts of the one* A E *and* E B, *is equal to the Rectangle conteined under the two parts* C E *and* E D, *of the other.*



Demonstration LEt the point of intersection E, in the first place be the center of the circle, as in the first figure, the four segments shall be equal, each of them being the semidiameter of the circle: Therefore it is manifest that the Rectangle conteined under the two parts of the one, is equal to the Rectangle conteined under the two parts of the other.

Secondly, Let C D only passe by the center, dividing A B into two equal parts in the point E, [a] and therefore at right angles, and joyn the right line B F: Forasmuch as C D is divided into two equal parts in the center F, and unequally in the point E. The Rectangle conteined under the unequal segments C E and E D, [b] with the square of the mean Section F E, is equal to the square of the halfe F D; that is to say, of F B: but the square of F B [c] is equal to the two squares of F E and E B: therefore the Rectangle conteined under C E and E D, with the square of F E, shall be also equal to the two squares of F E and E B; therefore if the common square of F E be taken away, there will remain the Rectangle conteined under C E and E D, equal to the square of E B; that is to say, to the Rectangle under A E and E B: For seeing that they are equal, the Rectangle conteined under the two is a square, according to the First Definition of the second Book. (See the Second Figure.)

a) 3. 3.
b) 5. 1.
c) 47. 1.

Thirdly, Let C D passing by the center F, divide A B in two parts unequally, in the point E, and let the same A B be divided in two equal parts in the point G, and let G F and F B be drawn; [d] F G shall be perpendicular to A B. Forasmuch then as C D is divided into two equal parts at F, and unequally at E, the Rectangle conteined under C E and E D, with the square of the mean Section F E, shall be equal to the square of the half F D; that is to say, to the square of F B its equal: But the square of F B [e] is equal to the two squares F G and G B: Therefore the Rectangle conteined under C E and E D, with the square of F E, is [f] also equal to the two squares of F G and G B, and the square of F E is equal also to the two squares of F G and G E. Therefore the Rectangle under C E and E D, with the two squares of F G and G E, is equal to the two squares of F G and G B, taking away therefore the common square of F G, there will remain the Rectangle conteined under C E and E D, with the square of G E, equal to the square of G B; but the Rectangle conteined under A E and E B, with the square of G E, is also equal to the square of G B: Seeing that A B is divided into two equal parts in the point G; and unequally in the point E. Therefore the Rectangle conteined under C E and E D, with the square of G E, is equal to the Rectangle under A E and E B, with the same square of G E, taking away therefore the common square of G E, there will remain the Rectangle conteined under C E and E D, equal to the Rectangle conteined under A E and E B; which is proposed. (See the third figure.)

d) 3. 3.
e) 47. 1.
f) 47. 1.

Fourthly, Let neither A B nor C D, (cutting one another in the point E,) passe by the center, and draw the diameter G H by the point E, (as in the fourth figure,) it shall be shewn as before, that the Rectangle conteined under A E and E B, is equal to the Rectangle under G E and E H, and the Rectangle under C E and E D, also equal to the Rectangle under G E and E H; and therefore equal to one another; whether the one of them A B and C D be divided in two equal parts or not: Therefore, If in a circle, &c. Which was to be demonstrated.

PROP. 36. THEOR. 30.

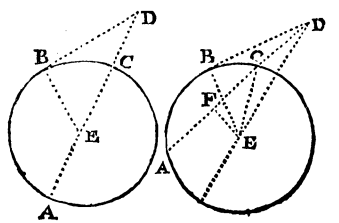
If there be taken some point D, *without a circle* A B C, *and from that point there fall two right lines to the circle, one of which* D A, *divides the circle, and the other* D B, *toucheth it; the Rectangle conteined under the whole line dividing* (*to wit,* A D) *and its part without* D C, *taken between the point* D, *and the convex circumference* C *is equal to the square of the touching line* D B.

Demon-

Demonſtration IN the firſt place, Let D A paſſe by the center E, and then joyn the right line E B, a which ſhall be perpendicular to D B. Foraſmuch as C A is divided into two equal parts in E, and to it is added D C, the Rectangle conteined under A D and D C, b with the ſquare of E C, that is to ſay, of E B, its equal, is equal to the ſquare of E D: But the ſquare of E D c is equal to the two ſquares of D B and B E: Therefore the Rectangle conteined under A D and D C, with the ſquare of B E ſhall be alſo equal to the two ſquares of D B and B E: Therefore taking away the common ſquare of E B, there will remain the Rectangle under A D and D C, equal to the ſquare of the touching line D B; which is propoſed.

a) 18. 3.
b) 6. 2.
c) 47. 1.

Secondly, Let the dividing line D A not paſſe by the center E, and divide A C into two equal parts in the point F, and joyn the right lines E B, E F, E C, and E D, the line E B ſhall be perpendicular to D B, and E F to A C: Foraſmuch then as A C d is divided into two equal parts in the point F, and to it is added D C, the Rectangle conteined under A D and D C, with the ſquare of F C, is equal to the ſquare of F D: Therefore adding the common ſquare of E F, the Rectangle conteined under A D and D C, with the two ſquares of C F and F E, e is equal to the two ſquares of D F and F E: But the ſquare of E C, that is to ſay, of E B its equal; e is equal to the two ſquares of C F and F E; therefore the Rectangle under A D and D C, with the ſquare of E B, is equal to the two ſquares of D F and F E, and the ſquare of D E is alſo equal to the two ſquares of D F and F E; therefore the Rectangle under A D and D C, with the ſquare of B E, ſhall be equal to the ſquare of D E, which is equal f to the two ſquares of D B and B E: Therefore the Rectangle under A D and D C, with the ſquare of B E, is equal to the two ſquares of D B and B E, taking away therefore the common ſquare of B E, there will remain the be Rectangle under A D and D C, equal to the ſquare D B. Which was to demonſtrated.



d) 6. 2.
e) 47. 1.
f) 47. 1.

COROLLARIE I.

From this Propoſition it is manifeſt that if from any point taken without the circle, there be drawn divers right lines, dividing the circle, the Rectangles compriſed under each of the whole lines, and its exterior part, are equal to one another, for if from the point A (*in the firſt figure,*) *there be drawn* A C, A D, *and* A E, *dividing the circle and* A B, *touching it, the Rectangle under* A C *and* A F, g *ſhall be equal to the ſquare of* A B: *in like manner, the Rectangle under* A D *and* A G, *and the Rectangle under* A E *and* A H; *and therefore equal to one another.*

g) 36. 3.

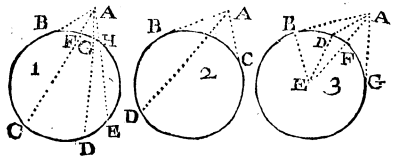
COROLLARIE II.

It is alſo manifeſt, that two right lines drawn from one and the ſame point, touching

touching the circle, are equal to one another, as are the two lines AB *and* AC, *which touch the circle in the points* B *and* C *(in the second figure;) I say they are equal to one another, for draw* AD *dividing the circle in the point* E, *as well the square of* AB *as the square of* AC, [h] *shall be equal to the Rectangle conteined under* AD *and* AE; *and therefore equal to one another; and therefore the lines* AB *and* AC *likewise equal to one another.* h) 36. 3.

COROLLARIE. III.

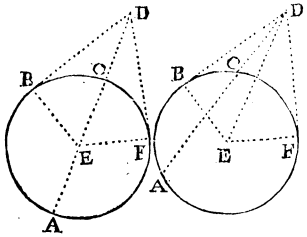
It is likewise manifest, that from a point taken without the circle, there can only be drawn two right lines, touching the circle; for if besides the two AB *and* CD, *there may be a third* AD *drawn, touching the same circle; having drawn the right lines* EB, *and* ED, *from the center* E; [i] *the angles* ABE *and* ADE *shall be right angles; and therefore equal to one another, which is absurd: for if you draw the right line* AE, [k] *the angle* ADE *shall be greater than the angle* ABE. i) 18. 3. k) 21. 1.



COROLLARIE IV.

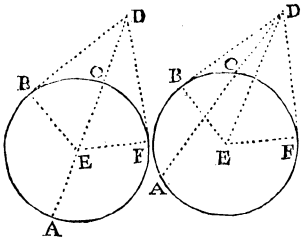
It is in the last place evident, that if two equal right lines be drawn from some point without the circle to the convex circumference, and that the one of them touch the circle, the other shall also touch it, &c.

PROP. 37. THEOR. 31.



If without the circle ABC, *there be taken some point* D, *and from that point there fall to the circle two right lines* DB *and* DA, *one of which* DA, *cuts the circle, and the other reacheth the circle; and that the Rectangle conteined under the whole dividing line* DA, *and its part without* DC, *taken between the point* D, *and the convex circumference* C, *be equal to the square of the line* DB, *which reacheth the circle, the same line* DB *shall touch the circle.*

Demonſtration FOr, [a] Let D F be drawn, touching the circle, and let the Center E be found, and joyn E B and E F: and if D A paſſe not by the center E, let D E be alſo joyned, the angle [b] D F E ſhall be a right angle; and foraſmuch as D F toucheth the circle, [c] the ſquare of the ſame D F ſhall be equal to the Rectangle conteined under D A and D C: But the ſquare of D B is equal to the ſame Rectangle under D A and D C, by Suppoſition; therefore the ſquares of D B and D F ſhall be equal to one another; and D B and D F alſo equal; But [d] E B and E F are equal; therefore the two ſides D B and B E of the triangle D B E, are equal to the two ſides D F and F E, of the triangle D F E, each to his correſpondent, and the baſe D E common, and the angles B and F ſhall be equal; but D F E is a right angle; therefore D B E ſhall be alſo a right angle: Therefore by the Corollary of the ſixteenth Propoſition of this Book, D B ſhall touch the circle, which was propoſed: Therefore if there be taken, &c. Which was to be demonſtrated.



a) 17. 3.
b) 18. 1.
c) 36. 3.
d) 15. def.

The End of the Third Element of EUCLIDE.

THE

THE FOURTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

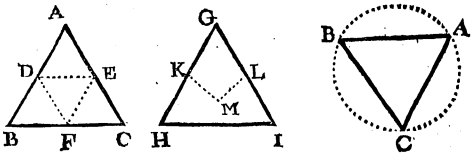
EUCLIDE treating in this Fourth Book of divers inſcriptions of figures in Circles, and the deſcriptions of the ſame figures about the Circle. Alſo of the Inſcriptions of the Circle in the ſame figures, and of the deſcriptions of the Circle about thoſe figures, he expoſeth in a few Definitions, what the meaning is, of a figure inſcribed in a figure, or to be deſcribed about a figure, beginning with right lined figures.

DEFINITIONS.

1 *A right lined figure is ſaid to be inſcribed in a right lined figure, when every one of the angles of the figure inſcribed, doth touch each ſide of the figure in which it is inſcribed.*

AS for Example, if each of the angles D, E, and F, of the interior triangle D E F doth touch each of the ſides of the exterior triangle A B C, the triangle D E F ſhall be ſaid to be inſcribed in the triangle

ABC. But forasmuch as the angle M of the triangle K L M, (a right line being drawn from K to L,) toucheth not the side H I of the triangle G H I, the triangle K L M shall not be said to be inscribed in the other, although it be all within, and that two of the angles of the one, do touch two sides of the other.

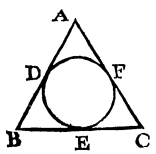


2 *In like manner, a figure is said to be described about a figure, when each side of the figure circumscribed, doth touch each angle of the figure about which it is described.*

CONtrarily, the triangle A B C shall be said to be described about the triangle D E F; Forasmuch as each of the sides of the one, doth touch each of the angles of the other. But the triangle G H I shall not be said to be described about the triangle K L M: Seeing that the side H I doth not touch the angle M, and the same ought to be understood of the inscriptions and circumscriptions of other right lined figures, &c. and right lined figures are properly said to be inscribed or circumscribed, when the number of the sides are equal, and although that be not absolutely necessary. (See the precedent figures.)

3 *A right lined figure is said to be inscribed in a circle, when each angle of the inscribed figure doth touch the circumference of the circle.*

AS if the angles A, B, and C, of the triangle A B C (See the precedent figure,) do touch the circumference of the circle A B C, the said triangle shall be said to be inscribed in the circle; and if some one of the angles do not touch the circumference, the triangle shall not be said to be inscribed in the circle.



4 *But a right lined figure is said to be described about a circle, when as each side of the figure circumscribed doth touch the circumference of the circle.*

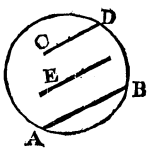
For

FOr Example, If the sides of the triangle A B C, do touch the circumference of the circle D E F: The triangle shall be said to be described about the circle.

5 *In like manner a circle is said to be inscribed in a right lined figure, when the circumference of the circle doth touch each side of the figure in which it is inscribed.*

6 *But a circle is said to be described about a figure, when the Circumference of the circle doth touch every angle of the figure about which it is described.*

IN like manner, the Circle D E F (in the figure of the fourth Definition) shall be said to be inscribed in the triangle A B C. But the Circle A B C, in the figure belonging to the second Definition, shall be said to be described about the triangle A B C; and the same is to be understood of other right lined figures, which are said to be inscribed in the Circle, or described about the Circle, or in which the Circle is said to be inscribed or described about.

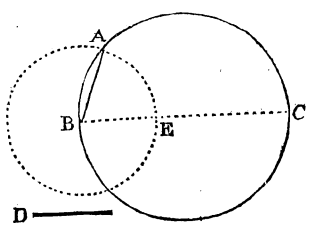


7 *A right line is said to be apted or fitted in a Circle, when the extremities thereof are in the Circumference of the Circle. As appears by this figure.*

PRO-

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. PROBLEM 1.



In a given circle A B C, *to fit a right line* A B, *equal to a given right line* D, *which may not be greater than the diameter* B C, *of the circle* A B C.

Construction. DRaw the diameter B C; if D be equal to B C, then is B C fitted in the circle, equal to D; but if D be leſſe than the diameter B C: Let B E be cut off [a] equal to D, and on B as a center, with the diſtance B E deſcribe the circle A E, cutting A B in the point A, and draw B A, which ſhall be the line fitted in the circle A B C, equal to D. [a] 3. 1.

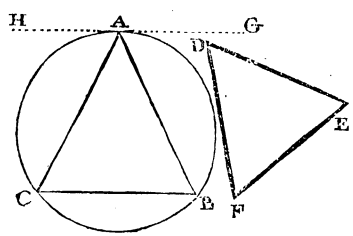
Demonſtration. FOr B A [b] is equal to B E, and D alſo equal to B E, by Conſtruction: therefore B A and D ſhall be equal to one another, and the extremities of B A are in the circumference, being drawn from the extremity of the diameter, in the point of Section of the circles: Therefore we have, &c. Which was to be done. [b] 25. def.

PROP. 2. PROBL. 2.

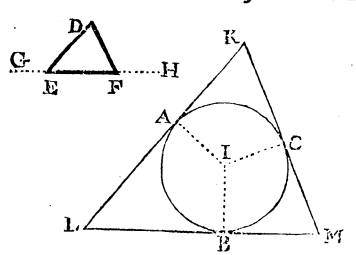
In a given circle A B C, *to inſcribe a triangle equiangled to a given triangle* D F E.

Conſtruction. DRaw H G, touching the circle in the point A, and make the angle G A B equal to the angle F, [a] and the angle H A C equal to the angle E, drawing the right lines A B and A C to the circumference, at the points B and C, and joyn the right line C B. I ſay, that the triangle A B C inſcribed in the given circle, is equiangled to the triangle D E F. [a] 17. 3.

Demonſtration. FOr [b] ſeeing that H G doth touch the circle, and A B cuts it, the angle G A B, which is made equal to the angle F, ſhall be equal to the angle C, in the alternate ſegment: therefore the angles C and F ſhall be equal to one another; In like manner, foraſmuch as the angle H A C (which is equal to the angle E, by Conſtruction,) is [c] equal to the angle B, in the alternate ſegment, the angles B and E ſhall be equal: therefore ſeeing that the two angles B and C of the triangle A B C are equal to the two angles E and F, of the triangle D E F, the two others [d] A and D ſhall be likewiſe equal to one another: Therefore the triangle A B C is equiangled to the triangle D E F: Therefore, In a Circle, &c. Which was to be done. [b] 32. 3. [c] 32. 3. [d] 32. 1.



PROP. 3. PROBL. 3.



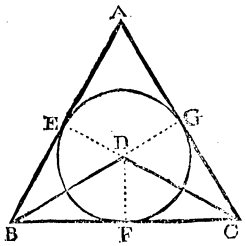
About a given circle A B C, *to deſcribe a triangle equiangled to a given triangle* D E F.

Conſtruction. HAving prolonged the ſide E F on both ſides, to the points G and H: Let the center I be taken, and draw I A at the contingent point A, and in the point I of the ſame line A I, make the angle A I B equal to the angle D E G, and B I C equal to D F H, and from the extremities A, B and C, draw at right angles the right lines K L, L M, and M K, which lines ſhall meet in the points K, L, and M: Foraſmuch as if you joyn a right line from A to B, the two angles L A B and L B A, ſhall be leſſe than two right angles, being no other than part of the two right angles I A L and I B L; therefore [a] A L and L B ſhall meet in the point L, and ſo of the reſt: Therefore the triangle L K M is deſcribed about the circle A B C, and is equiangled to the given triangle D E F. [a] 11. def. 1.

Demonſtration. FOr [b] all the ſides do touch the circle in the points A, B, and C, and foraſmuch as all the angles of the quadrilateral figure A I B L are equal to four right angles, as is ſhewn in the [b] C. 16. 3.

32 Propoſition of the firſt Book, and the two angles at the points A and B are right angles; the two angles A I B and A L B will remain equal to two right angles; and therefore equal to the two angles D E G and
c) 13. 1. D E F, which c are alſo equal to two right angles; therefore if the equal angles A I B and D E G be taken away, there will remaine A L B equal to D E F; even ſo the angle M will be ſhewn equal to the angle D F E: Therefore the third angle K ſhall be equal to the third angle D: Therefore the triangle K L M ſhall be equiangled to the triangle D E F: Therefore, &c. Which was to be done.

PROP. 4. PROBL 4.



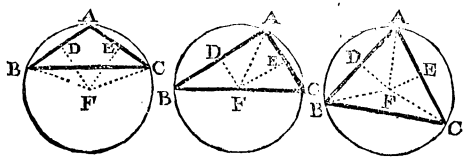
In a given triangle A B C, *to inſcribe a circle.*

a) 9. 1. *Conſtruction* DIvide a the two angles B and C, each in two equal parts, by the right lines B D and C D, meeting one another in the point D, and from that point draw the perpendiculars D G, D E, and D F, on the three ſides A B, B C, and C A.

Demonſtration FOraſmuch as the two angles D B F and D F B of the triangle D F B, are equal to the two angles D B E and D E B of the triangle D E B, each to its correſpondent, and the ſide D B com-
b) 26. 1. mon; b the two ſides D E and D F ſhall be equal, by the ſame reaſon, the ſides D F and D G ſhall be equal in the triangles D C F and D C G: Therefore the three lines D G, D F, and D E, ſhall be equal to one another, and the circle deſcribed on the point D as a center, at the diſtance
c) Co. 16. 3. D E, ſhall paſſe by the other points G and F, and ſhall c touch the ſides of the triangle in the points E, F, and G; ſeeing that the ſides are at right angles to the ſemidiameters D E, D F, D G: Therefore in a given triangle, &c. Which was to be done.

PROP. 5. PROBL. 5.

About a given triangle A B C, *to deſcribe a circle.*



a) 10. 1. *Conſtruction* DIvide a the two ſides A B and A C in two equal parts in the points D and E, from which points draw D F and E F,
b) 11. 1. b at right angles to thoſe ſides, and meeting in the point F.

Demonſtration FOr if a right line were drawn from D to E, the two angles F D E and F E D ſhould be leſſe than two right angles,)

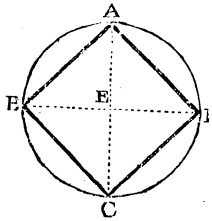
angles,) and the point F shall be in the triangle; or in the side, or without. Joyn then the right lines F A, F B, and F C: Forasmuch as the two sides A D and D F of the triangle A D F, are equal to the two sides B D and D F of the triangle B D F, and the angles at the point D right angles: [c] the bases F A and F B shall be equal, by the same reason, F A and F C shall be also equal; therefore the three sides F A, F B, and F C, shall be equal to one another: Therefore the circle described on the point F as a center, at the distance F A, shall also passe by the two other points B and C: Therefore about the triangle, &c. Which was to be done. c) 4. 1.

COROLLARIE.

It is manifest from this Demonstration, that if the center fall within the triangle, [d] *that all the angles are acute, being in the greatest segment of the circle; but if it fall on the side, the angle* [e] *opposite to that side, shall be a right angle in the semicircle: Lastly, if it fall without the triangle, the opposite angle* [f] *shall be obtuse, being in the least segment of the circle.* d) 31. 3. e) 31. 3. f) 31. 3.

Contrarily, If the triangle be an Oxigon, the center will fall within it, if a Rectangle, it will fall on the side which subtends the right angle, and if Ambligon, it will fall without.

PROP. 6. PROBL. 6.



In a given circle A B C, *to describe a square.*

Construction DRaw the two diameters A C and B D, which shall divide one another at right angles in the center E, and joyn the right lines A B, B C, C D, and D A: I say that A B C D is the square inscribed in the given circle.

Demonstration FOrasmuch as the two sides E A and E B of the triangle A E B are equal to the two sides E A and E D, of the triangle E A D, being all drawn from the center, and the angles conteined of those sides being right angles, [a] the bases A B and A D shall be equal; and in like manner C D and D A, and D A and A B; therefore A B C D shall be a square, having all the sides equal, and all the angles A, B, C, and D, right angles, [b] each of them being in the semicircle: Therefore, In a circle, &c. Which was to be done. a) 4. 1. b) 31. 3.

PROP. 7. PROBL. 7.

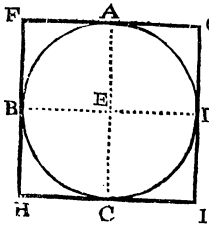
About a given circle A B C, *to describe a square.*

Construction DRaw two diameters, to divide one another at right angles in the center E, and by the points A, B, C, and D, draw the right lines F G, F H, H I, and I G, at right angles to the diameters, which shall meet with one another in the points F, H, I, and G: Forasmuch as if a right line be drawn from B to A, the two angles F B A and F A B, shall be lesse than two right angles, and so of the others.

I ſay that F G I H is a ſquare deſcribed about the given circle.

Demonſtration FOr ſeeing that the two angles A E B and F B E are right angles, F H and A C [a] ſhall be parallels, likewiſe G I and A C; therefore F H and [b] G I ſhall be parallels, and by the ſame reaſons F G and H I ſhall be alſo parallel.

a) 28. 1.
b) 30. 1.



c) 34. 1.

Now foraſmuch as A C H F is a parallelogram, the [c] oppoſite ſides A C and F H ſhall be equal, and the oppoſite angles C and F equal: But E C H is a right angle; therefore A F H ſhall be a right angle; by the ſame reaſon, the angles H, I, and G, ſhall be ſhewn to be right angles; and the ſides H I, I G, and G F, equal to the diameters A C and B D; and therefore equal to one another. Therefore F H I G ſhall have the four ſides equal, and the four angles right angles; and therefore ſhall be a ſquare, all whoſe ſides [d] do touch the circle: Therefore about a given circle, &c. Which was o be done.

d) C. 16. 3.

PROP. 8. PROBL. 8.

In a given ſquare F G H I, *to inſcribe a circle.*

Conſtruction HAving divided the ſides in two equal parts in the points B, C, D, and A, draw the right lines B D and C A, interſecting in the point E.

Demonſtration NOw [a] foraſmuch as F G and H I are equal and parallel, their halves F A and H C ſhall be equal and parallel; and therefore F H ſhall be equal and parallel to C A: In like manner, I G ſhall be equal and parallel to the ſame C A: Likewiſe F G and H I ſhall be equal and parallel to B D; therefore F E, E H, I E, and E G, ſhall be parallelograms; and therefore the lines E B, E C, E D, and E A, ſhall be equal to F A, B H, the equal lines G A, and F B; but thoſe lines are equal to one another; being the halves of F G, F H, &c. Therefore E B, E C, E D, and E A, are equal, therefore the circle deſcribed from the point E as a center, and at the diſtance E B: ſhall alſo paſſe by the points C, D, and A, and ſhall touch [b] all the ſides, ſeeing that the angles in the points B, C, D, and A, are right angles, and ſhall be inſcribed in the ſquare F I: Therefore in a given ſquare we have inſcribed a circle: Which was to be done.

a) 34. 1.
b) C. 16. 3.

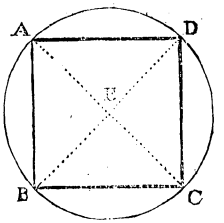
PROP. 9. PROBL. 9.

About a given ſquare A B C D, *to deſcribe a circle.*

Conſtruction DRaw the two diameters C A and B D, interſecting in the point E.

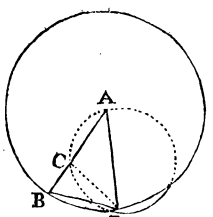
Demonſtration FOraſmuch as the ſides A B and A D, of the triangle A B D are equal, [a] the angles A B D and A D B ſhall be equal: But the angle B A D is a right angle in the ſquare, therefore [b] A B D and A D B ſhall be the halves of right angles: In like manner, we

a) 5. 1.



we ſhall ſhew that all the other angles, which are at the points A, B, C, and D, are the halves of right angles; and therefore equal to one another: Therefore ſeeing that the angles E A D and E D A are equal to one another, the lines E A and E D ſhall be equal, and by the ſame reaſon E A and E B ſhall be equal, and alſo E B and E C; likewiſe E C and E D: Therefore the circle deſcribed on the center E, at the diſtance E A, ſhall paſſe by the other points B, C, and D: Therefore about a given ſquare, &c. Which was to be done.

PROP. 10. PROBL. 10.



To conſtitute an Iſoſceles triangle A B D, *which may have each of the angles* B *and* D, *which are at the baſe* B D, *double to the other* A.

Conſtruction ASſume any line, as A B, which divide in the point C, in ſuch ſort as the Rectangle conteined under [a] A B and B C, may be equal to the ſquare of A C; then on the center A, with the diſtance A B, deſcribe a circle, [b] in which circle fit the right line B D, equal to A C, and joyn A D: I ſay, that the triangle A B D is an Iſoſceles triangle, and hath each of the angles A B D and A D B double to the other angle A.

a) 11. 2.

b) 1. 4.

Demonſtration FOr having drawn C D, deſcribe [c] about the triangle A C D the circle D C A, foraſmuch as the Rectangle conteined under A B and B C is equal to the ſquare of C A, that is to ſay, of B D, its equal; and A B cuts the circle [d] A C D, the line B D ſhall touch it in the point D: Therefore the angle B D C ſhall be [e] equal to the angle A in the alternate ſegment C A D, adding the common angle C D A, the whole angle A D B ſhall be equal to the two angles C A D and C D A; but the exterior angle D C B is [f] equal to the ſame angles A and A D C; therefore the angle B C D is equal to A D B; that is to ſay to [g] A B D, its equal; therefore [h] C D and D B ſhall be equal; But B D is equal to A C by Conſtruction: Therefore C D ſhall be equal to C A, and therefore [i] the two angles C A D and C D A ſhall be equal: Therefore the angle A D B, which is ſhewn equal to the two angles C A D and C D A, ſhall be double to the angle A; and therefore the angle A B D ſhall be alſo double to the ſame angle A: We have therefore conſtituted, &c. Which was to be done.

c) 5. 4.

d) 37. 3.

e) 32. 3.

f) 32. 1.

g) 5. 1.

h) 6. 1.

i) 5. 1.

COROLLARIE.

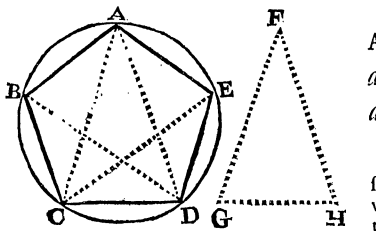
Now foraſmuch as the three angles [k] *of the triangle* A B D, *are equal to two right angles; that is to ſay, to five fifths of two right angles: It is manifeſt that*

k) 32. 1.

the angle A *is the fifth part of two right angles, and each of the two angles* D *and* B *is the two fifth parts of two right angles: Likewise* A *is the two fifth parts of a right angle, and either* B *or* D, *which of them you please is the four fifths;* [l] *forasmuch as all the three are equal to two right angles, that is to say, to ten fifths of a right angle.*

l) 32. 1.

PROP. 11. PROBL. 11.



In a given circle ABCD, *to describe an equilateral and equiangled Pentagon.*

Construction CONstitute the Isosceles triangle FGH; which may have each of the angles G and H double to the angle F, [a] and in the circle ABCD, inscribe the triangle ACD equiangled to the triangle FGH, [b] and divide both the angle ACD and ADC into two equal parts by the lines DB and CE, and joyn the right lines AB, BC, CD, DE, and EA: I say, that the Pentagon ABCDE inscribed in the circle is equilateral and equiangled.

a) 2. 4.

b) 9. 1.

Demonstration FOrasmuch as each of the angles ACD and ADC, is double to the angle CAD, they being each divided into two equal parts, the five angles ADB, BDC, CAD, DCE, and ECA shall be equal; Therefore [c] the Arches AB, BC, CD, DE, and EA, on which those angles are subtended, are equal: Therefore [d] the right lines AB, BC, CD, DE, and EA, which subtend them are equal: Therefore the Pentagon ABCD is equilateral: Again forasmuch as the arches AB and ED are equal, if you adde the common angle BCD, the whole ABCD shall be equal to the whole EDCB: Therefore the [e] angles AED and BAE, insisting on them shall be equal, in the same manner the other angles shall be equal, for they insist on equal arches, each of which is compounded of three equal arches: Therefore the Pentagon is equiangled, the which being also shewn to be equilateral: We have inscribed in a circle, &c. Which was to be done.

c) 26. 1.

d) 29. 1.

e) 27. 1.

COROLLARIE.

Hence it follows that the angle of the equilateral and equiangled Pentagon, is the three fifths of two right angles, or the $\frac{6}{5}$ *of one right angle:* [f] *For seeing that the three angles* BAC, CAD, *and* DAE *are equal, being they insist on the equal arches* BC, CD, *and* DE: *But* CAD *is* $\frac{1}{5}$ *part of two right angles, or* $\frac{2}{5}$ *parts of one right angle, the whole* BAE *shall be the* $\frac{3}{5}$ *of two right angles, or six fifths of one right angle.*

f) 27. 3.

PROP. 12. PROBL. 12.

About a given circle ABC, *to describe an equilateral and equiangled Pentagon.*

Con-

Construction INscribe [a] in the circle eqnilateral and equiangled Pentagon ABCDE; and let the center E be taken, and draw the right lines FA, FB, FC, FD, and FE, by the extremities of which lines draw GH, HI, IK, KL, and LG, at right angles to them, meeting in the points G, H, I, K, and L.

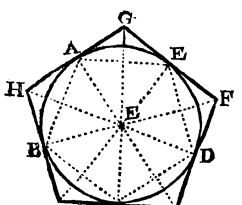
a) 11. 4.

For seeing that the angles GAE and GEA are lesse than two right angles, being part of the two right angles FAG and FEG, [b] the lines AG and EG shall meet with one another in the point G, and so of the others: And forasmuch as they [c] touch the circle, the Pentagon GHIKL shall be described about the circle, which I say is equilateral and equiangled.

b) 11. c. f.

c) C. 16. 3.

Demonstration FOr having drawn the right lines FG, FH, FI, FK, and FL, the two squares of FA and AH [d] shall be equal to the square of FH; and in like manner, the squares of FB and BH shall be equal to the same square of FH: therefore the two squares of FA and AH shall be equal to the two squares of FB and BH, taking away then the equal squares of the equal lines FA and FB, the squares of AH and BH will remaine equal; therefore the lines AH and BH are equal: And forasmuch as the two sides AH and FH of the triangle AFH are equal to BF and FH of the triangle BFH, each to his correspondent, and the base AH equal to the base BH, as is shewn, [e] the angle AFH shall be equal to the angle BFH, therefore the angles [f] AHF and BHF shall be also equal: therefore the angle AFB is double to the angle BFH, and the angle AHB double to the angle BHF.



d) 47. 1.

c) 8. 1.

f) 4. 1.

By the same discourse we shall shew that the angle BFC is double to the angle BFI, and the angle BIC double to the angle BIF: Seeing therefore [g] that the angles AFB and BFC are equal, as insisting on the equal circumferences AB and BC, [h] being subtended by the equal right lines AB and BC, their halves BFH and BFI, shall be equal.

g) 27. 3.

h) 28. 3.

Therefore seeing that the two angles BFH and HBF of the triangle BFH, are equal to the two angles BFI and IBF of the triangle IFB, and the side BF adjacent common, [i] the sides BH and BI shall be equal, and the angles BHF and BIF equal: Therefore the line HI is double to the line HB.

i) 26. 1.

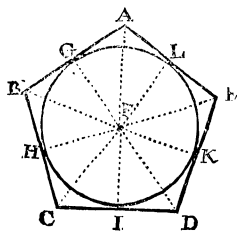
By the same reason, we shall shew that GH is double to HA; but HB and HA are shewn to be equal: Therefore their double, to wit, HI and HG shall be equal. In like manner, we shall shew, that the right lines IK, KL, and LG are equal to each of the right lines HI and HG: therefore the Pentagon GHIKL, is equilateral.

Again, forasmuch as it is shewn that the angles BHF and BIF are equal, and that they are the halves of the angles BHA and BIC; their doubles BHA and BIC shall be equal. By the same reason, the angles IKL, KLG, and LGH shall be equal to each of the angles BHA and BIC: Therefore the Pentagon GHIKL is equiangled:

There-

Therefore being ſhewn to be equilateral, we have deſcribed about a circle a Pentagon, equilateral and equiangled: Which was to be done.

PROP. 13. PROBL 13.



In a given Pentagon A B C D E, *which is equilateral and equiangled, to inſcribe a circle.*

Conſtruction Divide two of its angles B A E, and A B C, into two equal parts, by the right lines A F and B F, which ſhall meet with one another in the point F.

a) 11. c. 1. *Demonſtration* Seeing [a] that the two angles F A B and F B A are leſſe than two right angles, being the halves of the two angles A and B, that are leſſe than four right angles, as is demonſtrated in the 32 Propoſition of the firſt Book. Then draw the right lines F C, F D, and F E: Foraſmuch as the two ſides A B and B F of the triangle A B F, are equal to C B and B F of the triangle E B F, each to its correspondent, and A B F and

b) 4. 1. C B F conteined of them, equal by Conſtruction, [b] the baſes A F and C F, and the angles B A F and B C F are equal. And ſeeing that the angles A and C of the Pentagon are put equal, and B A F is the half of B A E by Conſtruction, the angle B C F ſhall be the half of B C D: Therefore B C D divided into two equal parts, by the ſame reaſon, we ſhall ſhew that the two other angles D and E are divided into two equal parts: Now draw from the

c) 12. 1. point F the [c] perpendicular lines F G, F H, F I, F K, and F L, on the ſides of the Pentagon: Foraſmuch as the two angles F G A and F A G of the triangle F A G; are equal to the two angles F L A and F A L of the triangle F A L, and the ſide A F oppoſite to one of the equal common angles, F G and F L, ſhall be equal.

In like manner, it ſhall be ſhewn that the other perpendiculars F H, F I, and F K, are equal to each of the other F G and F L: Therefore the circle deſcribed from the center F, at the diſtance F G, ſhall likewiſe paſſe by all the points H, I, K, and L; and foraſmuch as the ſides of

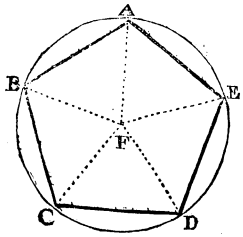
d) C. 16. 1. 0 the Pentagon do [d] touch the circle, ſeeing that they make right angles with the ſemidiameters F G, F H, &c. We ſhall have deſcribed a circle in the given Pentagon. Which was to be done,

PROP. 14. PROBL. 14.

About a given Pentagon A B C D E, *which is equilateral and equiangled, to deſcribe a circle.*

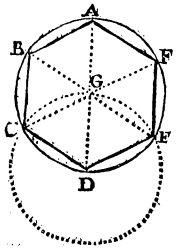
a) 9. 1. *Conſtruction* Having divided the angles B A E and A B C, into [a] two equal parts, by the right lines A I and B F, which ſhall meet with one another in the point F within the Pentagon, as hath been demonſtrated in the precedent Propoſition, and having joyned the right lines F C, F D, and F E, we ſhall ſhew, as in the precedent Problem, that the other angles C, D, and E, are divided into two equal parts: Therefore all the half angles ſhall be equal to one another, ſeeing that the whole angles are equal.

Demon-



Demonstration Forasmuch then as in the triangle A F B, the two angles F A B and F B A, are equal, the lines [b] F A and F B shall be equal, by the same reason, the other lines F C, F D, and F E, shall be equal to each of them: Therefore the circle described from the center F, at the distance of F A, shall also passe by the points B, C, D, and E: Therefore, About a Pentagon, &c. Which was to be done. b) 6. 1.

PROP. 15. PROBL. 15.



In a given circle A B C D E F, *to inscribe an equilateral and equiangled Hexagon.*

Construction HAving assumed the center G, and drawn the diameter A D, describe the circle C G E from the point D, which cutteth the given circle in the points C and E, from which points draw the right lines C F and E B, by the center G, and joyn the right lines A B, B C, C D, D E, E F, and F A. So shall the Hexagon A B C D E F be inscribed in the given circle, which I say is equilateral and equiangled.

Demonstration FOr by the definition of the circle, the three lines G C, C D, and D G shall be equal to one another, and the triangle C D G shall be equilateral; therefore [a] the three angles thereof shall be equal to one another, which [b] being equal to two right angles, which you please of the two, as C G D shall be the third part of two right angles: By the same reason, the angle D G E shall be the third part of two right lines: But the angles C G D, D G E, and E G F, are equal to two right angles: Therefore the other angle E G F shall be also the third part of two right angles: Therefore the three angles C G D, D G E, and E G F, [c] are equal to one another, to which the angles C G B, B G A, and A G B being [d] equal at the head, the six angles at the center G, shall be equal to one another: Wherefore [e] the circumferences on which they insist are equal; [f] and therefore the right lines A B, B C, C D, D E, E F, and F A, are equal. Wherefore the Hexagon A B C D E F shall be equilateral.

a) 5. 1.
b) 32. 1.
c) 13. 1.
d) 15. 1.
e) 26. 3.
f) 29. 3.

Again, forasmuch as the circumference B C is equal to A F, if you adde the common circumferences C D E F, the circumferences B C D E F and A F E D C shall be equal: Therefore [g] the angles A B C and B A F, which insist on them shall be equal: We shall shew in like manner, that the other angles of the Hexagon C, D, E, and F, are equal to each of the angles A and B, for that each of them insisteth as an arch compounded of four equal circumferences: Therefore the Hexagon is also equiangled: Therefore, In a given circle: Which was to be done.

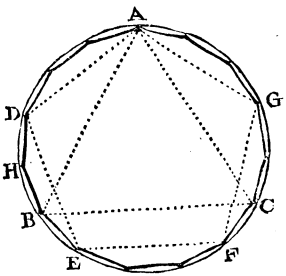
g) 27. 3.

C O-

COROLLARIE.

From this Proposition it is manifest, that the semidiameter of a circle is equal to the side of the Hexagon inscribed therein. For D G *the semidiameter, is equal to* D C *the side of the Hexagon, by the definition of the circle.*

PROP. 16. PROBL 16.



In a given cirtle A B C, *to inscribe an equilateral and equiangled Quindecagon.*

Construction INscribe in the circle, the equilateral triangle A B C, a the three arches A B, B C, and C A, shall be equal, because of the three sides, or the three equal angles A, B, and C.

a) 26, 28. 3.

b) 11. 4.

Again, b Inscribe in the same circle an equilateral and equiangled Pentagon A D E F G, having one of the angles applyed at the point A, c the five arches A D, D E, E F, F G, and G A, shall be equal; and seeing that the circumference of the circle ought to be divided into 15 equal parts: the arch A B shall contein five of them, and the arch A D three, seeing the arch A B is the third part of the whole circumference, and A D the fifth part: Therefore the rest D B, shall contein two parts. d Therefore having divided the arch D B into two equal parts in the point H, B H shall be the fifteenth part of the whole circumference: Therefore drawing the right line B H, it will subtend the fifteenth part of the whole circumference: Therefore e if you fit in the circle fourteen other right lines, equal to B A, there will be inscribed in the circle an equilateral Quindecagon, f the which is also equiangled, seeing that each of the angles insist on equal arches, each of which is compounded of thirteen equal arches, as is manifest Therefore, In a given circle we have described a Quindecagon. Which was to be done.

c) 28. 3.

d) 30. 3.

e) 1. 4.

f) 27. 3.

The End of the Fourth Element of EUCLIDE.

THE

THE FIFTH ELEMENT OF EUCLIDE.

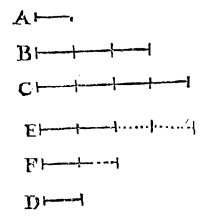
THE ARGUMENT.

IN the four precedent Books *EUCLIDE* hath treated of continued quantity considered, absolutely: But in the two following Books he disputeth of the same quantity, not absolutely; but in as much as the one refers it self to the other, that is to say, in as much as it is compared with another, it hath some Reason. In this Book he teaches the proportions of continued quantities in general, not referring them to any kind of quantity, as to a Line, a Superficie, or to any Body. But in the Sixth Book he shewes especially what reason lines have to one another, the angles, the circumferences of Circles, Triangles, and other plain figures, and to keep his Method, he defineth first the terms needful in the Demonstration.

DEFINITIONS.

1 *A magnitude is part of another magnitude, the lesser of the greater, when as the lesser doth measure the greater.*

HE ſaith therefore that a leſſer Magnitude which doth meaſure another greater Magnitude, is termed a part thereof; As for Example, foraſmuch as the Magnitude A taken three times, doth meaſure the Magnitude B, A ſhall be termed a part of B, and taken four times meaſureth C, the Magnitude A ſhall be alſo termed a part of C. But foraſmuch as D meaſureth neither E nor F, for taken twice, it exceeds F, and taken thrice it wanteth of E, and taken four times, it exceeds it; D ſhall not be termed a part of the Magnitudes E and F.



Now amongſt Mathematicians, there are two ſorts of parts, the one which meaſureth its whole, in ſuch ſort as being repeated, a certain number of times, it conſtituteth its whole, as 4 compared with 12, 16, 20, &c. is termed an aliquot part, the other doth not meaſure its whole, or being taken a certain number of times, it either exceeds or wants thereof, as 4 compared with 6, 7, 9, 18, 30, &c. is termed an aliquant part.

Therefore here *EUCLIDE* defineth only an aliquot part, as well becauſe it meaſureth only its whole, (for the aliquant part is not ſaid to meaſure its whole,) as for that (as in the Seventh Book,) the aliquant part in numbers is not by *EUCLIDE* called a part, but parts, for 4 is not a part of 6, but is the two third parts thereof, &c.

2 *A magnitude is multiplex of another leſſer magnitude, when the greater is meaſured of the leſſer.*

AS in the precedent Example, as well the Magnitude B, as C, is Multiplex of the Magnitude A, foraſmuch as the Magnitude A meaſureth as well the one as the other; to wit, B and C: But neither the Magnitude E nor F, ought to be termed Multiplex of the Magnitude D; foraſmuch as D meaſureth neither the one or the other: Therefore a part referreth it ſelf to Multiplex, and Multiplex to a part, in ſuch ſort as a leſſer quantity meaſuring a greater, is termed a part of the greater. But the greater, which is meaſured of the leſſer, is termed Multiplex of the leſſer.

It is then manifeſt enough, that the part heretofore defined, is that which exactly meaſures its whole, without any remainder.

Now when two leſſer magnitudes do meaſure equally two other greater Magnitudes, that is to ſay, that if one of the leaſt be conteined ſo many times exactly in one of the greater, as the other leſſer is conteined times in the other greater, thoſe two greater Magnitudes ſhall be termed equimultiplices of the two leſſer, and the ſame is to be ſaid, if divers leſſer Magnitudes do equally meaſure divers other greater Magnitudes.

3 *Reaſon, is an habitude of two magnitudes of the ſame kind, compared the one to the other, according to quantity.*

When

WHen two quantities of the ſame kind, as two numbers, two lines, two ſuperficies, two ſolids, &c. are compared to one another, according to quantity, that is to ſay, according as one is greater or leſſe than the other, or equal thereto, ſuch compariſon or mutual habitude is called reaſon, or (as others will have it,) proportion. Wherefore if you compare a line with a ſuperficie, or a number with a line, that compariſon cannot be called Reaſon, foraſmuch as, neither the line and the ſuperficie, nor the number and the line are quantities of the ſame kind.

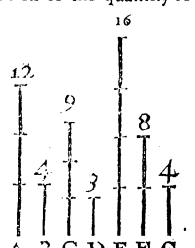
In like manner, if you compare a line with another line, or a ſuperficie with another ſuperfice, according to the quality, that is to ſay, according as the one is white, and the other black, or as the one is hot, and the other cold, &c. Although that both may be of one kind, nevertheleſſe ſuch compariſon ſhall not be termed Reaſon or Proportion, ſeeing that it is not made according to quantity.

Now although that Reaſon or Proportion is properly found betwixt quantities alone, nevertheleſſe the things which do receive in ſome manner the nature of the quantity, as Times, Sounds, Voices, Places, Motions, Weights, and Powers, are likewiſe ſaid to have Reaſon, if you conſider their habitude, according to quantity, as when we ſay that one time is greater than another time, or leſſe, or that two times are equal, &c. this habitude ſhall be called Proportion or Reaſon; for then the times are conſidered according to the manner of ſome quantities.

It remains that in all Proportion, the quantity that refers it ſelf to another, is termed by *EUCLIDE*, and other Geometricians, the Antecedent of the Reaſon. But that to which another refers it ſelf, is termed the conſequent of the Reaſon. As for Example, in the Reaſon of a line of 6 feet to a line of 3 feet, the line of 6 ſhall be termed the Antecedent of the Reaſon, and the line of 3 the conſequent, and contrarily, if you compare the line of 3 to the line of 6, the line of 3 ſhall be termed the Antecedent, and that of 6 the conſequent, otherwiſe the quantity put firſt ſhall be termed the Antecedent, and the ſecond the conſequent.

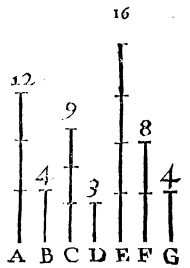
4 *Proportion is a ſimilitude or likeneſſe of Reaſons.*

THat which I call here Proportion, the Greeks call ἀναλογία, Analogie, and many Latines Proportionality. Therefore even as the habitude of two quantities to one another is termed Reaſon; ſo the compariſon of two or more Reaſons to one another, is called Proportion, as if the Reaſon of the quantity A to the quantity B, be like the Reaſon of C to D, the habitude between the Reaſons ſhall be called Proportion; in the ſame manner, if the Reaſon of E to F, be like to that of F to G, that ſimilitude ſhall be called Proportion.



Now there are divers ſorts of Proportion or habitude of Reaſons, (and we ſhall call the habitude of two quantities Reaſon; but the habitude of Reaſons Proportion,) deſcribed by Authors, principally by *Boeſius* and *Iordanus*, of which thoſe here have been

most in use amongst the Ancients; to wit, the proportion Arithmetical, Geometrical, Musical, or Harmonical. But *EUCLIDE* treateth only of Geometrical proportion in this Book. And of those there are two sorts, the one continued, in which the intermediate quantities are taken twice, in such sort as there is not made any interruption of Reasons. But each intermediate quantity is a consequent of the precedent quantity, and an antecedent to that which follows, as if you should say that there is the same reason of E to F, as of F to G, that proportion shall be called continued Proportion. But the other shall be termed discontinued proportion, in the which each of the intermediate quantities is taken only once, in such sort as there is made an interruption of Reasons, and not any quantity is Antecedent and Consequent, but Antecedent only, or Consequent only, as if you shall say that there is the same reason of A to B, as of C to D, this proportion shall be called Discontinual Proportion, or Proportion not continued.



Of the Division of the Reason.

Rational. *Irrational.* *EUCLIDE* divideth Reason in Rational and Irrational, Rational is that which may be expressed by numbers, as the reason of a line of 20 feet, to a line of 10 feet; for that reason is shewn by those two numbers 20 and 10. Irrational is that which cannot be expressed by numbers, as is the reason of the Diameter of a square, to the side of the same square, for that reason cannot be found in Numbers, as *EUCLIDE* doth demonstrate in his Tenth Book.

Commensurable quantities. Others say that Reason Rational, is that which is between any two quantities commensurable; But Irrational is that which is between two quantities incommensurable; Now those quantities are said to be commensurable, which have one common aliquot part, or are measured by one and the same common measure, as a line of 20 feet, and a line of 8, for a line of 4 feet is an aliquot part of both of them: In like manner, a line of 2 feet is an aliquot part of them: and therefore as well 4 as 2, shall measure as well the line of 20, as the line of 8.

Incommensurable. But Quantities incommensurable, are those which have no aliquot part, or any common measure that may measure them, as is the diameter of a square, and the side of the same square. For although that each of those lines have infinite aliquot parts, as the half, the third, and other parts: Nevertheless, not any aliquot part of the one, be it never so little, can possibly measure the other, as *EUCLIDE* doth demonstrate in the Tenth Book and last Proposition, in which Book are shewn divers other incommensurable lines besides those two. Therefore only, Reason Rational is found in numbers: But as well Rational as Irrational, is found in continued quantity.

Reason of equality. Reason is also divided into Reason of equality and Reason of inequality. Reason of equality is that which is between two equal quantities compared to one another, as between 20 and 20, between 100 and 100, be-

between a line of 12 feet, and a line of 12 feet; but Reason of inequality is that which is between two quantities unequal, compared to one another, as between 20 and 10, between 8 and 40, between a line of 6 feet, and a line of 2, &c.

Now these two kinds of Reason have this in common with the first, that all reason of equality is necessarily rational, and not contrarily. In like manner, all Reason Irrational, is necessarily reason of inequality, and not contrarily, &c.

Reason of equality. Again, Reason of inequality, (for we have left the reason of equality, considering that it cannot be subdivided; seeing that all equal quantities be they great or small, have alwayes the same reason of equality,) is subdivided into Reason of greater inequality, and of lesser inequality.

Inequality Major. Minor Reason of greater inequality, is when the greatest quantity is compared to the least, as the reason of 20 to 10, also a line of 8 feet to a line of 6 feet, &c. Reason of lesser Inequality is when the lesser quantity is compared to the greater, as the reason of 10 to 20, also of a line of 6 feet to a line of 8 feet, &c. Now this reason is not vain and superfluous, as divers doubt: for there is not the same reason of 4 to 2, as of 2 to 4, but much different to one another, their use being divers, as is manifest to those that are any wise versed in Geometry and in Algebra.

These are the general Divisions of reason considering it as it conteins all the reasons, none excepted. Now we will subdivide as well the reason of the greater inequality, as of the lesser inequality, for that they only comprehend the Reasons Rational; forasmuch as we ought to speak in the Tenth Book of quantities which have Reason Irrational.

Therefore Reason Rational, of the greater inequality is divided into five Kinds, to wit, into Reason Multiplex, Super-particular, Super-partient, Multiplex Super-particular, and Multiplex Super-partient: In like manner, the reason of the lesser inequality is divided according to the same kinds, provided that to each term of the reason there be put before it this Præposition *(Sub)* that is to say, under, as in the Reason Sub-Multiplex, Subsuper-particular, &c.

Now of these five kinds, the three first are simple, but the two last are compounded of those three first, as is manifest.

Of Reason Multiplex.

Reason Multiplex is an habitude of a greater quantity to a lesser, when the greater conteineth the lesser, a certain number of times precisely, as 2 times, 3 times, 10 times, 100 times. In such sort as the lesser measureth the greater, as is the reason of this number 20 to the number 4, for 20 conteins 4 five times, also the reason of a line of 30 feet to a line of 5 feet, &c.

Now this Reason conteineth under it infinite kinds, for if the greater quantity of the Reason Multiplex doth contein the lesser only twice, it shall be called Reason Double: if thrice, Triple, if ten times, Decuple, if a hundred times, Centuple, &c.

From these things, we shall easily define all the kinds of Reason Multiplex; for the Reason Octuple shall be no other thing then an habitude of a greater quantity to a lesser, when the greater doth contein the lesser, 8 times. In the same manner, we may define the other Reasons Multiplices, as

as the Reaſon Quintuple, ſuch as is 40 to 8, of which Reaſon, the greater conteins the leſſer five times, alſo the Reaſon of a line of 10 feet, to a line of 5 feet, is that in which the greater quantity conteineth the leſſer twice, and ſo of the others.

Of Reaſon Super-particular.

Reaſon Super-particular is an habitude of a greater quantity to a leſſer, when the greater conteines the leſſer only once and over and above an aliquot part of the ſaid leſſer, to wit the half, the third or the quarter &c. As is the Reaſon of 3 to 2, for 3 conteins 2 once and over and above Unite which is the half of 2. So in like manner, a line of 12 feet hath Reaſon Super-particular to a line of 9 feet, for the firſt line 12 conteineth the laſt 9 once, and over above a line of 3 feet, which is the third part of 9 feet.

This Reaſon is divided in like manner into infinit kinds: for if the aliquot part conteined in the greateſt quantity, be the halfe of the leſſer quantity, the Reaſon conſtituted ſhall be called Seſquialtera, if it be the third part, it ſhall be called Seſquitertia: if the fourth Seſquiquarta, and if the Thouſandth part, Sequimileſima.

Therefore by this ſame term the definitions of all the Reaſons, Super-particular ſhall be eaſie, as the Reaſon ſeſquioctava, is when the greater quantity conteins the leſſer, once and over and above the eighth part of the leſſer, as is the Reaſon of 9 to 8, alſo of 45 to 40, and the ſame is to be underſtood of the reſt.

Of Reaſon Super-partients.

Reaſon Super-partients is the habitude of a greater quantity to a leſſer, when the greater conteins the leſſer once only, and over and above ſome Aliquot parts of the ſame leſſer, which being taken together, do not make an Aliquot part, as is the proportion of 8 to 5, for 8 conteins 5 once, and over and above 3 Unites, each of which is an Aliquot part, to wit, the fifth part of this number 5, but the number 3 compounded of 3 Unites, is not an Aliquot part of 5.

I have ſaid that theſe Aliquot parts ought not to conſtitute an Aliquot part, becauſe of divers reaſons which at firſt ſeeme to be Super-partients, which nevertheleſſe are Super-particulars, as is the reaſon between 10 and 8, for although that the number 10 conteineth 8 once, and over and above 2 Unites, each of which is the eighth part of 8, yet notwithſtanding, forasmuch as the number of 2, compounded of theſe 2 Unites, is the fourth part of 8: this reaſon ought not to be called Super-partients, but Super-particular, to wit Seſquiquarta: therefore, to the end that two quantities may be ſaid to have Reaſon ſuper-partients, it is neceſſary that the greater quantity do contein the leſſer once, and over and above divides Aliquot parts of the ſame leſſer, the which taken together, ought not to conſtitute an Aliquot part; which divers not conſidering do confound many kindes of Reaſons one among another.

Of Reaſon Multiplex Superparticular.

Reaſon Multiplex Super-particular, is the habitude of a greater quantity to a leſſer, when the greater doth contein the leſſer, a certain number of times, as 2, 3, 4, &c. and over and above an Aliquot part of the leſſer, as is the Reaſon of 9 to 4, for 9 conteins 4 twice, and over and above, (in which

which it agrees with the Multiplex as with the double) doth contein Unity, which is the fourth part of the leſſer number (and in that, this Reaſon is like to the Superparticular, to wit, to the Seſquiquarta) in ſuch ſort as this reaſon ſhall be rightly ſaid to be compounded of the Multiplex and of the Superparticular.

Now this Reaſon, even ſo as is the Multiplex, is divided into ſeveral kindes, as into double Superparticular triple Superparticular, &c. conſidering that the greater quantity conteineth the leſſer quantity twice or thrice and over and above an Aliquot part of the leſſer quantity.

Again each of thoſe kindes is ſubdivided into divers, having regard to the Reaſon Superparticular, for Example, the reaſon triple Superparticular, conteineth under it the triple Seſquialtera (to wit, when the greater quantity doth contein thrice the leſſer and over and above the halfe of the ſame leſſer) the triple Seſquitertia, the triple Seſquiquarta and ſo of others.

Of Reaſon Multiplex Superpartients.

Laſtly, Reaſon Multiplex Superpartients is the habitude of a greater quantity to a leſſer, when the greater doth contein the leſſe, a certain number of times, and over and above ſome Aliquot parts of the leſſer, the which being taken together do not make an Aliquot part; as is the reaſon of 11 to 3, I have ſaid [not making an Aliquot part] for the reaſon alleaged in the reaſon Superpartients. For if the ſame Aliquot parts ſhould make an Aliquot part, it ſhould not be Reaſon Multiplex Superpartients but Multiplex Superparticular.

As the reaſon of 20 to 6 ought not to be termed Multiplex Superpartients Sixths, although that 20 contein 6 three times and two Sixths, for as much as $\frac{2}{6}$ parts make $\frac{1}{3}$ part. Wherefore it ſhall be called reaſon triple Seſquitertia.

Now this reaſon is divided, firſt of all, having regard to the reaſon Multiplex; as in Multiplex double, Superpartients triple Superpartients, &c. Again, each of them, (regard being had to the number of the parts,) conteineth under it, infinite kindes: As under the triple Superpartients, is conteined the triple Superbipartients, &c.

Laſtly, each of them, in conſideration of the denomination of the Aliquot parts is again divided into infinite kindes: As the triple Supertripartients is divided into triple Supertripartients fourths and triple Supertripartients fifths, &c. Of all which the definitions and the Examples will be eaſie by what hath been ſaid.

Of Reaſons rational, of leſſer inequality.

All that hath been ſaid hitherto, of the 5 kindes of reaſons Rational, of the greater inequall, ought in like manner to be underſtood of the five Kindes correſponding, of the leſſer inequallity: Nevertheleſſe alwayes adding this Propoſition (*Sub*) that is to ſay under as hath been ſaid.

For if in the Examples heretofore propoſed, we compare the leſſer quantities with the greater, we ſhall have the Reaſons of the leſſer inequality correſponding, for even ſo as the Reaſon of 100 to 1 is Centuple, ſo the Reaſon of 1 to 100, is Subcentuple, and as the Reaſon of 11 to 3, is Triple Superbipartient thirds, ſo the Reaſon of 3 to 11, is Subtriple, and Superbipartients thirds, and ſo of others.

Of

Of the Denominators of Reaſons Rational.

Foraſmuch as the uſe of the Denominators of Reaſons Rational before expoſed, is not a little profitable, it will not be amiſſe here to ſhew by what numbers they are denominated. The Denominator of any Reaſon, is the number which doth openly and diſtinctly expreſſe the habitude of one quantity to another, as the Denominator of the Reaſon Octuple is 8, for that the number 8 doth ſhew that the greater quantity of the Reaſon Octuple conteineth the leſſer 8 times. In like manner, the Denominator of the Reaſon Seſquiquinta is $\frac{1}{5}$; foraſmuch as this number doth ſignifie that the greater quantity of the Reaſon Seſquiquinta, conteineth the leſſer once, and the fifth part of the ſame leſſer: the ſame ought to be underſtood of the Denominators of the other Reaſons.

Wherefore (in my opinion) *EUCLIDE* in the Sixth Book, and ſundry other Mathematicians, do call the Denominator of any Reaſon whatſoever the quantity thereof: for the Denominator (as we have ſaid) ſhews how great one quantity is in reſpect of another, to wit, of hat with which it is compared, as appears by the Examples propoſed.

Now from theſe things which we have ſaid, may eaſily be gathered the Denominator of any Reaſon, for the Denominator of the Reaſon Multiplex whatſoever it be, is a whole number, which doth contein as many unites as the greater quantity doth contein times the leſſer: As for Example, the Denominator of the Reaſon Double 2, of the Reaſon Octuple 8, the Centuple is 100, &c. but the Denominators of the Reaſons Submultiplices, correſpondent to the Multiplices, are aliquot parts, denominated of the Denominators of the Reaſons Multiplices, to which the denominated do correſpond. As the Denominator of the Reaſon Subdupla, is $\frac{1}{2}$, Subquintuple is $\frac{1}{5}$, Subcentuple is $\frac{1}{100}$, &c. and ſo we ſhall find the Denominators of the other Reaſons Submultiplices. Therefore the Denominator of any Reaſon Submultiplex is a broken number, or a fraction of which the Numerator is alwayes an Unite: but the Denominator is a number denominating the Reaſon Multiplex correſponding, as appears by theſe Examples. Therefore it will not be difficult to find the Denominator of any Reaſon Multiplex, or Submultiplex, if what hath been ſaid be well underſtood; foraſmuch as the ſame prolation demonſtrateth the Denominator of the Reaſon, as is manifeſt by the Examples propoſed.

The Denominator of any Reaſon Super-particular is Unity with that aliquot part that the greater quantity ought to contein over and above the leſſer: As the Denominator of the Reaſon Seſquialtera is 1 and $\frac{1}{2}$, Seſquioctava is 1 and $\frac{1}{8}$, Seſquicenteſima is 1 and $\frac{1}{100}$, &c. and it will not be difficult to find the Denominator of any Reaſon Superparticular; ſeeing that the ſame prolation of Reaſon doth expreſſe the Denominator by its aliquot part, as is evident by the Examples propoſed.

But the Denominators of the Reaſons Subſuper-particulars correſpondents, are fractions, whoſe Numerators are leſſe than their Denominators by Unity only, each to his correſpondent, as the Denominator of the Reaſon Subſeſquialtera is $\frac{2}{3}$, Subſeſquioctava is $\frac{8}{9}$, Subſeſquicenteſima $\frac{100}{101}$. Now the Denominator of any Reaſon Super-particular may be found, if for the Numerator of the fraction you take the Denominator of the aliquot part expreſſed in the reaſon, and for the Denominator of the ſame fraction, you take a number greater than Unity: As the Denominator of

the

the Reaſon Subſeſquidecimal is $\frac{10}{11}$; ſeeing that the Numerator of this fraction is the number denominating the tenth part, to wit 10, but the Denominator of the ſame fraction exceedeth the Numerator by Unity, the Denominator of the Reaſon Subſeſquiſeptimal is $\frac{7}{8}$, that of the reaſon ſubſeſquiquinta is $\frac{5}{6}$, &c.

In like manner, we ſhall find the Denominator of any Reaſon Subſuperparticular, if we reduce the Denominator of the Reaſon Subſuperparticular, correſpondent in a fraction, of the which fraction the Numerator ſhall exceed alwayes the Denominator by unity, the which is alſo of the aliquot part, whereof mention is made in the Reaſon propoſed, ſeeing we tranſpoſe the terms of this fraction, in ſuch ſort as the Numerator may be made Denominator, and the Denominator Numerator; we ſhall have the Denominator of the reaſon Subſuperparticular propoſed. As if the Reaſon Subſeſquiſeptimal be propoſed; foraſmuch as the Denominator of the Reaſon Seſquiſeptimal which is correſpondent thereto, is 1 and $\frac{1}{7}$, which is reduced into this fraction $\frac{8}{7}$, whoſe Numerator is greater than the Denominator by the aliquot part of Unity; wherefore if we tranſpoſe this fraction in this manner $\frac{7}{8}$, we ſhall ſay that the Denominator of the Reaſon Subſeſquiſeptimal is $\frac{7}{8}$, &c.

The Denominator of any Reaſon Superpartients is Unity with the aliquot parts, (not making an aliquot part) which the greateſt quantity ought to contein over and above the leſſer; As the Denominator of the Reaſon Supertripartient ſeptimal is 1 and $\frac{3}{7}$, Supertripartient 20 is $1\frac{3}{17}$, &c. and is eaſie to be found, foraſmuch as the prolation of the Reaſon doth ſhew the proper Denominator, as appeares by the Examples before mentioned.

But the Denominator of any Reaſon Super-partient, is a Fraction, whoſe Numerator is leſſe than the Denominator by ſo many unites, as the greater quantity of the Reaſon propoſed, doth contein aliquot parts, over and above the leſſer, as the Denominator of the Reaſon Subſupertripartient ſeptimal is $\frac{17}{20}$ of the Subſupertripartient 20 is $\frac{17}{20}$, &c. Now the Denominator of any Reaſon Subſupertripartient ſhall be found, if for the Denominator of the fraction, you take the Denominator of the aliquot parts conteined in the reaſon propoſed, to which if you adde the number of the ſame parts, it will give you the Denominator of that fraction.

As the Denominator of the Reaſon Subſupertripartient 11, to wit 11, to which is added 4, the number of four aliquot parts, to the end that 15 be made the Denominator of the ſame fraction.

But the Denominator of the Reaſon Subſupertripartient fifths, is this fraction $\frac{5}{8}$, for the Numerator thereof is the number denominating the fifth parts, to wit 5: But the Denominator of the ſame fraction 8, is made of the Numerator 5, and of 3, the number of the three aliquot parts, and ſo of the other Denominators, which may be alſo found in this manner.

Let the Denominator of the Reaſon Subſuperpartient, correſpondent in a fraction be reduced, whoſe Numerator may exceed the Denominator, which denominateth alſo the aliquot parts expreſſed, by ſo many unites as there are aliquot parts.

For the numbers of this fraction being tranſpoſed in ſuch ſort as the Numerator be made Denominator, you ſhall have the Denominator of the Reaſon propoſed Subſuperpartient, as the Denominator of the Reaſon Subſupertripartient fifths is $\frac{5}{8}$, for that the Denominator of the

Reason Supertripartient fifths is $1\frac{3}{5}$, which is reduced into this fraction $\frac{8}{5}$, which makes $\frac{5}{8}$ in transposing the numbers, and the same ought to be done for finding the other numbers, &c.

The Denominator of any Reason Multiplex Superparticular, is a whole number denominating the Reason Muliplex proposed, with the aliquot part that the greater quantity ought to contein, over and above the lesser; as the Denominator of the Reason Tripleseſquiseptimal is $3\frac{1}{7}$, the Quintuple seſquinoncuple is $5\frac{1}{9}$, &c, and is very easie to expresse the Denominator of any Reason Multiplex Superparticular, forasmuch as the prolation of the Reason sheweth distinctly both the Denominator of the Reason Multiplex, and the aliquot part as may be seen by the examples proposed. Now the Denominator of any Reason Submultiplex superparticular, is a fraction, whereof the Numerator is a number denominating the aliquot parts in the Reason, as the Denominator of the Reason Subtriple seſquiseptimal is $\frac{7}{13}$, the Subquintuple seſquinonal is $\frac{9}{46}$, &c.

Now this Denominator will be found, if for the Numerator of the fraction you take the Denominator of the aliquot part, which being multiplyed by the Denominator of the Reason Multiplex, if you adde unity to the product, you shall have the Denominator of the same fraction.

As for Example, the Denominator of the Reason subquadruple seſquisexta is $\frac{6}{17}$; forasmuch as the Numerator 6 doth denominate the sixth parts, and the same multiplyed by 4 the Denominator of the Reason quadruple, and having added to the product 24 unites, to the end the Denominator of the same fraction may be made 25, &c. otherwise you should find the same denominator, if you reduce into one fraction the Denominator of the Reason Multiplex superparticular, corresponding as is before said.

As if the Reason Subquadruple seſquisexta were proposed; forasmuch as the Denominator of the Reason quadruple sexta correspondent is $4\frac{1}{6}$, which shall be reduced to $\frac{25}{6}$; therefore if you change the order of the terms, of the same fraction you will have $\frac{6}{25}$, for the Denominator of the Reason Subquadruple seſquisexta, and so of the others.

The Denominators of any Reason Multiplex superpartients, is a whole number denominating the Reason Multiplex expressed therein, with the aliquot parts which the greater quantity ought to contein over and above the lesser, which said aliquot parts do not constitute an aliquot part, as the Denominator of the Reason Triple Superquinpartient octa is $3\frac{5}{8}$, of the quadruple superbipartient fifth is $4\frac{2}{5}$.

And to find the Denominators, there is no difficulty, seeing they are expressed distinctly and openly in each Reason proposed Multiplex superpartient, to wit, as well the Denominator of the Reason Multiplex conteined therein, as the aliquot parts, as is manifest by the Examples proposed. But the Denominator of any Reason Submultiple superpartient, is a fraction, of which the Numerator is a number denominating the aliquot parts of the same Reason.

As the Denominator of the Reason Subtriple superquintapartient is $\frac{8}{29}$, subquadruple superbipartient is $\frac{7}{27}$. Now the Denominator of those Reasons submultiplices superpartients will be found, if for the Numerator of the fraction you take the Denominator of the aliquot parts, which being multiplyed by the Denominator of the Reason Multiplex, and having added to the product the number of the aliquot parts, you will obtein the Denominator of the same fraction.

As the Denominator of the Reason subduple superoctupartient is $\frac{11}{19}$, for that

that the Numerator of this fraction 13 doth denominate the thirteenth part which multiplyed by 2, the Denominator of the Reason Duple, and to the number produced 26 there be added the number of 8 parts, it will make 34, the Denominator of the same fraction.

Otherwise there will be found the same Denominator, reducing the Denominator of the Reason correspondent in a fraction, and transposing the terms as is before said.

As for Example, Let the Reason Subquintuple supertripartient tenths be proposed, to find the Denominator, you shall reduce into a fraction the Denominator of the reason correspondent quintuple supertripartient tenth, which is $5\frac{1}{10}$, and you shall have this fraction $\frac{51}{10}$; and in changing the terms, that is to say putting 10 instead of 53, and 53 instead of 10, you will have $\frac{10}{53}$ for the Denominator of the reason subquintuple supertripartient tenth, and so of the rest.

Lastly, the Denominator of the Reason of equality is alwayes unity; forasmuch as in this reason the quantities ought to be equal to one another, and therefore the one ought to contein the other once and no more, which is signified by unity.

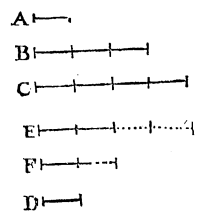
5 *Magnitudes are said to have reason to one another, which being multiplyed may exceed one another.*

FOrasmuch as *EUCLIDE* in the third Definition, hath termed the habitude of two magnitudes of the same Kind, Reason, he teacheth now in this fifth Definition, what thing is required, to the end that two quantities of the same kind may be said to have Reason to one another; saying that Magnitudes are said to have Reason to one another, of the which (which you please) being multiplyed, may be augmented in such sort as that it may exceed the other, as the diameter and the side of one and the same square, are said to have Reason, although it be Irrational, which cannot be expressed by any number; forasmuch as the side multiplyed by 2 or taken twice, doth exceed the diameter; for seeing that the two sides of the square, with the diameter do constitute an Isosceles triangle, the two sides of the square shall be greater than the diameter thereof.

So in like manner, the circumference of a circle and the diameter thereof shall have Reason, (although it be not as yet known;) forasmuch as the diameter multiplyed by 4, or taken 4 times, doth exceed the circumference, seeing that every circumference of a circle (as is demonstrated by *Archimedes*) doth contein only three times the diameter of the said circle, and over and above a particle; a little lesse than the seventh part of the diameter.

In the same manner, divers curviline figures will have Reason with right lined figures as *Proclus* hath shewn on the First of *EUCLIDE*. Now as divers Interpreters do say that neither all Lines, nor all Plain Angles have Reason to one another, according to this Definition: for say they, a finite line hath no Reason to an infinite; seeing that in what manner soever it be multiplyed, it cannot exceed the infinite line, neither hath a right lined angle Reason to an angle of contingence, for the angle of contingence cannot exceed a right lined angle.

HE saith therefore that a lesser Magnitude which doth measure another greater Magnitude, is termed a part thereof; As for Example, forasmuch as the Magnitude A taken three times, doth measure the Magnitude B, A shall be termed a part of B, and taken four times measureth C, the Magnitude A shall be also termed a part of C. But forasmuch as D measureth neither E nor F, for taken twice, it exceeds F, and taken thrice it wanteth of E, and taken four times, it exceeds it; D shall not be termed a part of the Magnitudes E and F.



Now amongst Mathematicians, there are two sorts of parts, the one which measureth its whole, in such sort as being repeated, a certain number of times, it constituteth its whole, as 4 compared with 12, 16, 20, &c. is termed an aliquot part, the other doth not measure its whole, or being taken a certain number of times, it either exceeds or wants thereof, as 4 compared with 6, 7, 9, 18, 30, &c. is termed an aliquant part.

Therefore here *EUCLIDE* defineth only an aliquot part, as well because it measureth only its whole, (for the aliquant part is not said to measure its whole,) as for that (as in the Seventh Book,) the aliquant part in numbers is not by *EUCLIDE* called a part, but parts, for 4 is not a part of 6, but is the two third parts thereof, &c.

2 *A magnitude is multiplex of another lesser magnitude, when the greater is measured of the lesser.*

AS in the precedent Example, as well the Magnitude B, as C, is Multiplex of the Magnitude A, forasmuch as the Magnitude A measureth as well the one as the other; to wit, B and C: But neither the Magnitude E nor F, ought to be termed Multiplex of the Magnitude D; forasmuch as D measureth neither the one or the other: Therefore a part referreth it self to Multiplex, and Multiplex to a part, in such sort as a lesser quantity measuring a greater, is termed a part of the greater. But the greater, which is measured of the lesser, is termed Multiplex of the lesser.

It is then manifest enough, that the part heretofore defined, is that which exactly measures its whole, without any remainder.

Now when two lesser magnitudes do measure equally two other greater Magnitudes, that is to say, that if one of the least be conteined so many times exactly in one of the greater, as the other lesser is conteined times in the other greater, those two greater Magnitudes shall be termed equimultiplices of the two lesser, and the same is to be said, if divers lesser Magnitudes do equally measure divers other greater Magnitudes.

3 *Reason, is an habitude of two magnitudes of the same kind, compared the one to the other, according to quantity.*

When

WHen two quantities of the same kind, as two numbers, two lines, two superficles, two solids, &c. are compared to one another, according to quantity, that is to say, according as one is greater or lesse than the other, or equal thereto, such comparison or mutual habitude is called reason, or (as others will have it,) proportion. Wherefore if you compare a line with a superficie, or a number with a line, that comparison cannot be called Reason; forasmuch as, neither the line and the superficie, nor the number and the line are quantities of the same kind.

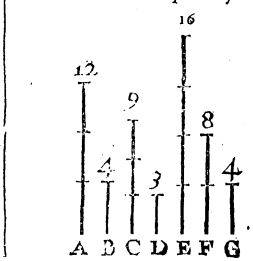
In like manner, if you compare a line with another line; or a superficie with another superfice, according to the quality, that is to say, according as the one is white, and the other black, or as the one is hot, and the other cold, &c. Although that both may be of one kind, nevertheless such comparison shall not be termed Reason or Proportion, seeing that it is not made according to quantity.

Now although that Reason or Proportion is properly found betwixt quantities alone, nevertheless the things which do receive in some manner the nature of the quantity, as Times, Sounds, Voices, Places, Motions, Weights, and Powers, are likewise said to have Reason, if you consider their habitude, according to quantity, as when we say that one time is greater than another time, or lesse, or that two times are equal, &c. this habitude shall be called Proportion or Reason; for then the times are considered according to the manner of some quantities.

It remains that in all Proportion, the quantity that refers it self to another, is termed by *EUCLIDE*, and other Geometricians, the Antecedent of the Reason. But that to which another refers it self, is termed the consequent of the Reason. As for Example, in the Reason of a line of 6 feet to a line of 3 feet, the line of 6 shall be termed the Antecedent of the Reason, and the line of 3 the consequent, and contrarily, if you compare the line of 3 to the line of 6, the line of 3 shall be termed the Antecedent, and that of 6 the consequent, otherwise the quantity put first shall be termed the Antecedent, and the second the consequent.

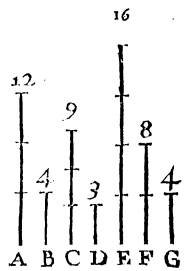
4 *Proportion is a similitude or likenesse of Reasons.*

THat which I call here Proportion, the Greeks call ἀναλογία, Analogie, and many Latines Proportionality. Therefore even as the habitude of two quantities to one another is termed Reason; so the comparison of two or more Reasons to one another, is called Proportion, as if the Reason of the quantity A to the quantity B, be like the Reason of C to D, the habitude between the Reasons shall be called Proportion; in the same manner, if the Reason of E to F, be like to that of F to G, that similitude shall be called Proportion.



Now there are divers sorts of Proportion or habitude of Reasons, (and we shall call the habitude of two quantities Reason; but the habitude of Reasons Proportion,) described by Authors, principally by *Boesius* and *Iordanus*, of which those here have been

most in use amongst the Ancients; to wit, the proportion Arithmetical, Geometrical, Musical, or Harmonical. But *EUCLIDE* treateth only of Geometrical proportion in this Book. And of those there are two sorts, the one continued, in which the intermediate quantities are taken twice, in such sort as there is not made any interruption of Reasons. But each intermediate quantity is a consequent of the precedent quantity, and an antecedent to that which follows, as if you should say that there is the same reason of E to F, as of F to G, that proportion shall be called continued Proportion. But the other shall be termed discontinued proportion, in the which each of the intermediate quantities is taken only once, in such sort as there is made an interruption of Reasons, and not any quantity is Antecedent and Consequent, but Antecedent only, or Consequent only, as if you shall say that there is the same reason of A to B, as of C to D, this proportion shall be called Discontinual Proportion, or Proportion not continued.



Of the Division of the Reason.

Rational. *Irrational.* *EUCLIDE* divideth Reason in Rational and Irrational, Rational is that which may be expressed by numbers, as the reason of a line of 20 feet, to a line of 10 feet; for that reason is shewn by those two numbers 20 and 10. Irrational is that which cannot be expressed by numbers, as is the reason of the Diameter of a square, to the side of the same square, for that reason cannot be found in Numbers, as *EUCLIDE* doth demonstrate in his Tenth Book.

Commensurable quantities. Others say that Reason Rational, is that which is between any two quantities commensurable; But Irrational is that which is between two quantities incommensurable; Now those quantities are said to be commensurable, which have one common aliquot part, or are measured by one and the same common measure, as a line of 20 feet, and a line of 8, for a line of 4 feet is an aliquot part of both of them: In like manner, a line of 2 feet is an aliquot part of them: and therefore as well 4 as 2, shall measure as well the line of 20, as the line of 8.

Incommensurable. But Quantities incommensurable, are those which have no aliquot part, or any common measure that may measure them, as is the diameter of a square, and the side of the same square. For although that each of those lines have infinite aliquot parts, as the half, the third, and other parts: Nevertheless, not any aliquot part of the one, be it never so little, can possibly measure the other, as *EUCLIDE* doth demonstrate in the Tenth Book and last Proposition, in which Book are shewn divers other incommensurable lines besides those two. Therefore only, Reason Rational is found in numbers: But as well Rational as Irrational, is found in continued quantity.

Reason of equality. Reason is also divided into Reason of equality and Reason of inequality. Reason of equality is that which is between two equal quantities compared to one another, as between 20 and 20, between 100 and 100, between a line of 12 feet, and a line of 12 feet; but Reason of inequality is that which is between two quantities unequal, compared to one another, as between 20 and 10, between 8 and 40, between a line of 6 feet, and a line of 2, &c.

Now these two kinds of Reason have this in common with the first, that all reason of equality is necessarily rational, and not contrarily. In like manner, all Reason Irrational, is necessarily reason of inequality, and not contrarily, &c.

Reason of equality. Again, Reason of inequality, (for we have left the reason of equality, considering that it cannot be subdivided, seeing that all equal quantities be they great or small, have alwayes the same reason of equality,) is subdivided into Reason of greater inequality, and of lesser inequality.

Inequality Major. Minor Reason of greater inequality, is when the greatest quantity is compared to the least, as the reason of 20 to 10, also a line of 8 feet to a line of 6 feet, &c. Reason of lesser Inequality is when the lesser quantity is compared to the greater, as the reason of 10 to 20, also of a line of 6 feet to a line of 8 feet, &c. Now this reason is not vain and superfluous, as divers doubt: for there is not the same reason of 4 to 2, as of 2 to 4, but much different to one another, their use being divers, as is manifest to those that are any wise versed in Geometry and in Algebra.

These are the general Divisions of reason considering it as it conteins all the reasons, none excepted. Now we will subdivide as well the reason of the greater inequality, as of the lesser inequality, for that they only comprehend the Reasons Rational; forasmuch as we ought to speak in the Tenth Book of quantities which have Reason Irrational.

Therefore Reason Rational, of the greater inequality is divided into five Kinds, to wit, into Reason Multiplex, Super-particular, Super-partient, Multiplex Super-particular, and Multiplex Super-partient: In like manner, the reason of the lesser inequality is divided according to the same kinds, provided that to each term of the reason there be put before it this Præposition *(Sub)* that is to say, under, as in the Reason Sub-Multiplex, Subsuper-particular, &c.

Now of these five kinds, the three first are simple, but the two last are compounded of those three first, as is manifest.

Of Reason Multiplex.

Reason Multiplex is an habitude of a greater quantity to a lesser, when the greater conteineth the lesser, a certain number of times precisely, as 2 times, 3 times, 10 times, 100 times. In such sort as the lesser measureth the greater, as is the reason of this number 20 to the number 4, for 20 conteins 4 five times, also the reason of a line of 30 feet to a line of 5 feet, &c.

Now this Reason conteineth under it infinite kinds, for if the greater quantity of the Reason Multiplex doth contein the lesser only twice, it shall be called Reason Double: if thrice, Triple, if ten times, Decuple, if a hundred times, Centuple, &c.

From these things, we shall easily define all the kinds of Reason Multiplex; for the Reason Octuple shall be no other thing then an habitude of a greater quantity to a lesser, when the greater doth contein the lesser, 8 times. In the same manner, we may define the other Reasons Multiplices, as

as the Reaſon Quintuple, ſuch as is 40 to 8, of which Reaſon, the greater conteins the leſſer five times, alſo the Reaſon of a line of 10 feet, to a line of 5 feet, is that in which the greater quantity conteineth the leſſer twice, and ſo of the others.

Of Reaſon Super-particular.

Reaſon Super-particular is an habitude of a greater quantity to a leſſer, when the greater conteines the leſſer only once and over and above an aliquot part of the ſaid leſſer, to wit the half, the third or the quarter &c. As is the Reaſon of 3 to 2, for 3 conteins 2 once and over and above Unite which is the half of 2. So in like manner, a line of 12 feet hath Reaſon Super-particular to a line of 9 feet, for the firſt line 12 conteineth the laſt 9 once, and over above a line of 3 feet, which is the third part of 9 feet.

This Reaſon is divided in like manner into infinit kinds: for if the aliquot part conteined in the greateſt quantity, be the halfe of the leſſer quantity, the Reaſon conſtituted ſhall be called Seſquialtera, if it be the third part, it ſhall be called Seſquitertia: if the fourth Seſquiquarta, and if the Thouſandth part, Sequimileſima.

Therefore by this ſame term the definitions of all the Reaſons, Super-particular ſhall be eaſie, as the Reaſon ſeſquioctava, is when the greater quantity conteins the leſſer, once and over and above the eighth part of the leſſer, as is the Reaſon of 9 to 8, alſo of 45 to 40, and the ſame is to be underſtood of the reſt.

Of Reaſon Super-partients.

Reaſon Super-partients is the habitude of a greater quantity to a leſſer, when the greater conteins the leſſer once only, and over and above ſome Aliquot parts of the ſame leſſer, which being taken together, do not make an Aliquot part, as is the proportion of 8 to 5, for 8 conteins 5 once, and over and above 3 Unites, each of which is an Aliquot part, to wit, the fifth part of this number 5, but the number 3 compounded of 3 Unites, is not an Aliquot part of 5.

I have ſaid that theſe Aliquot parts ought not to conſtitute an Aliquot part, becauſe of divers reaſons which at firſt ſeeme to be Super-partients, which nevertheleſſe are Super-particulars, as is the reaſon between 10 and 8, for although that the number 10 conteineth 8 once, and over and above 2 Unites, each of which is the eighth part of 8, yet notwithſtanding, forasmuch as the number of 2, compounded of theſe 2 Unites, is the fourth part of 8: this reaſon ought not to be called Super-partients, but Super-particular, to wit Seſquiquarta: therefore, to the end that two quantities may be ſaid to have Reaſon ſuper-partients, it is neceſſary that the greater quantity do contein the leſſer once, and over and above divides Aliquot parts of the ſame leſſer, the which taken together, ought not to conſtitute an Aliquot part; which divers not conſidering do confound many kindes of Reaſons one among another.

Of Reaſon Multiplex Superparticular.

Reaſon Multiplex Super-particular, is the habitude of a greater quantity to a leſſer, when the greater doth contein the leſſer, a certain number of times, as 2, 3, 4, &c. and over and above an Aliquot part of the leſſer, as is the Reaſon of 9 to 4, for 9 conteins 4 twice, and over and above, (in which

which it agrees with the Multiplex as with the double) doth contein Unity, which is the fourth part of the leſſer number (and in that, this Reaſon is like to the Superparticular, to wit, to the Seſquiquarta) in ſuch ſort as this reaſon ſhall be rightly ſaid to be compounded of the Multiplex and of the Superparticular.

Now this Reaſon, even ſo as is the Multiplex, is divided into ſeveral kindes, as into double Superparticular triple Superparticular, &c. conſidering that the greater quantity conteineth the leſſer quantity twice or thrice and over and above an Aliquot part of the leſſer quantity.

Again each of thoſe kindes is ſubdivided into divers, having regard to the Reaſon Superparticular, for Example, the reaſon triple Superparticular, conteineth under it the triple Seſquialtera (to wit, when the greater quantity doth contein thrice the leſſer and over and above the halfe of the ſame leſſer) the triple Seſquitertia, the triple Seſquiquarta and ſo of others.

Of Reaſon Multiplex Superpartients.

Laſtly, Reaſon Multiplex Superpartients is the habitude of a greater quantity to a leſſer, when the greater doth contein the leſſe, a certain number of times, and over and above ſome Aliquot parts of the leſſer, the which being taken together do not make an Aliquot part; as is the reaſon of 11 to 3, I have ſaid [not making an Aliquot part] for the reaſon alleaged in the reaſon Superpartients. For if the ſame Aliquot parts ſhould make an Aliquot part, it ſhould not be Reaſon Multiplex Superpartients but Multiplex Superparticular.

As the reaſon of 20 to 6 ought not to be termed Multiplex Superpartients Sixths, although that 20 contein 6 three times and two Sixths, for as much as $\frac{2}{6}$ parts make $\frac{1}{3}$ part. Wherefore it ſhall be called reaſon triple Seſquitertia.

Now this reaſon is divided, firſt of all, having regard to the reaſon Multiplex; as in Multiplex double, Superpartients triple Superpartients, &c. Again, each of them, (regard being had to the number of the parts,) conteineth under it, infinite kindes: As under the triple Superpartients, is conteined the triple Superbipartients, &c.

Laſtly, each of them, in conſideration of the denomination of the Aliquot parts is again divided into infinite kindes: As the triple Supertripartients is divided into triple Supertripartients fourths and triple Supertripartients fifths, &c. Of all which the definitions and the Examples will be eaſie by what hath been ſaid.

Of Reaſons rational, of leſſer inequality.

All that hath been ſaid hitherto, of the 5 kindes of reaſons Rational, of the greater inequall, ought in like manner to be underſtood of the five Kindes correſponding, of the leſſer inequallity: Nevertheleſſe alwayes adding this Propoſition (*Sub*) that is to ſay under as hath been ſaid.

For if in the Examples heretofore propoſed, we compare the leſſer quantities with the greater, we ſhall have the Reaſons of the leſſer inequality correſponding, for even ſo as the Reaſon of 100 to 1 is Centuple, ſo the Reaſon of 1 to 100, is Subcentuple, and as the Reaſon of 11 to 3, is Triple Superbipartient thirds, ſo the Reaſon of 3 to 11, is Subtriple, and Superbipartients thirds, and ſo of others.

Of

Of the Denominators of Reasons Rational.

Forasmuch as the use of the Denominators of Reasons Rational before exposed, is not a little profitable, it will not be amisse here to shew by what numbers they are denominated. The Denominator of any Reason, is the number which doth openly and distinctly expresse the habitude of one quantity to another, as the Denominator of the Reason Octuple is 8, for that the number 8 doth shew that the greater quantity of the Reason Octuple conteineth the lesser 8 times. In like manner, the Denominator of the Reason Sesquiquinta is $\frac{1}{5}$; forasmuch as this number doth signifie that the greater quantity of the Reason Sesquiquinta, conteineth the lesser once, and the fifth part of the same lesser: the same ought to be understood of the Denominators of the other Reasons.

Wherefore (in my opinion) *EUCLIDE* in the Sixth Book, and sundry other Mathematicians, do call the Denominator of any Reason whatsoever the quantity thereof: for the Denominator (as we have said) shews how great one quantity is in respect of another, to wit, of hat with which it is compared, as appears by the Examples proposed.

Now from these things which we have said, may easily be gathered the Denominator of any Reason, for the Denominator of the Reason Multiplex whatsoever it be, is a whole number, which doth contein as many unites as the greater quantity doth contein times the lesser: As for Example, the Denominator of the Reason Double 2, of the Reason Octuple 8, the Centuple is 100, &c. but the Denominators of the Reasons Submultiplices, correspondent to the Multiplices, are aliquot parts, denominated of the Denominators of the Reasons Multiplices, to which the denominated do correspond. As the Denominator of the Reason Subdupla, is $\frac{1}{2}$, Subquintuple is $\frac{1}{5}$, Subcentuple is $\frac{1}{100}$, &c. and so we shall find the Denominators of the other Reasons Submultiplices. Therefore the Denominator of any Reason Submultiplex is a broken number, or a fraction of which the Numerator is alwayes an Unite: but the Denominator is a number denominating the Reason Multiplex corresponding, as appears by these Examples. Therefore it will not be difficult to find the Denominator of any Reason Multiplex, or Submultiplex, if what hath been said be well understood; forasmuch as the same prolation demonstrateth the Denominator of the Reason, as is manifest by the Examples proposed.

The Denominator of any Reason Super-particular is Unity with that aliquot part that the greater quantity ought to contein over and above the lesser: As the Denominator of the Reason Sesquialtera is 1 and $\frac{1}{2}$, Sesquioctava is 1 and $\frac{1}{8}$, Sesquicentesima is 1 and $\frac{1}{100}$, &c. and it will not be difficult to find the Denominator of any Reason Superparticular; seeing that the same prolation of Reason doth expresse the Denominator by its aliquot part, as is evident by the Examples proposed.

But the Denominators of the Reasons Subsuper-particulars correspondents, are fractions, whose Numerators are lesse than their Denominators by Unity only, each to his correspondent, as the Denominator of the Reason Subsesquialtera is $\frac{2}{3}$, Subsesquioctava is $\frac{8}{9}$, Subsesquicentesima $\frac{100}{101}$. Now the Denominator of any Reason Super-particular may be found, if for the Numerator of the fraction you take the Denominator of the aliquot part expressed in the reason, and for the Denominator of the same fraction, you take a number greater than Unity: As the Denominator of the

the Reason Subsesquidecimal is $\frac{10}{11}$; seeing that the Numerator of this fraction is the number denominating the tenth part, to wit 10, but the Denominator of the same fraction exceedeth the Numerator by Unity, the Denominator of the Reason Subsesquiseptimal is $\frac{7}{8}$, that of the reason subsesquiquintina is $\frac{5}{6}$, &c.

In like manner, we shall find the Denominator of any Reason Subsuperparticular, if we reduce the Denominator of the Reason Subsuperparticular, correspondent in a fraction, of the which fraction the Numerator shall exceed alwayes the Denominator by unity, the which is also of the aliquot part, whereof mention is made in the Reason proposed, seeing we transpose the terms of this fraction, in such sort as the Numerator may be made Denominator, and the Denominator Numerator; we shall have the Denominator of the reason Subsuperparticular proposed. As if the Reason Subsesquiseptimal be proposed; forasmuch as the Denominator of the Reason Sesquiseptimal which is correspondent thereto, is 1 and $\frac{1}{7}$, which is reduced into this fraction $\frac{8}{7}$, whose Numerator is greater than the Denominator by the aliquot part of Unity; wherefore if we transpose this fraction in this manner $\frac{7}{8}$, we shall say that the Denominator of the Reason Subsesquiseptimal is $\frac{7}{8}$, &c.

The Denominator of any Reason Superpartients is Unity with the aliquot parts, (not making an aliquot part) which the greatest quantity ought to contein over and above the lesser; As the Denominator of the Reason Supertripartient septimal is 1 and $\frac{1}{7}$, Supertripartient 20 is $1\frac{1}{2}$, &c. and is easie to be found, forasmuch as the prolation of the Reason doth shew the proper Denominator, as appeares by the Examples before mentioned.

But the Denominator of any Reason Super-partient, is a Fraction, whose Numerator is lesse than the Denominator by so many unites, as the greater quantity of the Reason proposed, doth contein aliquot parts, over and above the lesser, as the Denominator of the Reason Subsupertripartient septimal is $\frac{7}{8}$ of the Subsupertripartient 20 is $\frac{17}{18}$, &c. Now the Denominator of any Reason Subsupertripartient shall be found, if for the Denominator of the fraction, you take the Denominator of the aliquot parts conteined in the reason proposed, to which if you adde the number of the same parts, it will give you the Denominator of that fraction.

As the Denominator of the Reason Subsupertripartient 11, to wit 11, to which is added 4, the number of four aliquot parts, to the end that 15 be made the Denominator of the same fraction.

But the Denominator of the Reason Subsupertripartient fifths, is this fraction $\frac{5}{8}$, for the Numerator thereof is the number denominating the fifth parts, to wit 5: But the Denominator of the same fraction 8, is made of the Numerator 5, and of 3, the number of the three aliquot parts, and so of the other Denominators, which may be also found in this manner.

Let the Denominator of the Reason Subsuperpartient, correspondent in a fraction be reduced, whose Numerator may exceed the Denominator, which denominateth also the aliquot parts expressed, by so many unites as there are aliquot parts.

For the numbers of this fraction being transposed in such sort as the Numerator be made Denominator, you shall have the Denominator of the Reason proposed Subsuperpartient, as the Denominator of the Reason Subsupertripartient fifths is $\frac{5}{8}$, for that the Denominator of the

Reaſon Supertripartient fifths is $1\frac{3}{5}$, which is reduced into this fraction $\frac{8}{5}$, which makes $\frac{5}{8}$ in tranſpoſing the numbers, and the ſame ought to be done for finding the other numbers, &c.

The Denominator of any Reaſon Multiplex Superparticular, is a whole number denominating the Reaſon Muliplex propoſed, with the aliquot part that the greater quantity ought to contein, over and above the leſſer; as the Denominator of the Reaſon Tripleſeſquiſeptimal is $3\frac{1}{7}$, the Quintuple ſeſquinoncuple is $5\frac{1}{9}$, &c, and is very eaſie to expreſſe the Denominator of any Reaſon Multiplex Superparticular, foraſmuch as the prolation of the Reaſon ſheweth diſtinctly both the Denominator of the Reaſon Multiplex, and the aliquot part as may be ſeen by the examples propoſed. Now the Denominator of any Reaſon Submultiplex ſuperparticular, is a fraction, whereof the Numerator is a number denominating the aliquot parts in the Reaſon, as the Denominator of the Reaſon Subtriple ſeſquiſeptimal is $\frac{7}{13}$, the Subquintuple ſeſquinonal is $\frac{9}{44}$, &c.

Now this Denominator will be found, if for the Numerator of the fraction you take the Denominator of the aliquot part, which being multiplyed by the Denominator of the Reaſon Multiplex, if you adde unity to the product, you ſhall have the Denominator of the ſame fraction.

As for Example, the Denominator of the Reaſon ſubquadruple ſeſquiſexta is $\frac{6}{17}$; foraſmuch as the Numerator 6 doth denominate the ſixth parts, and the ſame multiplyed by 4 the Denominator of the Reaſon quadruple, and having added to the product 24 unites, to the end the Denominator of the ſame fraction may be made 25, &c. otherwiſe you ſhould find the ſame denominator, if you reduce into one fraction the Denominator of the Reaſon Multiplex ſuperparticular, correſponding as is before ſaid.

As if the Reaſon Subquadruple ſeſquiſexta were propoſed, foraſmuch as the Denominator of the Reaſon quadruple ſexta correſpondent is $4\frac{1}{6}$, which ſhall be reduced to $\frac{25}{6}$; therefore if you change the order of the terms, of the ſame fraction you will have $\frac{6}{25}$, for the Denominator of the Reaſon Subquadruple ſeſquiſexta, and ſo of the others.

The Denominators of any Reaſon Multiplex ſuperpartients, is a whole number denominating the Reaſon Multiplex expreſſed therein, with the aliquot parts which the greater quantity ought to contein over and above the leſſer, which ſaid aliquot parts do not conſtitute an aliquot part, as the Denominator of the Reaſon Triple Superquinpartient octa is $3\frac{1}{7}$, of the quadruple ſuperbipartient fifth is $4\frac{1}{3}$.

And to find the Denominators, there is no difficulty, ſeeing they are expreſſed diſtinctly and openly in each Reaſon propoſed Multiplex ſuperpartient, to wit, as well the Denominator of the Reaſon Multiplex conteined therein, as the aliquot parts, as is manifeſt by the Examples propoſed. But the Denominator of any Reaſon Submultiple ſuperpartient, is a fraction, of which the Numerator is a number denominating the aliquot parts of the ſame Reaſon.

As the Denominator of the Reaſon Subtriple ſuperquintapartient is $\frac{5}{14}$, ſubquadruple ſuperbipartient is $\frac{4}{11}$. Now the Denominator of thoſe Reaſons ſubmultiplices ſuperpartients will be found, if for the Numerator of the fraction you take the Denominator of the aliquot parts, which being multiplyed by the Denominator of the Reaſon Multiplex, and having added to the product the number of the aliquot parts, you will obtein the Denominator of the ſame fraction.

As the Denominator of the Reaſon ſubduple ſuperoctupartient is $\frac{11}{34}$, for that

that the Numerator of this fraction 13 doth denominate the thirteenth part which multiplyed by 2, the Denominator of the Reason Duple, and to the number produced 26 there be added the number of 8 parts, it will make 34, the Denominator of the same fraction.

Otherwise there will be found the same Denominator, reducing the Denominator of the Reason correspondent in a fraction, and transposing the terms as is before said.

As for Example, Let the Reason Subquintuple supertripartient tenths be proposed, to find the Denominator, you shall reduce into a fraction the Denominator of the reason correspondent quintuple supertripartient tenth, which is $5\frac{3}{10}$, and you shall have this fraction $\frac{53}{10}$; and in changing the terms, that is to say putting 10 instead of 53, and 53 instead of 10, you will have $\frac{10}{53}$ for the Denominator of the reason subquintuple supertripartient tenth, and so of the rest.

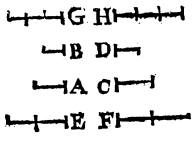
Lastly, the Denominator of the Reason of equality is alwayes unity; forasmuch as in this reason the quantities ought to be equal to one another, and therefore the one ought to contein the other once and no more, which is signified by unity.

5 *Magnitudes are said to have reason to one another, which being multiplyed may exceed one another.*

FOrasmuch as *EUCLIDE* in the third Definition, hath termed the habitude of two magnitudes of the same Kind, Reason, he teacheth now in this fifth Definition, what thing is required, to the end that two quantities of the same kind may be said to have Reason to one another; saying that Magnitudes are said to have Reason to one another, of the which (which you please) being multiplyed, may be augmented in such sort as that it may exceed the other, as the diameter and the side of one and the same square, are said to have Reason, although it be Irrational, which cannot be expressed by any number; forasmuch as the side multiplyed by 2 or taken twice, doth exceed the diameter; for seeing that the two sides of the square, with the diameter do constitute an Isosceles triangle, the two sides of the square shall be greater than the diameter thereof.

So in like manner, the circumference of a circle and the diameter thereof shall have Reason, (although it be not as yet known;) forasmuch as the diameter multiplyed by 4, or taken 4 times, doth exceed the circumference, seeing that every circumference of a circle (as is demonstrated by *Archimedes*) doth contein only three times the diameter of the said circle, and over and above a particle; a little lesse than the seventh part of the diameter.

In the same manner, divers curviline figures will have Reason with right lined figures as *Proclus* hath shewn on the First of *EUCLIDE*. Now as divers Interpreters do say that neither all Lines, nor all Plain Angles have Reason to one another, according to this Definition: for say they, a finite line hath no Reason to an infinite; seeing that in what manner soever it be multiplyed, it cannot exceed the infinite line, neither hath a right lined angle Reason to an angle of contingence, for the angle of contingence cannot exceed a right lined angle.



6 *Magnitudes* A, B, C, *and* D, *are said to be in the same Reason, the first* A *to the second* B, *and the third* C *to the fourth* D, *when the equimultiplices* E *and* F, *of the first* A, *and the third* C, *to the equimultiplices* G *and* H *of the second* B, *and the fourth* D, *by whatever multiplication, are either deficient together, or are equal together, or do exceed together each to other, if those be taken which do answer one another.*

EUCLIDE shews in this place what conditions are required in Magnitudes, that they may be said to have the same Reason, which to expose, he is constrained to serve himself with their equimultiplices, to the end he may comprehend all the Reasons of Magnitudes, as well Rational as Irrational.

Now if you compare the one to the other, the magnitudes which have been taken equimultiplices, to wit, those which answer to one another; as the Multiplex of the first and the Multiplex of the second, which are E and G, also the Multiplex of the third, and the Multiplex of the fourth, to one another, to wit, F and H, and if it be always found that they have such relation to one another, if E the Multiplex of the first Magnitude A, be lesse than G the Multiplex of the second Magnitude B, that F the Multiplex of the third Magnitude C, be also lesse than H the Multiplex of the fourth C: or if E be equal to G, also F is equal to H; or lastly, if E be greater than G, also F is greater than H, which is to be deficient together the one and the other, of the one and the other, or to be equal together, or to exceed together, in such sort as the contrary cannot be found in any kind of Multiplices; that is to say, that E be never lesse than G, but F also be lesse than H, and that E be never equal to G, if F also be not equal to H; and lastly, that E be never greater than G, if that F also be not greater than H.

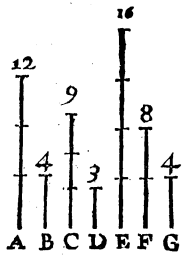
If therefore it be found that the equimultiplex taken in what manner soever, have relation perpetually to one another, as aforesaid, it will be said that there is the same Reason of the first Magnitude A, to the second B, as of the third magnitude C, to the fourth D; as if it were sometimes found, even in one only kind of Multiplices, that the Multiplex E wanteth of the Multiplex of G; but the Multiplex of F wanteth not of H, or that E is equal to G, but F is not equal to H: Or lastly, E doth exceed G, but F exceedeth not H, although in infinite other sorts of Multiplices, the above said condition be found in some sort, the Magnitudes shall not be said to have the same Reason to one another, but divers, as shall be made manifest by the Eighth Definition.

Contrarily, if there be the same Reason of the first to the second, as of the third to the fourth, it will follow that the equimultiplices of the first and

and the third, shall be both equal to the Multiplices of the second and the fourth, or both lesse, or both greater in any Multiplication whatsoever.

And if there were not the same Reason of the first to the second, as of the third to the fourth, also the equimultiplices of the first and the third exceed not together the equimultiplices of the second and the fourth, or shall not be equal, or lesse, &c.

Now here is to be noted, that this Definition ought also to be taken of three Magnitudes, provided that the second be taken twice, to the end you may have 4, &c.



7 *Magnitudes which have the same Reason, are called proportionals.*

AS if of the Magnitudes A, B, C, and D, there is the same reason of A to B, as of C to D, those Magnitudes A, B, C, and D, shall be termed proportional. In like manner, if there be the same reason of E to F, as of F to G, the Magnitudes E, F, and G, shall be termed proportional. Now there be some Magnitudes continually proportional, between which the proportion continued is found, as are the Magnitudes E, F, and G; but some are not continually proportional, but seperately, as are the Magnitudes A, B, C, and D, for in these there is an interruption of Reasons, but in the others not, as is said in the fourth Definition.

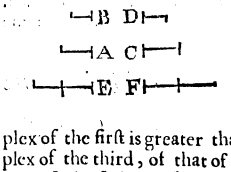
8 *But when of the equimultiplices the multiplex of the first magnitude shall exceed that of the second. But the multiplex of the third magnitude shall not exceed that of the fourth, then the first magnitude shall be said to have greater Reason to the second, than the third to the fourth.*

EUCLIDE declareth in this Definition, what condition four magnitudes ought to have, to the end that the first may be said to have greater reason to the second, than the third to the fourth; saying that if the equimultiplices of the first and the third be taken, and likewise other equimultiplices of the second and the fourth, and that there be sometimes found (but not always,) that the Multiplex of the first Magnitude be greater than the Multiplex of the second: but the Multiplex of the third is not greater than the Multiplex of the fourth, but lesse or equal, it will be said that there is greater reason of the first Magnitude to the second, then of the third to the fourth, as is manifest by the following figure proposed, in which are taken the double of the first Magnitude A, and of the third C, to wit, E and F; but of the second B and the fourth D, hath been taken the triples G and H. And forasmuch as E the Multiplex of the first, is greater than G the Multiplex of the second. But F

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the Multiplex of the third, is not greater than H, the Multiplex of the fourth, but leſſe, it may be ſaid that there will be greater Reaſon of A the firſt Magnitude, to B the ſecond Magnitude, then of C the third, to D the fourth.

Now it is not neceſſary (to the end that of four Magnitudes, the firſt be ſaid to have greater Reaſon to the ſecond, than the third to the fourth,) that the equimultiplices according to any Multiplication whatſoever, have ſuch habitude, as the Multiplex of the firſt exceed the Multiplex of the ſecond; But that the Multiplex of the third exceed not the Multiplex of the fourth. But it ſufficeth that according to any Multiplication whaſoever, they have ſuch an habitude, for it may happen ſometimes that as well the Multiplex of the firſt is greater than the Multiplex of the ſecond as the Multiplex of the third, of that of the fourth: In like manner, as the Multiplex of the firſt is leſſe than the Multiplex of the ſecond, that that of the third is alſo leſſe than that of the fourth: Nevertheleſſe, ſeeing that, that happeneth not in every Multiplication, but that ſometimes the Multiplex of the firſt exceeds that of the ſecond, but the Multiplex of the third is leſſe than, or equal to the Multiplex of the fourth, for this cauſe the firſt Magnitude ſhall be ſaid to have greater Reaſon to the ſecond, than the third to the fourth, and not the ſame as is ſeen in the Example propoſed, where the third hath greater Reaſon to the ſecond, than the fourth hath to the third.



15, 9, 12, 3, 2, 10, 18, 14.
20, 12, 16, 4, 3, 15, 12, 21.

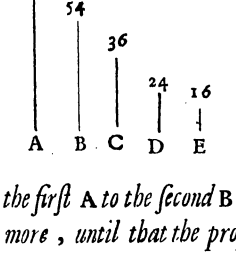
Therefore to the end that four Magnitudes may be ſaid to be proportional, it is neceſſary that the equimultiplices of them taken, according to any Multiplication whatſoever, be deficient together, equal together, or do exceed together, as hath been expoſed in the Sixth Definition.

But to the end that the firſt may be ſaid to have greater Reaſon to the ſecond, than the third to the fourth, it ſufficeth that according to ſome Multiplication, the Multiplex of the firſt exceed the Multiplex of the ſecond; but that of the third exceedeth not that of the fourth, although that according to infinite other Multiplications, the equimultiplices of the firſt and of the third, exceed together the equimultiplices of the ſecond, and of the fourth.

But if contrarily, the Multiplex of the firſt Magnitude wanteth of the Multiplex of the ſecond, the Multiplex of the third wanteth not of the Multiplex of the fourth, the firſt Magnitude ſhall be ſaid to have leſſer Reaſon to the ſecond, than the third to the fourth; although that according to divers other Multiplications, the equimultiplices of the firſt and of the third do want together, of the equimultiplices of the ſecond and of the fourth, as may be ſeen in the ſame numbers of the Example propoſed, the Reaſon of 2 to 3 ſhall be leſſe than that of 3 to 4, &c.

9 *Proportion cannot be conſtituted in leſſe than three terms.*

FOraſmuch as all Analogie which is termed Proportion, is a ſimilitude or likeneſſe of Reaſons, and ſeeing that in all Reaſons there is the term antecedent, and the term conſequent, it is neceſſary, that in all proportion there may be at leaſt two terms antecedents, and two conſequents: Wherefore if the proportion be not continued, there will be required at leaſt four terms or Magnitudes: But if the Proportion be continued, there will be at leaſt three terms; foraſmuch as the middle term is taken twice; ſeeing that it is the conſequent of one Reaſon, and the antecedent the other, and this here is the leaſt number of the terms of proportion; for in any two terms is only found Reaſon, and not Proportion.



10 *When there are three magnitudes A, B, C, proportional, the firſt A to the third C, is ſaid to have dupliciate reaſon of the firſt A to the ſecond B: But when four magnitudes are proportional, the firſt A to the fourth D, is ſaid to have triplicate reaſon of the firſt A to the ſecond B, and alwayes after the ſame order, one more, until that the proportion be accompliſhed.*

AS if the Magnitudes A, B, C, D, and E, are continually proportional' in ſuch ſort, as there may be the ſame reaſon of A to B, as of B to C, and of C to D, and of D to E; the reaſon of the firſt Magnitude A, to the third Magnitude C, ſhall be termed the duplicate of the reaſon that the firſt Magnitude A hath to the ſecond Magnitude B: Foraſmuch as between A and C, there are two reaſons, which are equal to the reaſon of A to B; to wit, the reaſon of A to B, and of B and to C, in ſuch ſort as the reaſon of A to B is doubled, that is to ſay, placed twice in order. But the reaſon of the firſt A to the fourth D, is ſaid to be triplicate of the reaſon that the firſt Magnitude A hath to the ſecond Magnitude B; foraſmuch as between A and D are three reaſons, which are equal to the reaſon of A to B; to wit, the reaſon of A to B, of B to C, and of C to D, inſomuch as the reaſon of A to D encloſeth or comprehendeth in ſome ſort the reaſon of A to B triplicated, that is to ſay, put three times in order. So in like manner the reaſon of A to E, ſhall be ſaid to be quadruplicate of the reaſon of A to B; foraſmuch as four reaſons are found between H and F, which are equal to the reaſon of A to B, &c.

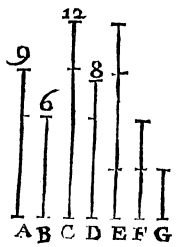
And if contrarily, there be the ſame reaſon of E to D, as of D to C, of C to B, as of B to A, the reaſon of E to C ſhall be termed the duplicate of the reaſon of E to D: But the reaſon of E to B ſhall be ſaid to be the triplicate of the reaſon of E to D: So in like manner, the reaſon of E to A ſhall be ſaid to be quadrupled of the reaſon of E to D.

11 *Magnitudes are ſaid to be Homologal, or of like reaſon, the antecedents to the antecedents, and the conſequents to the conſequents.*

He

HE hath defined heretofore that proportion is a ſimilitude of reaſons, and now he teacheth that not only in ſome proportion the Reaſons are ſaid to be alike, but alſo the terms themſelves or quantities, are ſaid to be alike or homologal, ſaying that the Magnitudes antecedents are termed homologal, or alike to one another in reaſon, and the ſame of the conſequents to one another, to the end that it may be underſtood in ſeveral Demonſtrations what ſides of figures compared to one another, ought to be the antecedents of the reaſons, and which the conſequents, as ſhall be declared in the Sixth Book.

If there be then the ſame reaſon of A to B, as of C to D, the quantity A ſhall be ſaid to be alike to the quantity C and B alike to D, for becauſe of the ſimilitude of Reaſons, it is neceſſary that both the one and the other Magnitude antecedent be equal to one and the other conſequent, or in the ſame ſort, greater or leſſe, otherwiſe both the one and the other conſequent, or in the ſame ſort greater or leſſe, otherwiſe both the one and the other antecedent ſhould not have the ſame reaſon to the one and the other conſequent: Behold the Example.



In the Magnitudes propoſed, to the which the antecedents are in the ſame ſort greater than the conſequents, to wit, by the halfe: See another Example.

In the Magnitudes continually proportional E, F, and G, where as well E and F are homologal as F and G, as is manifeſt, and for this cauſe *EUCLIDE* in the ſixth and eighth Definition, enjoynes to take the equi-multiplices of the firſt and third Magnitude, that is is to ſay the antecedents.

In like manner, to take the other Multiplices of the ſecond and the fourth Magnitudes, to wit, the conſequents, for they are alike to the Magnitudes proportioual, as is manifeſt by this Definition, but to the Magnitudes that are not proportional, they are unlike.

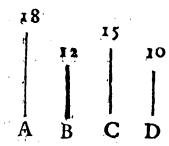
12 *Reaſon alternate, is to take the antecedent compared to the antecedent, and the conſequent to the conſequent.*

HE here unfoldeth ſome certaine kinds of arguing in proportions, whoſe uſe is very frequent amongſt Geometricians, which kinds of arguings are ſix in number; The firſt is termed alternate reaſon or reaſon by permutation, the ſecond is termed inverſe, or contrary reaſon, the third compoſition of reaſon, or conjunct proportion, the fourth diviſion of reaſon, or diſjunct or ſeparated proportion, the fifth converſion of reaſon, or reverſed proportion: Laſtly, the ſixth is termed reaſon of equality, or equal proportion.

Therefore, (ſaith he,) alternate reaſon, or reaſon by permutation, is when four proportional Magnitudes being propoſed, it be inferred that there is the ſame reaſon of the antecedent of the firſt reaſon, to the antecedent of the laſt, as of the conſequent of the firſt, to the conſequent of the laſt; as if we propoſe the ſame reaſon of A to B, as of C to D; and

and therefore conclude that there is the same Reason of A to C as of B to D; we are said to argue according to Permutation of Reason.

The Greek Authours do use in this kinde of arguing; neer this manner of expression; to wit, as A is to B, so C is to D: Therefore alternately, or contrarily, A shall be to C as B to D. Now we shall demonstrate the truth of this Illation at the sixteenth proposition of this Book. And in this kinde of arguing, it is necessary that all the four Magnitudes be of one and the same kind, to the end they may have reason to one another, being taken two and two at pleasure, for it is no arguing to say as the line A is to the line B, so the number C is to the number D; then alternately, as the line A is to the number C, so the line B is to the number D, considering that there is no proportion of a line to a number, and contrarily, as is manifest by the fifth definition.



But as to the other following kindes of arguing, the first Magnitudes may be of one kinde of quantity and the last Magnitudes of another, as shall be made manifest by the demonstrations of this fifth Book.

13 *Inverse or transposed Reason, is to take the consequent as antecedent, to compare it to the antecedent, as if it were the consequent.*

AS if (seeing that A is to B as C is to D) we should inferre that B is to A as D is to C (see the preceedent figure) we shall be said to argue according to reason Inverse, in this kinde of arguing Authours expresse themselves after this manner, to wit, as A is to B, so C is to D then inversely B shall be to A as D to C, which manner of arguing shall be demonstrated to be true by the Corollarie of the eighth Proposition of this Book.

Lastly, the two first Magnitudes, may be of one kinde and the latter of another, for it may be very well inferred, as the line A is to the line B, so the Triangle or the number C, is to the Triangle or number D, then inversely, as the line B is to the line A, so the Triangle or the number D, is to the Triangle or number C, as shall be made evident by the Corollary of the fourth Proposition of this Book.

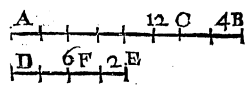
14 *Composition of reason is when the antecedent with the consequent are taken together as one to be compared to the same consequent.*

AS if there were the same reason of A C to C B, as of D F, to F E; and from that may be gathered, that there is the same reason, of the whole A B, to wit, of the antecedent with the consequent, to C B the consequent; as of the whole D E (to wit, the antecedent with the consequent,) to F E the consequent, this sort of arguing shall be called Composition of reason for as much as, of the antecedent and of the consequent there is compounded another new antecedent.

Now the Greek Authours use this manner of expression in this kind of arguing, to wit, as A C is to C B, so D F is to F E: then in compoundings

as AB ſhall be to CB, ſo DE to FE, this manner of arguing ſhall be demonſtrated in the eighteenth Propoſition of this Book.

To this, there may yet be added two other means of arguing, the firſt may be termed compoſition of Reaſon converſe, to wit, when the antecedent and the conſequent are taken as one, to be compared to the antecedent, as if (forasmuch as AC is to CB, ſo DF to FE) we inferre then as AB compounded of the antecedent, and of the conſequent, is to the antecedent AC; ſo DE compounded of the antecedent, and of the conſequent, is to the antecedent DF, which manner of arguing we ſhall ſhew to be of worth by the eighteenth Propoſition of this Book, in which we ſhall uſe this manner of expreſſion, when we ſhall ſpeak of compoſition of converſe reaſon.



The other manner of arguing, may be termed compoſition of contrary reaſon, which is done when you compare one and the ſame Magnitude the antecedent, to the antecedent, and to the conſequent, as to one only; as if AC be to CB, ſo DF is to FE, and we inferre by compoſition of contrary reaſon: Therefore as AC antecedent, is to the whole AB, compounded of the antecedent, and of the conſequent; ſo DF antecedent ſhall be to DE, compounded of the antecedent, and of the conſequent; and we ſhall alſo demonſtrate this kind of arguing to be of worth in the eighteenth Propoſition of this Book.

15 *Diviſion of Reaſon is when you take the exceſſe by which the antecedent ſurmounts the conſequent, to compare it to the ſame conſequent.*

AS if you ſhall ſay, that there is the ſame reaſon of the whole AB to to CB, as of the whole DE to FE (ſee the precedent figure): Therefore there will be the ſame reaſon of AC, (the exceſſe by which the antecedent ſurmounts the conſequent,) to CB conſequent, as of DF, (the exceſſe, by which the antecedent ſurmounts the conſequent,) to FE conſequent.

Now in this Diviſion of Reaſon Authors ſpeak thus: [Therefore in dividing, &c.] But this Illation ſhall be demonſtrated in the ſeventeenth Propoſition of this Book.

Now to this manner of arguing there may be added two others, the firſt of which may be called Diviſion of Converſe Reaſon, to wit, when the conſeqnent is compared to the exceſſe, by which the antecedent ſurmounts the conſequent, as if AB be to CB as DE is to FE, and we conclude by Diviſion of Reaſon converſe; Therefore as CB conſequent ſhall be to AC (the exceſſe by which the antecedent ſurmounts the conſequent,) ſo FE conſequent ſhall be to DF the exceſſe by which the antecedent ſurmounts the conſequent; but we ſhall demonſtrate the certainty of this manner of arguing, in the ſeventeenth Propoſition of this Book.

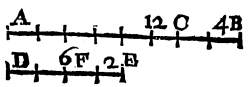
Now it is manifeſt that to the end that both the one and the other of thoſe arguings may take place, (to wit, this, and that of *EUCLIDE*) that the antecedent ought to be greater than the conſequent, otherwiſe the Definition cannot be made. The

The other manner of arguing may be called Diviſion of Contrary Reaſon, which is made when the antecedent is compared with the exceſſe by which the conſequent ſurmounts the antecedent, as when we ſay, as AC is to AB, ſo DF is to DE: Then by Diviſion of Contrary Reaſon AC the antecedent ſhall be in like manner to CB (the exceſſe by which the conſequent ſurmounts the antecedent,) as DF the antecedent is to FE the exceſſe by the which the conſequent ſurmounts the antecedent; which manner of arguing we ſhall alſo demonſtrate in the ſeventeenth Propoſition of this Book.

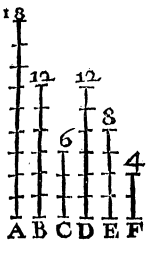
Laſtly, it is manifeſt, that in this diviſion of contrary Reaſon, the conſequent ought to be greater than the antecedent, to the end the exceſſe may be taken, by which the conſequent ſurmounts the antecedent.

16 *Converſion of Reaſon, is to take the antecedent to compare it to the exceſſe, by which the antecedent ſurmounts the ſame conſequent.*

AS if we ſay after this manner, as the whole Magnitude AB is to CB, ſo the whole DE is to FE; therefore alſo the ſame AB ſhall be to AC, the exceſſe by which the antecedent ſurpaſſeth the conſequent, as DE to DF; we ſhall be ſaid to argue according to converſion of Reaſon: Therefore Authors do expreſſe themſelves almoſt after this manner: [Therefore by Converſion of Reaſon, &c.] But this manner of arguing ſhall be confirmed in the Corollary of the nineteenth Propoſition of this Book. See the precedent figure.



¶ It is manifeſt alſo than in this manner of arguing by converſion of Reaſon, the antecedent ought to exceed the conſequent, to the end the exceſſe by which the antecedent ſurmounts the conſequent may be taken.

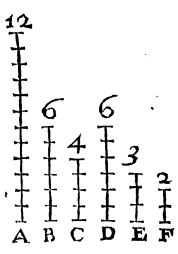


17 *Equal Reaſon, or Reaſon of equality, is when there be divers magnitudes, and other magnitudes equal to them in multitude, which may be taken two and two, and in the ſame reaſon: When as in the firſt magnitudes, the firſt is to the laſt of the ſame magnitudes; ſo in the ſecond magnitudes, the firſt is to the laſt of the ſame. Otherwiſe, it is to take the extreams by the ſubtraction of the meanes.*

LEt A B C be ſeveral Magnitudes of one part, and D, E, and F, as many Magnitudes of the other part, which being taken two and two, in the ſame Reaſon; that is to ſay, as A is to B, ſo D is to E, and as B is to C, ſo E is to F, if therefore for this cauſe it be inferred, that there is the ſame Reaſon of A the firſt of the firſt Magnitudes, to C the laſt of the ſame Magnitudes, as of D the firſt of the ſecond Magnitudes, to F the laſt of the ſame Magnitudes, this manner of arguing ſhall be called equal Reaſon or Reaſon of equality, to wit, in which the extream Magnitudes being ſubſtracted from the meanes, are ſaid to have one and the ſame Reaſon to one another, as appears by the other Definitions.

But forasmuch as it is permitted to argue in proportions by two ſeveral manners, by the equality; to wit, in the one, when we take the Magnitudes two and two in the ſame Reaſon, in proceeding by order, and in the other, when the order is changed. *EUCLIDE* explaines in the two following Definitions, what is meant by Ordinate Proportion, and Perturbated or troubled Proportion.

18 *Ordinate proportion is, when as the antecedent is to the conſequent, as the antecedent is to the conſequent; and as the conſequent is to ſome other, ſo the conſequent is alſo to ſome other.*



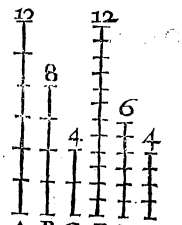
AS if A be to B, as D is to E; and again, as B the conſequent is to ſome other, to wit, to C, ſo E the conſequent is to ſome other, as to F; ſuch manner of arguing ſhall be termed Ordinate: and forasmuch as the ſame order is obſerved, as well in the firſt Magnitudes, as in the ſecond; ſeeing that on both parts the firſt is firſt of all compared with the ſecond, and after that the ſecond with the third. Therefore when in the manner of arguing by equality, the Ordinate Proportion is obſerved and kept, it is demonſtrated in the two and twentieth Propoſition of this Book, that this manner of arguing is good.

19 *Perturbated or troubled Proportion is, when as three magnitudes are propoſed on one part, and other magnitudes, equal unto them in multitude, and as in the firſt magnitudes, the antecedent is to the conſequent; ſo in the ſecond magnitudes the antecedent is to the conſequent: But as in the firſt magnitudes, the conſequent is to ſome other, ſo in the ſecond magnitudes, ſome other is to the antecedent.*

That

THat is to ſay, when three Magnitudes are propoſed on the one part and as many on the other, the firſt of the firſt is to the ſecond of the ſame; as the ſecond of the ſeconds, is to the third of the ſame; and as the ſecond of the firſt is to the third of the ſame; ſo the firſt of the ſeconds is to the ſeconds of the ſame.

As if there be the ſame Reaſon of A to B, as of E to F; then afterward in the firſt Magnitudes, as B conſequent, is to ſome other, to wit C; ſo in the ſecond Magnitudes, ſome other, to wit D, is to E the antecedent. And this ſort of arguing ſhall be termed diſturbed Proportion, forasmuch as the ſame order in the proportions of the Magnitudes is not obſerved; ſeeing that in the firſt magnitudes, the firſt is compared with the ſecond: but in the ſecond Magnitudes, the ſecond is compared with the third: And again, in the firſt Magnitudes, the ſecond is compared to the third, but in the ſecond Magnitudes, the firſt is compared with the ſecond. Therefore, when in the manner of arguing by equality, the proportion diſturbed is kept and obſerved, it is demonſtrated that this manner of arguing is good, as in the three and twentieth Propoſition of this Book.



Laſtly, as well the diſturbed Proportion, as the Ordinate, inferre alwayes by equality, that there is the ſame proportion of the extreams, although more than three Magnitudes be propoſed, as ſhall be made manifeſt by the two and twentieth and three and twentieth Propoſitions of this Book.

PRO-

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

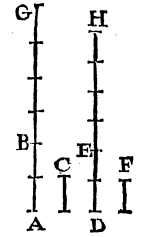


If there be a number of magnitudes AB *and* CD, *as many as you pleaſe, equal in multitude to other magnitudes* E *and* F, *and equimultiplices to them, each to his correſpondent, as the one of the magnitudes* AB, *ſhall be multiplex of one* E, *ſo the whole* AB *and* CD, *ſhall be Multiplex of the whole* E *and* F.

Demonſtration FOr ſeeing that AB and CD are equimultiplices of E and F, if AB be divided according to the Magnitudes AG, GH, and HB, equal each of them to E, and CD in like manner according to CI, IK, and KD, equal each to F. Now each of them may be divided into parts totally equal to E and F; ſeeing that AB and CD are equimultiplices of E and F; and therefore E ſhall be conteined exactly as many times in AB, as F in CD, as appears by the ſecond Definition of this Book; the Magnitudes AG, GH, and HB, ſhall be as many in number as CI, IK, and KD. But foraſmuch as AG and E, are equal to one another, if to them be added the equal Magnitudes CI and F, AG and CI together, they ſhall be equal to E and F together. In like manner, GH and IK together ſhall be equal to E and F together; and alſo HB and KD to the ſame E and F; therefore as many times as E is contained in AB, or F in CD, ſo many times E and F together ſhall be conteined in AB and CD together; therefore as AB is Multiplex of E, ſo AB and CD together, ſhall be Multiplex of E and F together, by the ſecond Definition of this Book: Therefore, If there be, &c. Which was to be demonſtrated.

PROP.

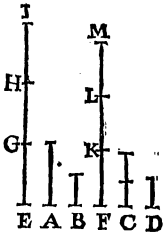
PROP. 2. THEOR. 2.



If the firſt AB, *be as much multiplex of the ſecond* C, *as the third* DE *is of the fourth* F, *and that the fifth* BG *be alſo as much Multiplex of the ſecond* C, *as the ſixth* EH *is of the fourth* F. *The magnitude* AG, *compounded of the firſt* AB, *and of the fifth* BG, *ſhall be as much multiplex of the ſecond* C, *as the magnitude* DH, *compounded of the third* DE, *and of the ſixth* EH, *is of the fourth* F.

Demonſtration FOr ſeeing that AB and DE are equimultiplices of C, and of F, there will be in AB as many Magnitudes equal to C, as there are in DE, equal to F: In like manner, there will be in BG as many Magnitudes equal to C, as there are in EH, equal to F. If therefore to the equal Multitudes AB and DE, the equal Multitudes BG and EH, all the Multitudes AG and DH ſhall be equal in number: Therefore C ſhall be conteined as many times in AG as F is conteined in DH: Therefore AG (compounded of the firſt and fifth,) ſhall be as much Multiplex of C the ſecond, as DH (compounded of the third and the ſixth,) is of F the fourth: Therefore, If the firſt, &c. Which was to be demonſtrated.

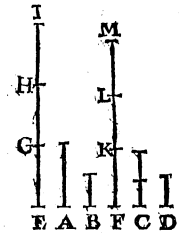
PROP. 3. THEOR. 3.



If the firſt A, *be as much Multiplex of the ſecond* B, *as the third* C, *is of the fourth* D, *and there be taken the equimultiplices* E *and* F, *of the firſt* A, *and of the third* C, *in equal reaſon, the magnitude* E, *taken multiplex of the firſt* A, *ſhall be alſo ſo much multiplex of the ſecond as the magnitude taken* F, *multiplex of the third* C, *ſhall be of the fourth* D.

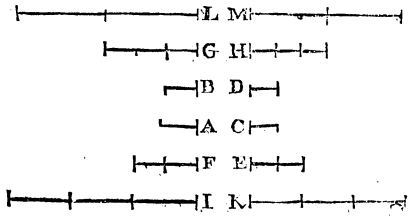
Demonſtration FOr ſeeing that E and F are equimultiplices of A and C, if E be divided into Magnitudes equal to A, as into EG, GH, and HI, alſo F into Magnitudes, equal to C, as into FK, KL, and

and L M; there will be in E as many parts equal to A, as there are in F, equal to D; forasmuch therefore as E G is equal to A, and F K equal to C: But A and C are equimultiplices of B and D, by supposition; E G and F K shall be equimultiples of the same B and D: In like manner, G H, and K L, and H I and L M shall be equimultiplices of the same B and D: forasmuch therefore as E G the first magnitude, is so much Multiplex of the second B, as F K the third is Multiplex of the fourth D, also G H the fifth, is as much Multiplex of B the second, as K L the sixth is Multiplex of the fourth D; also G H the fifth is as much Multiplex of B the second, as K L the sixth is Multiplex of the fourth D; E H compounded of the first and of the fifth, shall be as much Multiplex of the second B, as F L compounded of the third, and the sixth is Multiplex of the fourth D. Again, forasmuch as E H the first is as much Multiplex of B the second, as F L the third, is of D the fourth, as we even now demonstrated: But H I the fifth is so much Multiplex of the second B, as L M the sixth is of D, the fourth E I, compounded of the first and fifth, shall be as much Multiplex of the second B, as F M compounded of the third, and of the sixth, is of D the fourth, and so of the others, if there be more parts in E and F: Therefore, If, &c. Which was to be demonstrated.



PROP. 4. THEOR. 4.

If the first A, *hath the same reason to the second* B, *as the third* C, *to the fourth* D, *also the equimultiplices* E *and* F, *of the first* A, *and of the third* C, *shall have the same reason to the equimultiplices* G *and* H, *of the second* B, *and of the fourth* D, *according to any Multiplication, if they be taken so as that they answer to one another.*



Demonstration FOr if you constitute E the first Magnitude, and G the second, F the third, and H the fourth, and that I and K any what ever equimultiplices of E and F, the first and third also L and M, some equimultiplices of G the second, and H the fourth; forasmuch as

as E the first is as much Multiplex of A the second, as F the third, is of G the fourth, and that I and K be taken equimultiplices of E and F, the first and third, in equal reason, [a] I and K shall be also equimultiplices of A and C, the second and fourth. By the same reason L and M are equimultiplices of B and D; and forasmuch as A is to B, as C to D, and that I and K are taken equimultiplices of A and C, the first and third, [b] if I the Multiplex of the first A, wanteth of L the Multiplex of the second B; also K the Multiplex of the third C, shall want of M the Multiplex of the fourth D: if I be equal to L, also K shall be equal to M: and lastly, if I exceed L, also K shall exceed M, according to any whatsoever Multiplication they be taken. Therefore, seeing that I and K are equimultiplices of E the first, and F the third, also L and M equimultiplices of G the second, and H the fourth, and that it hath been demonstrated that if I the Multiplex of E the first, be lesse than L the Multiplex of G the second, that K the Multiplex of F the third, is lesse than M the Multiplex of H the fourth, and if I be equal to L, K is equal to M, and if I be greater than L, K is greater than M, and that this doth happen in every Multiplication, there [c] shall be the same reason of E the first, to G the second, as of F the third, to A the fourth; Therefore if the first hath the same reason to the second, as the third to the fourth, &c. Which was to be demonstrated.

a) 3.

b) 6. def.

c) 6. def.

COROLLARIE.

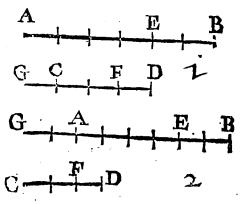
By this Demonstration the proof of inverse reason explained by EUCLIDE, *is manifest;* [d] *which is if four Magnitudes are proportional, they shall be the same taken backwards, or contrarily, to wit, if* A *be to* B, *as* C *to* D, *in inverse reason as* B *shall be to* A, *so* C *to* D; *for seeing that as* A *is to* B, *so* C *is to* D; [e] E *and* F *equimultiplices of* A *and* C *first and third, shall be both greater, equal, or lesse than* G *and* H, *equimultiplices of* B *and* D, *the second and fourth, in any Multiplication whatever: Therefore contrarily,* G *and* H *shall be both lesse, equal, or greater than* E *and* F. *Therefore* [f] *there will be the same reason of* B *to* A, *as of* D *to* C.

d) 13. def.

e) 6. def.

f) 6. def.

PROP. 5. THEOR. 5.



If a magnitude AB *be as much Multiplex of a magnitude* CD, *as the part cut off* AE, *is of the part cut off* CF; *also the remainder* EB *shall be as much Multiplex of the remainder* FD, *as the whole* AB, *is of the whole* CD.

Demonstration ASsume CG, of which EB may be so much Multiplex, as AE is of CF, or the whole AB of the whole CD: forasmuch as AE and EB are equimultiplices of CF and GC, the [a] whole AB, shall be so much Multiplex of the whole GF, as AE is of CF; that is to say, the whole of the whole, as one only Magnitude, of one only Magnitude. But AB is proposed so much Multiplex of CD,

a) 1. 5.

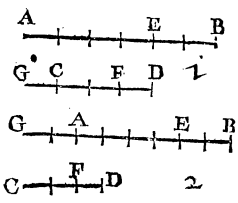
as AE of CF, therefore AB shall be as much Multiplex of GF, as of CD, therefore b GF and GD are equal; therefore taking away the common CF, there remains DF, equal to GC; therefore EB shall be as much Multiplex of FD, as of CG, but EB is as much Multiplex of CG, by supposition, as AE is of CF; that is to say, as the whole AB of the whole GD: Therefore the remainder EB shall be as much Multiplex of the remainder FD, as the whole AB, of the whole CD. Which was proposed.

b) 6. c. f.

Otherwise, Let AB be as much Multiplex of the whole CD, as the part cut off AE, of the part cut off CF. I say, that the rest EB, is as much Multiplex of the rest FD, as the whole AB, of the whole CD.

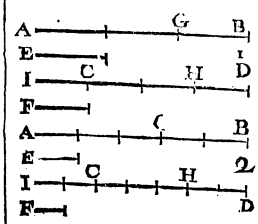
For having taken GA as much Multiplex of FD, as AE is of CF, or the whole AB of the whole CD: Forasmuch as AE and GA are equimultiplices of GF and FD, c the whole GE shall be as much Multiplex of the whole CD, as AE of CF; But AB is in like manner, also as much Multiplex of CD, as AE of CF, by supposition: Therefore GE and AB are equimultiplices of CD; d and therefore equal to one another, taking away then the common AE, there will remain GA and EB equal; and therefore equimultiplices of FD; seeing that GA is taken as much Multiplex of FD as AE of CF, or the whole AB of the whole CD, therefore the remainder EB shall be as much Multiplex of the remainder FD, as the whole AB of the whole CD, which was proposed: Therefore, If a Magnitude, &c. Which was to be demonstrated.

c) 1. 5.



d) 6. c. f.

PROP. 6. THEOR. 6.



If two magnitudes A B *and* C D, *are equimultiplices of two other magnitudes* E *and* F, *and there be cut off from them some equimultiplices* A G *and* C H, *of the same magnitudes* E *and* F, *either the remainders* G B *and* H D, *shall be equal to the same* E *and* F, *or equimultiplices of the same* E *and* F.

Demonstration. FOr in the first place, Let GB be equal to E: I say, that HD shall be equal to F: Let CI be put equal to F; forasmuch as AG the first, is as much Multiplex of E the second, as CH the third, is of F the fourth, & that GB the fifth, is equal to E, and IC the sixth, equal to F the fourth, a AB compounded of the first and of the fifth, shall be as much Multiplex of

a) 2. 5.

of E the second, as IH compounded of the third and sixth, is of F the fourth. But CD is as much Multiplex of F by Supposition, as AB is of E: therefore HI and CD are equimultiplices of F. Therefore b equal to one another, therefore taking away the common part CH, there will remain CI and HD equal: Now seeing that CI is put equal to F, HD shall be likewise equal to F, which was proposed. In like manner we shall demonstrate, that if GB be Multiplex of E, that HD shall be as much Multiplex of F, in putting IC as much Multiplex of F, as GB is of E, &c.

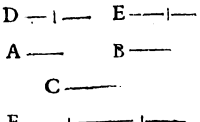
b) 6. c. f.

Otherwise, forasmuch as AB and CD are equimultiplices of E and F, there will be in AB as many Magnitudes equal to E, as there are in CD equal to F. Again, forasmuch as AG and CH are equimultiplices of the same E and F, c there will be in like manner in AG as many magnitudes equal to E, as there are in CH equal to F.

c) 1, 2. def.

If therefore from the equimultiplices AB and CD, you take away the equimultiplices AG and CH, there will remaine the equimultiplices GB and HD equimultiplices. Therefore GB shall contein as many times E, as HD shall contein F, if once only, GB and HD shall be equal to E F, if more than once, they shall be equimultiplices. Which was proposed.

PROP. 7. THEOR. 7.



Equal magnitudes A *and* B, *have the same reason to one and the same magnitude* C, *and one and the same magnitude* C *hath the same reason to the equal magnitudes* A *and* B.

Demonstration. FOr assume D and E the equimultiplices of the equal Magnitudes A and B, a the same D and E shall be equal to one another. Again, Let F be any Multiplex of C; forasmuch then as D and E are equal, and equimultiplices of A and B, the first and third, both the one and the other shall be lesse than F, or equal, or greater, after what Multiplication soever they be taken.

a) 6. c. f.

Therefore seeing that D and E equimultiplices of the first A, and of the third B, are both greater than F, the Multiplex of the second and of the fourth, (for C is put as for two Magnitudes) or equal, or lesse; b there will be the same reason of A the first, to C the second, as of B the third, to D the fourth.

b) 6. def. 5.

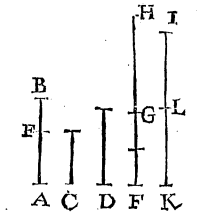
In the same manner, we shall demonstrate that F is lesse than each of the Magnitudes D and E, or equal, or greater: Therefore seeing that F the Multiplex of the first, and of the third C, is lesse than either D or E, equimultiplices of the second, and the fourth A and B, or equal, or greater, there will be likewise the same reason of the first C, to the second A, as of the third C, to the fourth B. Which was proposed.

SCHOLIUM.

This second part may be more briefly demonstrated by Inverse Reason, it having been already demonstrated that there is the same reason of A *to* C, *as of* B *to* C, *alternately,* c *there shall be the same reason of* C *to* A, *as of* C *to* B. *Therefore the equal Magnitudes, &c. Which was to be demonstrated.*

c) Cor. 4. 5

PROP. 8. THEOR. 8.



Of unequal magnitudes A, B *and* C, *the greater* A B, *hath greater reason to one and the same magnitude* D, *than the lesser* C, *and one and the same magnitude* D, *hath greater reason to the lesser* C, *than to the greater* A B.

Demonstration FOr from the greatest A B cut off A E equal to C; and let A B be proposed the first Magnitude, C the third, and D the second and fourth, then take H G and G F, equimultiplices of A E and E B, in such sort as G H the Multiplex of A E be greater than D, and F G the Mutiplex of E B, be not lesse than the same D, but also greater.

Now forasmuch as H G and G F are equimultiplices of A E and E B; a) 2. 5. a the whole H F shall be as much Multiplex of the whole A B, as G H is of A E; that is to say of C, equal to A E. In like manner, Let I K be taken Multiplex of D the second and fourth, in such sort as that it be greater than H G, but lesse than the whole H F, which will be easily done; for seeing that D is lesse than F G, and also lesse than G H, you have only to adde D so many times as its product I K exceeds H G; by this meanes I K shall be lesse than the whole H F, and greater than G H, for having cut off K L equal to D, it shall be lesse than F G, (F G being taken greater than D,) and the remainder L I lesse than G H, or equal, (the same G H being put greater than D,) and D added only so many times as its product I K exceeds G H. Therefore I K shall be greater than G H, and lesse than the whole H F. Therefore seeing that F H and H G are equimultiplices of A B the first, and C the third, and I K Multiplex of D, put for the second and the fourth; but F H the Multiplex of the first, is not b) 8. def. greater than I K the Multiplex of the fourth, but lesse by supposition, (b for I K the Multiplex of D, is taken greater than H G,) there shall be greater reason of A B the first, to D the second, then of C the third, to D the fourth.

Secondly, forasmuch as I K the Multiplex of D the first; (Let D be taken for first, and third, and C the second, and A B fourth,) is greater than H G Multiplex of C the second: But I K the Multiplex of D the third, is not greater than F H Multiplex of A B the fourth, but lesse; F H being greater than I K, as hath been shewn, there will be greater reason of D the first, to C the second, than of D the third, to A B the fourth, which was proposed: Therefore, The Magnitudes, &c. Which was to be demonstrated.

PROP. 9. THEOR. 9.

Magnitudes A *and* B, *which have the same reason to one and the same magnitude* C, *are equal to one another: and those mag-*

magnitudes A B, *to which one and the same magnitude* C, *hath the same reason, are equal to one another.*

Demonstration OTherwise, they should be unequal, and the one greater than the other. Let A then be the greater, and B the lesser, (if possible:) Therefore a there will be greater reason of A the greater, to C, then of B the lesser, to the same C, which is contrary to Supposition: Therefore A and B are not unequal, but equal. a) 8. 5.

Secondly, Let C have the same reason to A, as to B. I say again, that A and B are equal to one another: for if the one were greater, to wit A, and B the lesser, C b would have greater reason to B the lesser, b) 8. 5. then to A the greater, which is contrary to Supposition: Therefore A shall not be greater than B, but equal thereto. Therefore Magnitudes, &c. Which was to be demonstrated.

A ——
B ——
C ——

PROP. 10. THEOR. 10.

Of magnitudes A, *and* B, *which have reason to one and the same magnitude* C, *that which hath the greatest reason* A, *is the greater; but that* B *to which one and the same magnitude* C, *hath the greatest reason, is the least.*

Demonstration FIrst, Let A have greater reason to C, then B to the same C: I say that A is greater than B, for if A were equal to B, the same a A and B would have the same reason to C, and if A were a) 7. 5. lesse than B, b the greatest B would have greater reason to C, than the b) 8. 5. least A, which is contrary to supposition; therefore A is not equal to B, nor lesse, but greater.

Secondly, Let C have greater reason to B then to A. I say that B is lesse than A; c for then C would have the same reason to A as to B, which is c) 7. 5. contrary to supposition: B also shall not be greater then A, forasmuch as C should have greater reason to the least A, then to B the greater, which is contrary to supposition: Therefore B is lesse than A, which was proposed: Therefore the Magnitudes, &c. Which was to be demonstrated.

PROP. 11. THEOR. 11.

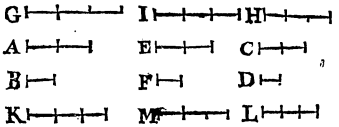
Reasons which are the same to one and the same reason, are also the same to one another.

LEt the reason of A to B, be as that of E to F, and E be to F, as C to D: I say that as A is to B, so C is to D.

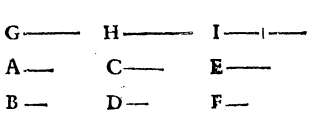
Demonstration FOr let there be taken any equimultiplices whatsoever, of all the antecedents A, E, and C, which may be G, I, and H: In like manner, Let there be also taken any whatsoever equimultiplices

K,

K, M, and L, of the conſequents B, F, and D, foraſmuch as A the firſt is
a) 6. def. 5. to B the ſecond, as E the third, to F the fourth, [a] if G the Multiplex of A the firſt, wanteth of K the Multiplex of B the ſecond, I the Multiplex of E the third, ſhall alſo want of M the Multiplex of F the fourth, and if G be equal to K, I ſhall be equal to M, and if G be greater than K, I, ſhall be greater than
b) 6. def. 5. M. [b] But (as ſhall be demonſtrated in the ſame manner,) if I, be leſſe than M, or equal, or greater: Likewiſe H ſhall be leſſe than L, or equal, or greater; foraſmuch as by Suppoſition there is the ſame reaſonof E the firſt, to F the ſecond, as of C the third, to D the fourth. Wherefore if G the Multiplex of A the firſt, be leſſe than K the Multiplex of B the ſecond; H the Multiplex of C the third, ſhall be leſſe than L the Multiplex of D the fourth, and if G be equal, or greater than K, alſo H ſhall be equal or greater than L, and ſhall be demonſtrated that it will happen ſo in any whatever
c) 6. def. 5. other equimultiplices: Therefore [c] there ſhall be the ſame reaſon of A the firſt, to B the ſecond, as of C the third, to D the fourth. Therefore, Reaſons which are the ſame, &c. Which was to be demonſtrated.



PROP. 12. THEOR. 12.

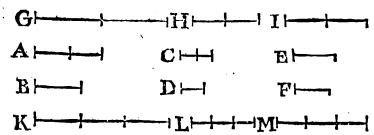


If as many magnitudes as you pleaſe A, B, C, D E, *and* F, *are proportional, as one of the antecedents* A, *ſhall be to one of the conſequents* B; *ſo all the antecedents* A, C, *and* E, *ſhall be to all the conſequents* B, D, *and* F.

Demonſtration FOr aſſume G, H, and I, equimultiplices of the antecedents A, C, and E, and K, L, and M, equimultiplices of
a) 1. 5. the conſequents B, D, and F, [a] the whole G, H, and I, together, ſhall be as much Multiplex of the whole A, C, and E, together, as one of them alone, is of one alone, to wit, as G is of A, and the whole K, L, and M together, ſhall be as much Multiplex of the whole B, D, and F, together, as one alone, is of one alone, to wit, as K is of B: but foraſmuch as by ſuppoſition there is the ſame reaſon of A the firſt, to B the ſecond, as of C the third, to D the fourth, and as another E the third, to F the fourth, it will happen that
6. def. 5. if G the Multiplex of A the firſt, doth want of K the Multiplex of B the ſecond. In like manner, H the Multiplex of C the third, will want of L the Multiplex of D the fourth, and I of M, and if G be equal to K, or greater. Likewiſe, H ſhall be equal to L, and I to M, or greater; and therefore if G be leſſe or equal, or greater than K, the whole G, H, and I, together, ſhall be leſſe than the whole K, L, and M, together, or equal, or greater. Wherefore as

as A the firſt, is to B the ſecond, ſo A, C, and E, the third, ſhall be to B, D, and F, the fourth: Therefore if there be, &c. Which was to be demonſtrated. 6. def. 5.

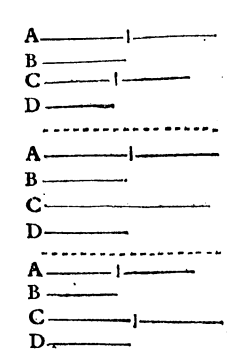
PROP. 13. THEOR. 13.



If the firſt A, *hath the ſame reaſon to the ſecond* B, *as the third* C, *hath to the fourth* D; *But the third* C, *hath greater reaſon to the fourth* D, *than the fifth* E, *to the ſixth* F, *alſo the firſt* A, *ſhall have greater reaſon to the ſecond* B, *than the fifth* E, *to the ſixth* F.

Demonſtration FOr, aſſume G, H, and I, equimultiplices of the antecedents A, C, and E, and K, L, and M, equimultiplices of the conſequents B, D, and F: foraſmuch as there is the ſame reaſon of A the firſt, to B the ſecond, as of C the third, to D the fourth, [a] it will happen, that if G, the Multiplex of A the firſt, exceed K, the Multiplex of B the ſecond, that H the Multiplex of C the third, will exceed L the Multiplex of D the fourth, &c. But when H exceeds L, [b] it is not neceſſary that I exceed M, but ſhall be ſometimes equal, or leſſe; foraſmuch as there is greater reaſon of C the firſt, to D the ſecond, then of E the third, to F the fourth; therefore if G exceed K, of neceſſity I ſhall exceed M: and [c] therefore there is greater reaſon of A the firſt, to B the ſecond, then of E the fifth, to F the ſixth: Therefore, &c. Which was to be demonſtrated.
a) 6. def. 3.
b) 6. def. 5.
c) 8. def. 5.

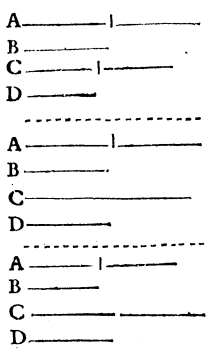
PROP. 14. THEOR. 14.



If the firſt A, *hath the ſame reaſon to the ſecond* B, *as the third* C, *to the fourth* D, *and that the firſt* A, *be greater than the third* C, *the ſecond* B, *ſhall be alſo greater than the fourth* D, *and if the firſt* A, *be equal to the third* C, *alſo the ſecond* B, *ſhall be equal to the fourth* D, *and if leſſe, leſſe.*

Demonſtration IN the firſt place, Let A the firſt, be greater than C the third,

third, there [a] will be greater reason of A the greater, to B, then of C the leſſer, to the ſame B: Therefore foraſmuch as C the firſt, is to D the ſecond, as A the third, to B the fourth: But the reaſon of A the third, to B the fourth, is greater than the reaſon of C the fifth, to B the ſixth, as we have ſhewn; [b] there will be therefore greater reaſon of C the firſt, to B the ſecond, then of C the fifth, to B the ſixth; therefore D ſhall be leſſe than B, therefore B ſhall be greater than D, which was propoſed.

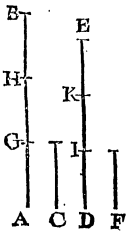


Secondly, Let A be equal to C, there [c] will be the ſame reaſon of A to B, as of C to B; foraſmuch then as the reaſons of C to D, and of C to B, are the ſame as the reaſon of A to B, [d] the reaſons of C to D, and of C to B, ſhall be the ſame to one another: Therefore B and D, ſhall be equal, which was propoſed.

Laſtly, Let A be leſſe than C, [e] there will be greater reaſon of the greateſt C, to B, then of the leaſt A, to the ſame B. Foraſmuch then as C the firſt, is to D the ſecond, as A the third, to B the fourth. But the reaſon of A the third, to B the fourth, is leſſe than that of C the fifth, to B the ſixth; there [f] will be likewiſe leſſe reaſon of C the firſt, to D the ſecond, than of C the fifth, to B the ſixth: Therefore B ſhall be leſſe than D, which was propoſed: [g] Therefore, If, &c. Which was to be demonſtrated.

a) 8. 5.
b) 13. 5.
c) 7. 5.
d) 11. 5.
c) 8. 5.
f) 13. 5.
g) 10. 5.

PROP. 15. THEOR. 15.



The parts C *and* F *are to one another; as are their equimultiplices* AB *and* DE, *to one another, if they be taken ſo as they mutually anſwer to one another.*

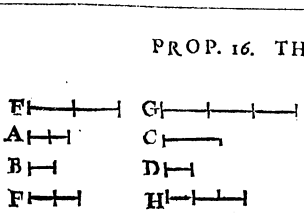
Demonſtration FOr ſeeing that A B is as much Multiplex of C, as D E is of F, the Magnitude C ſhall be as oftentimes conteined in A B, as F is conteined times in D E.

Let therefore A B be divided according to the parts A G, G H, and H B, each equal to C, and D E according to the parts D I, I K, and K E, each equal to F. And foraſmuch as A G and C, are equal, and D I and F, alſo equal, A G [a] ſhall be to D I, as C ſhall be to F, by the ſame reaſon, G H ſhall be to I K, and H B to K E, as C to F; and therefore A G, G H, and H B, ſhall have the ſame reaſon to D I, I K, and K E: Wherefore as A G ſhall be to D I, that is to ſay, as C to F, ſo [b] A B ſhall be to D E, to wit, all the antecedents together, A G, G H, and H B, to all the conſequents together D I, I K, and K E, which was propoſed. Therefore, The parts, &c. Which was to be demonſtrated.

a) 7. 5.
b) 12. 5.

PROP.

PROP. 16. THEOR. 16.



If four magnitudes A, B, C, *and* D, *be proportional, they ſhall be alſo proportional alternately.*

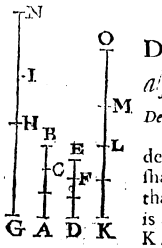
Demonſtration FOr, Let A be to B, as C is to D; I ſay that alternately, or by permutation, A ſhall be to C, as B to D.

Aſſume E and F equimultiplices of A and B, the firſt and ſecond; Alſo G and H equimultiplices of C the third, and D the fourth, E ſhall be to F, as A is to B; [a] Seeing that E and F are equimultiplices of A, and by the ſame reaſon, G ſhall be to H, as C to D.

Now ſeeing that the reaſons of E to F, and C to D, are the ſame as the reaſon of A to B, [b] they ſhall be the ſame to one another. Again, foraſmuch as the reaſons of E to F, and G to H, are the ſame as the reaſon of C to D, [c] they ſhall be alſo the ſame to one another: That is to ſay, as E the firſt, ſhall be to F the ſecond, ſo G the third, to H the fourth: Therefore [d] if E the firſt be greater than G, the third, F the ſecond, ſhall be greater than H the fourth, and if E be equal or leſſe than G, alſo F ſhall be equal or leſſe than H, in whatſoever Multiplication they be taken E F and G H. Therefore [e] A the firſt, ſhall be to C the ſecond, as B the third, to D the fourth; ſeeing that E and F, equimultiplices of the firſt and third A and B, are both greater than G and H, equimultiplices of the ſecond and the fourth C and D, or both equal, or both leſſe, &c. Which was propoſed. Therefore, If four Magnitudes, &c. Which was to be demonſtrated.

a) 15. 5.
b) 11. 5.
c) 11. 5.
d) 14. 5.
e) 6. def. 5.

PROP. 17. THEOR. 17.

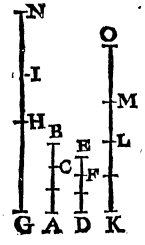


If Compounded Magnitudes AB, CB, DE, *and* FE, *are proportional, they ſhall be alſo proportional, being divided.*

Demonſtration FOr, Aſſume the equimultiplices of AC, CB, DF, and FE, of the ſame order, to wit, GH, HI, KL, and LM, and [a] GI ſhall be as much Multiplex of AB, as GH is of AC; that is to ſay, as KL is Multiplex of DF, [b] KM is likewiſe Multiplex of DE: Therefore GI and KM are equimultiplices of AB and DE. Again, aſſume IN and MO, equimultiplices of CB and FE. Foraſmuch as HI the firſt, is as much Multiplex of CB the ſecond, as LM the third, is of FE the fourth, alſo IN the fifth, as much multiplex of CB the ſecond, as MO the ſixth, is of FE the fourth, HN compounded of the firſt and the fifth, ſhall be as much multiplex of the ſecond CB, as LO compounded of the third and ſixth, is of FE the fourth. Therefore ſeeing that AB the firſt, is to CB the ſecond, as DE the third, to FE the

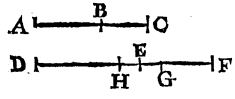
a) 1. 5.
b) 1. 5.

fourth, and that the equimultiplices of the firſt and of the third, A B and D E, have been aſſumed; to wit, G I and K M. In like manner, of the ſecond, and the fourth, C B and F E, the equimultiplices H N and L O,
c) 6. def. 5. c it will happen that if G I the Multiplex of the firſt A B, wanteth of H N the Multiplex of the ſecond C B, alſo K M the Multiplex of the third D E, ſhall want of L O the Multiplex of the fourth F E, and if it be equal, equal; and if greater, greater. And if as well G I wanteth of H N, as K M of L O, having taken away the common magnitudes H I and L M; in the like manner, G H will want of I N, and K L of M O; and if G I be equal to H N, and K M to L O, having taken away the common magnitudes H I and L M; G H ſhall be equal to I N, and K L to M O. And if laſtly, G I exceed H N; and K M, L O, taking away the common parts H I and L M; G H ſhall likewiſe exceed I N, and K L, M O. Therefore ſeeing that G H and K L have been aſſumed equimultiplices of A C the firſt, and of D F the third. In like manner, I N and M O, equimultiplices of C B the ſecond, and F E the fourth, and it is ſhewn in any multiplication whatſoever, that the equimultiplices of the firſt and third, do want together of the equimultiplices of the ſecond and of the fourth, or together are equal, or do
d) 6. def. 5. exceed them. d As A C the firſt, to C B the ſecond, ſo D F the third, ſhall be to F E the fourth. Which was propoſed. Therefore, If the magnitudes compounded, &c. Which was to be demonſtrated.



PROP. 18. THEOR. 18.

If Magnitudes divided A B, B C, D E, *and* E F, *are proportional, being compounded they ſhall be alſo proportional.*



Demonſtration LEt it be, as A B is to B C, ſo D E to E F: I ſay, that being compounded, they ſhall be proportional, that is to ſay, as A C is to B C, ſo D F is to E F.

For if A C be not to B C, as D F is to E F and D F, ſhall have the ſame reaſon to ſome other magnitude, leſſe than E F, or greater than the ſame E F, as A C to B C.

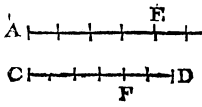
17. 5. Firſt, Let D F have the ſame reaſon to G F, leſſe than E F, as A C to B C, (if it may be done.)

Foraſmuch as A C is to B C, as D F to G F, by dividing, as A B is to B C, ſo D G ſhall be to G F: But as A B to B C, ſo D E to E F, by ſup-
11. 5. poſition. Therefore as D G the firſt, to G F the ſecond, ſo D E the third, to E F the fourth; and ſeeing that D G the firſt, is greater than D E the third; likewiſe G F the ſecond, ſhall be greater than E F the fourth, the part than the whole, which is impoſſible.

Secondly, Let D F have the ſame reaſon to H F, greater than E F, (if it may be) as A C hath to B C. Foraſmuch as A C is to B C, as D F to
17. 5. H F; by dividing, as A B ſhall be to B C, ſo D H to H F. But as A B to B C,

BC; so is DE to EF, by supposition: Therefore as DH the first, to HF the second, so DE the third, to EF the fourth. And seeing DH the first, is lesse than DE the third. Likewise, HF the second, shall be lesse than EF the fourth, the whole than its part, which is absurd: Therefore DF shall not have the same reason to GE the lesser, as to EF, or to HF the greater, as AC to BC: Therefore DF shall be to EF, as AC to BC; Which was proposed. Therefore, If the Magnitudes, &c. Which was to be demonstrated. 11. 5.

PROP. 19. THEOR. 19.



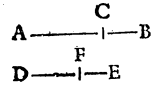
If as the whole AB, *is to the whole* CD, *so the part cut off* AE, *to the part cut off* CF, *then the remainder* EB, *shall be to the remainder* FD, *as the whole* AB, *to the whole* CD.

Demonstration FOr seeing that AB is to CD, as AE to CF, alternately, AB shall be to AE, as CD to CF: Therefore dividedly, EB shall be to AE, as CD to CF; therefore again alternately, EB shall be to FD, as AE to CF; that is to say, as the whole AB, to the whole CD; seeing that AB is put to CD, as AE to CF: Therefore, If as as the whole, &c. Which was to be demonstrated. 16. 5. 17. 5.

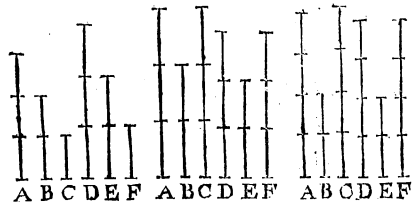
COROLLARIE.

Hence shall be demonstrated the manner of arguing in proportions, which is taken from the conversion of reason, according to the sixteenth Definition.

For, Let it be as AB *to* CB, *so* DE *to* FE. *I say, by conversion of reason, as* AB *is to* AC, *so* DE *to* DF: *For seeing that as* AB *is to* CB, *so* DE *is to* FE, *in dividing* AE *shall be to* CF, *as* DF *to* FE. *Therefore, alternately, as* CE *is to* EC, *so* FE *to* DF, *and therefore in compounding, as* AB *shall be to* AC, *so* DE *to* DF. *Which was proposed.* 17.



PROP. 20. THEOR. 20.



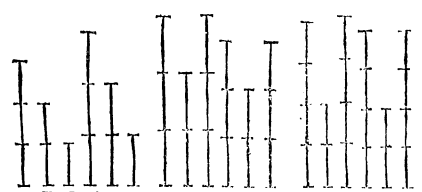
If there be three magnitudes A, B, *and* C, *and others equal to them in number* D, E, *and* F, *which being taken two and two, and in the same reason, as* A *to* B, *so* D *to* E,

and B *to* C, *as* E *to* F, *and that in equal reason, the first* A, *be greater than the third* C, *also the fourth* D *shall be greater than the sixth* F, *if equal, equal, and if lesse, lesse.*

13. 5. *Demonstration* FOr seeing that A is greater than C, there will be greater reason of A to B, than of C to B. But as A is to B, so D is to E, there will be therefore greater reason of D to E, then of C to B. But as C is to B, so F to E; (for seeing that as B to C, so E to F; alternately, as C shall be to B, so F shall be to E,) there will be therefore in
10. 5. like manner, greater reason of D to E, than of F to E. Therefore D shall be greater than F. Which was proposed.
7. 5. Secondly, Let A be equal to C: I say that D shall be equal to F. For seeing that A is equal to C, as A shall be to B, so C to B; but as A is to B,
11. 5. so D is to E: Therefore D shall be to E, as C to B; but as C is to B, so F
9. 5. is to E, (by inverse reason, as before) therefore D shall be likewise to E, as F to E; therefore D and F shall be equal; which was proposed.

Thirdly, Let A be lesse than C: I say that D is lesse than F: For seeing that A is lesse than C, there will be lesse reason of A to B than of C to B: but
8. 5. as A is to B, so D is to E; there will be therefore lesse reason of D to E,
15. 3. than of C to B; but by inverse reason as before, as C is to B, so F is to E:
10. 5. Therefore there is lesse reason of D to E, then of F to E, therefore D shall be lesse than F. Which was proposed. Therefore, If there be three magnitudes, &c. Which was to be demonstrated.

PROP. 21. THEOR. 21.



If there be three magnitudes A, B, *and* C, *and others equal to them in number,* D, E, *and* F, *which being taken two and two, and in the same reason, and that their proportion be perturbate, or without order, (that is, as* A *to* B, *so* E *to* F, *and as* B *to* C, *so* D *to* E:) *But that in equal reason, the first* A *be greater than the third* C, *the fourth* D, *shall be also greater than the sixth* F, *if equal, equal, if lesser, lesser.*

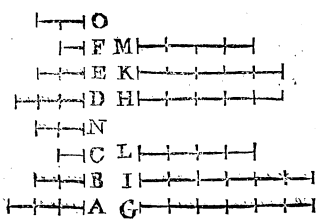
5. c. 13. 5. *Demonstration* FOr seeing that A is greater than C, there will be greater reason of A to B, than of C to B, but as A is to B, so E is to F; there will be therefore in like manner, greater reason of E to F, than of G to B. But seeing that as B is to C, so D is to E, by inverse reason, as

as C is to B, so E is to D. Wherefore there will be also greater reason of E to F, than of E to D; and therefore D shall be greater than F. Which was proposed. 10. 5.

Secondly, Let A be equal to C; I say that D shall be equal to F. For seeing that A is equal to C, as A shall be to B, so C shall be to B; but as 7. 5.
A is to B, so E is to F; Therefore as C is to B, so E shall be to F; but by 11. 5.
inverse reason, as C is to B, so E is to D, as before; therefore as E shall 9. 5.
be to F, so E shall be to D; and therefore D shall be equal to F. Which was proposed.

Lastly, Let A be lesse than C: I say that D shall be lesse than F: for seeing that A is lesse than C, there will be lesse reason of A to B, than of 8. 5.
C to B: But as A is to B, so E is to F; therefore there will be lesser reason of E to F, than of C to B: But forasmuch as before, by inverse reason, as C is to B, so E is to D, there will be lesser reason of E to F, then of E 10. 1.
to D. Wherefore D shall be lesse than F. Which was proposed. Therefore, If there be, &c. Which was to be demonstrated.

PROP. 22. THEOR. 22.



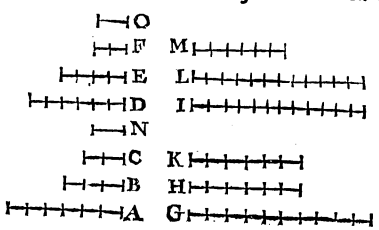
If there be as many magnitudes as you please, A, B, *and* C, *and others equal to them in number* D, E, *and* F, *which being taken two and two, and in the same reason,* (*that is as* A *to* B, *so* D *to* E, *and* B *to* C, *as* D *to* F,) *those magnitudes in equal reason, shall be proportional, that is as* A *to* C, *so* D *is to* F.

Demonstration FOr, assume G and H equimultiplices of A and D, and I and K, equimultiplices of B and E, and L and M of C and F: And seeing that A the first, is to B the second, as D the third, is to E the 4. 5.
fourth. In like manner, G the Multiplex of A the first, shall be to I the Mul-
tiplex of B the second, as H the Multiplex of D the third, to K the Multi- 4. 5.
plex of E the fourth; by the same reason, seeing that B the first, is to C the second, as E the third, to F the fourth, I the Multiplex of B the first, shall be to L the Multiplex of C the second, as K the Multiplex of E the third, shall be to M the Multiplex of F the fourth: Forasmuch then as there are three magnitudes G, I, and L, and other three H, K, and M, which are 20. 5.
taken two and two, and in the same reason. It will happen, that if G the first, exceed L the third, that H the fourth, will exceed M the sixth; and if equal, equal, and if lesse, lesse. Wherefore seeing that G and H, the equimultiplices of the first A, and of the third D, either want together, of L and M, equimultiplices of C the second, and of F the fourth, or together are equal, or together do exceed, according to whatsoever multiplications those equi-
multiplices be taken; A the first, shall be to C the second, as D the third, 6. def. 5.
is to F the fourth, which was proposed.

Again,

Again, Let them be more than three magnitudes, in ſuch ſort as C be alſo to N, as F to O, I ſay moreover, that as A ſhall be to N, ſo D ſhall be to O; for ſeeing it hath been already ſhewn in three magnitudes that A is to C, as D to F. But C is put to N, as F to O, there will be three magnitudes to A, C, and N, and other three D, F, and O, which are taken two and two, in the ſame reaſon: Therefore in equal reaſon demonſtrated in three magnitudes: Again, as A ſhall be to N, ſo D to O. In like manner, it may be ſhewn the ſame in five magnitudes by four, as it hath been demonſtrated in four magnitudes by three, and ſo of divers: Therefore, If there be as many Magnitudes, &c. Which was to be demonſtrated.

PROP. 23. THEOR. 23.

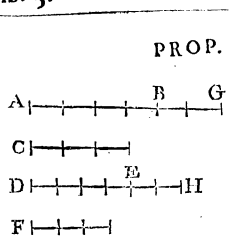


If there be three magnitudes A, B, *and* C, *and others equal to them in numbers* D, E, *and* F, *taken two and two, and in the ſame reaſon, and that their proportion be perturbed (that is to ſay, as* A *to* B, *ſo* E *to* F, *and* B *to* C, *as* D *to* E,) *in equal reaſon, they ſhall be alſo in the ſame reaſon (that is, as* A *to* C, *ſo* D *to* F.)

15. 5. 11. 5. *Demonſtration* ASſume G, H, and I, equimultiptices of A, B, and D. Likewiſe K, L, and M, equimultiplices of C, E, and F, as A ſhall be to B, ſo G ſhall be to H; ſeeing that G and H are equimultiplices of A and B. But as A is to B, ſo E is to F: Therefore as G is to H, 15. 5. ſo E is to F: But as E is to F, ſo L is to M; ſeeing that L and M are equi- 11. 8. multiplices of E and F. Therefore G ſhall be likewiſe to H, as L to M. Again, foraſmuch as B the firſt, is to C the ſecond, as D the third, to E the fourth; likewiſe H the Multiplex of the firſt B, ſhall be to K the Multiplex of the ſecond C, as I the Multiplex of D the third, is to L the 4. 5. Multiplex of E the fourth. Now foraſmuch as there are three magnitudes G, H, and K, and other three I, L, and M, which taken two and 21. 5. two in the ſame reaſon, their proportion being perturbated: Seeing that it is ſhewn, that G is to H, as L is to M, and as H is to K, ſo I is to L, if G the firſt, exceed K the third, I the fourth ſhall likewiſe exceed M the ſixth, and if equal, equal, and if leſſe, leſſe, therefore ſeeing that G and I equimultiplices of the firſt A, and of the third D, are both greater, or equal, or leſſe, then K and M equimultiplices of the ſecond C, and of the fourth F, as A the firſt, ſhall be to C the ſecond, ſo D the third, ſhall be to F the fourth. Which was propoſed. Therefore, If there be three Magnitudes, &c. Which was to be demonſtrated.

PROP.

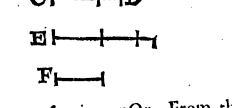
PROP. 24. THEOR. 24.



If the firſt AB, *hath the ſame reaſon to the ſecond* C, *as the third* DE, *hath to the fourth* F, *and that the fifth* BG, *hath alſo the ſame reaſon to the ſecond* C, *as the ſixth* EH, *hath to the fourth* F, *alſo* AG *the compound of the firſt* AB, *and the fifth* BG, *ſhall have the ſame reaſon to the ſecond* C, *as the compound* DH, *of the third* DE, *and of the ſixth* EH, *hath to the fourth* F.

Demonſtration FOr ſeeing that BG is to C, as EH to F, alternately, as C ſhall be to BG, ſo F ſhall be to EH; therefore foraſmuch as AB is to C, as DE is to F, and that C is to BG, as F is to EH, 22. 5. in equal reaſon, AB ſhall be to BG, as DE is to EH. Therefore by compounding, the whole AG ſhall be to BG, as the whole DH to EH, and BG to C, as EH is to F; in equal reaſon, AG ſhall be to C, as 22. 5. DH is to F. Which was propoſed. Therefore, If the firſt, &c. Which was to be demonſtrated.

PROP. 25. THEOR. 25.

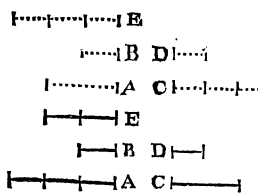


If four magnitudes are proportional, AB *to* CD, *as* E *to* F, *the greateſt* AB, *and the leaſt* F, *are greater than the two others* CD *and* E.

Demonſtration FOr, From the magnitude AB cut off AG equal to E, and from CD cut off alſo CH, equal to F. Then AG ſhall be to CH, as E to F; that is to ſay, as AB to CD. Therefore ſeeing that the whole AB, is to the whole CD, as the part cut off AG, is to the part cut off CH. In like manner, as the whole AB, to the whole 91. 5. CD, ſo the remainder GB, to the remainder HD. But AB (ſeeing that it is the greateſt of all,) is greater than CD. Therefore GB ſhall be alſo greater than HD. But foraſmuch as AG and E are equal, if you adde to them the equals F and CH; to wit, F to AG, and CH to E; AG and F together, ſhall be equal to E and CH together: Therefore adding the unequals GB and HD, AB and F together, ſhall be greater than E and CD together; ſeeing that GB is greater than HD. Which was propoſed. Therefore, If four Magnitudes, &c. Which was to be demonſtrated.

PROP

PROP. 26. THEOR. 26.



If the first A, *hath greater reason to the second* B, *than the third* C, *hath to the fourth* D, *by inverse reason, the second* B, *shall have lesse reason to the first* A, *than the fourth* D, *hath to the third* C.

Demonstration FOr, Let it be understood that E is to B, as C is to D, the reason of A to B shall be likewise greater than that of E to B; and [a] therefore A shall be greater than E: Wherefore [b] there will be lesse reason of B to A the greater, than of B to E the lesser: But as B is to E, so by inverse reason, D is to C. Therefore the reason of B to A, is likewise lesse than that of D to C. Which was proposed. Therefore, If the first, &c. Which was to be demonstrated.

a) 10. 5.
b) 8. 5.

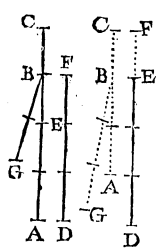
PROP. 27. THEOR. 27.

If the first A, *hath greater reason to the second* B, *than the third* C, *hath to the fourth* D; *also alternately, the first* A, *shall have greater reason to the third* C, *than the second* B, *shall have to the fourth* D.

Demonstration FOr, Let E be put to B, as C to D, A shall likewise have greater reason to B, than E to B. Wherefore [a] A shall be greater than E, and [b] therefore there will be greater reason of A to C, than of E to C; but [c] forasmuch as alternately as E is to C, so B is to D; (E being put to B, as C to D.) Therefore A shall also have greater reason to C, then B hath to D, which was proposed. Therefore, If the first, &c. Which was to be demonstrated. (As by the precedent figure is manifest.)

a) 10. 5.
b) 8. 5.
c) 6. 5.

PROP. 28. THEOR. 28.



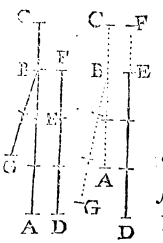
If the first AB, *hath greater reason to the second* BC, *than the third* DF, *hath to the fourth* EF, *the compounded* AC, *of the first* AB, *with the second* BC, *shall have also greater reason to the second* BC, *than the compounded* DF, *of the third* DE, *with the fourth* EF, *hath to the fourth* EF.

Demon-

Demonstration FOr, Let G B be put to B C, as D E to E F, there will be also greater reason of A B to B C, then of G B to B C: Therefore [a] A B shall be greater than G B, adding therefore the common part BC, A C shall be greater than GC; and therefore A C shall have greater reason to B C, than G C to B C; but in compounding, G C is to B C, as D F to E F; (seeing G B is put to B C, as D E to E F;) therefore A C shall have greater reason to B C, than D F to E F. Which was proposed. Therefore, &c. Which was to be demonstrated.

a) 10. 5.
17. 5.

PROP. 29. THEOR. 29.

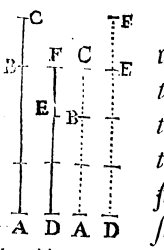


If the compound AC, *of the first* AB, *with the second* BC, *hath greater reason to the second* BC, *than the compound* DF, *of the third* DE, *with the fourth* EF, *hath to the fourth* EF. *In dividing, the first* AB, *shall have also greater reason to the second* BC, *than the third* DE, *to the fourth* EF.

Demonstration FOr, Let G C be put to B C, as D F to E F, the reason of A C to B C, shall be also greater than of G C to B C: Wherefore [a] A C shall be greater than G C, Therefore taking away the common part B C, there will remain A B, greater than G B. [b] Therefore there will be greater reason of A B to B C, than of G B to B C. But in dividing, as G B is to B C, so D E is to E F, (G C being put to B C, as D F to E F,) there will be therefore also greater reason of A B to B C, than of D E to E F. Which was proposed. Therefore, If the compound, &c. Which was to be demonstrated.

a) 10. 5.
b) 17. 5.

PROP. 30. THEOR. 30.

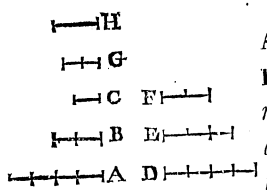


If the compound AC, *of the first* AB, *with the second* BC, *hath greater reason to the second* BC, *than the compound* DF, *of the third* DE, *with the fourth* EF, *hath to the fourth* EF, *by conversion of reason, the first* AB, *with the second* BC; (*that is* AC) *shall have lesser reason to the first* AB, *than the third* DE, *with the fourth* EF, (*that is* DF,) *hath to the third* DE.

Demonſtration FOr A C having greater reaſon to B C, than D F to E F; [a] in dividendo, A B ſhall have greater reaſon to B C, than D E to E F. Therefore by converſion of reaſon, there ſhall be leſſe reaſon of B C to A B, than of E F [b] to D E, and in compounding, there will be leſſer reaſon of the whole A C to A B, than of the whole D F to D E. Which was propoſed. Therefore, If the compound, &c. Which was to be demonſtrated.

a) 29. 5.
b) 26. 5.

PROP. 31. THEOR. 31.



If there be three magnitudes A, B, *and* C, *and other three* D, E, *and* F, *equal to them in number, and that there be greater reaſon of the firſt* A *of the firſt, to the ſecond* B, *then of the firſt* D, *of the laſt, to the ſecond* E. *Likewiſe that there be greater reaſon of the ſecond* B, *of the firſt, to the third* C, *than of the ſecond* E, *of the laſt, to the third* F. *In equal reaſon there will be alſo greater reaſon of the firſt* A, *of the firſt, to the third* C, *than of the firſt* D, *of the laſt, to the third* F.

Demonſtration FOr, Let G be put to C, as E to F: Therefore there ſhall be alſo greater reaſon of B to C, than of G to C: Wherefore [a] B ſhall be greater than G: Therefore [b] there will be greater reaſon of A to G the leſſer, then of A to B the greater: But the reaſon of A to B, is put greater than of D to E: Therefore there will be yet greater reaſon of A to G, then of D to E.

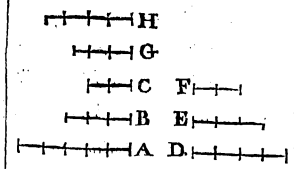
a) 10. 5.
b) 8. 5.

Again, Let H be put to G, as D to E, there will be alſo therefore greater reaſon of A to G, then of H to G: Therefore [c] A ſhall be greater than H: Therefore [d] A the greateſt, ſhall have greater reaſon to C, then H the leſſer, ſhall have to the ſame C. But as H is to C, ſo D is to F, in equal reaſon, (ſeeing that as D is to E, ſo H is to G, and as E is to F, ſo G is to C;) Therefore there ſhall be alſo greater reaſon of A to C, then of D to F. Which was propoſed. Therefore, If there be three Magnitudes, &c. Which was to be demonſtrated.

c) 10. 5.
d) 8. 5.
e) 22. 5.

PROP. 32. THEOR. 32.

If there be three magnitudes A, B, *and* C, *and others equal to them in number* D, E, *and* F, *and that there be greater reaſon of the firſt* A, *of the firſt, to the ſecond* B, *then of the ſecond* E,



E, *of the laſt, to the third* F; *likewiſe that there be greater reaſon of the ſecond* B, *of the firſt, to the third* C, *then of the firſt* D, *of the laſt, to the ſecond* E: *In equal reaſon, there will be alſo greater reaſon of the firſt of the firſt* A, *to the third* C, *then of the firſt of the laſt* D, *to the third* F.

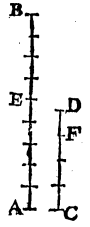
Demonſtration FOr put G to C, as D to E; therefore alſo B ſhall have greater reaſon to C, than G to C; therefore [a] B ſhall be greater than G. Wherefore [b] there will be greater reaſon of A to G the leſſe, than of the ſame A to B the greater. But there is greater reaſon of A to B, than of E to F: Therefore there will be yet greater reaſon of A to G, than of E to F.

a) 10. 5.
b) 8. 5.

Again, put H to G, as E to F; there will be then greater reaſon of A to G, than of H to G. Therefore [c] A ſhall be greater than H. Wherefore [d] A ſhall have greater reaſon to C, than H the leſſer, ſhall have to the ſame C: But [e] as H is to C, ſo D is to F, in equal reaſon, (D being to E, as G to C, and as E to F, ſo H to G:) there is therefore alſo greater reaſon of A to C, than of D to F. Which was propoſed. Therefore, If there be three Magnitudes, &c. Which was to be demonſtrated.

c) 10. 5.
d) 8. 5.
e) 23. 5.

PROP. 33. THEOR. 33.



If there be greater reaſon of the whole AB, *to the whole* CD, *then of the part cut off* AE, *to the part cut off* CF; *there will be alſo greater reaſon of the remainder* EB, *to the remainder* FD, *then of the whole* AB, *to the whole* CD.

Demonſtration FOr ſeeing that there is greater reaſon of AB to CD, than of AE to CF; [a] by permutation, there ſhall be alſo greater reaſon of AB to AE, than of CD to CF; therefore [b] by converſion of reaſon, there will be leſſer reaſon of AB to EB; than of CD to FD. Therefore [c] alternately, there ſhall be alſo leſſer reaſon of AB to CD, than of EB to FD; to wit, the remainder EB, ſhall have greater reaſon to the remainder FD, than the whole AB, to the whole CD. Which was propoſed. Therefore, If there be greater reaſon, &c. Which was to be demonſtrated.

a) 27. 5.
b) 30. 5.
c) 27. 5.

PROP. 34. THEOR. 34.



If there be as many magnitudes as you please A, B, *and* C, *and others equal to them in number* D, E, *and* F, *and that there be greater reason of the first of the first* A, *to the first of the last* D, *than of the second* B, *to the second* E; *and of the second* B, *to the second* E, *then of the third* C, *to the third* F, *and so on; all the first* A, B, *and* C *together, shall have greater reason to all the last* D, E, *and* F *together; then all the first* B *and* C, *the first* A *being taken away, to all the last* E F, *the first* D *also taken away; But lesse reason than the first of the first* A, *to the first of the last* D; *and lastly, also greater reason than the last of the first* C, *to the last of the last* F.

Demonstration. FOr seeing that there is greater reason of A to D, than of B to E; [a] by permutation, there shall be greater reason of A to B, then of D to E; therefore [b] in compounding, there shall be greater reason of A and B together to B, then of D and E together to E; and [c] again, by permutation there will be greater reason of A & B together, to D and E together, than of B to E. Therefore the whole A & B having greater reason to the whole D & E; than the part cut off B, to the part cut off E, [d] the remainder A shall also have greater reason to the remainder D, than the whole A & B, to the whole D & E. In like manner, there will be greater reason of B to E, than of the whole B and C, to the whole E and F; therefore there will be yet greater reason of A to D, than of the whole B and C, to the whole E and F. Therefore [e] by permutation, there will be greater reason of A to B and C, than of D to E and F. Therefore [f] in compounding, there will be greater reason of the whole A, B, and C, to B and C, than of the whole D, E, and F, to E and F, and again, [g] by permutation, there will be greater reason of the whole A, B, and C, together, to the whole D, E, and F, together, then of B and C to E and F. Which was in the first place proposed.

Therefore there being greater reason of the whole A, B, and C, to the whole D, E, & F than of the part cut off B and C, to the part cut off E and F; There [h] will be also greater reason of the remainder A, to the remainder D, then of the whole A, B, and C, to the whole D, E, and F. Which was in the second place proposed.

But forasmuch as there is greater reason of B to E, than of C to F; alternately, [i] there will be also greater reason of B to C, than of E to F, and in compounding, greater reason of the whole B and C to C, than of the whole

a) 27. 5.
b) 28. 5.
c) 27. 5.
d) 33. 5.
e) 27. 5.
f) 28. 5.
g) 27. 5.
h) 33. 5.
i) 28. 5.

whole E & F to F; and again, by permutation, greater reason of B and C, to E & F, than of C to F. But there is greater reason of A, B and C, to D, E, & F, as hath been shewn, than of B and C to E and F: Therefore there will be yet greater reason of the whole A, B, and C, to the whole D, E, and F, than of the last C, to the last F. Which was in the third place proposed.

Let there be afterward proposed four magnitudes on both parts, by the same supposition; that is to say, that there be greater reason of C the third, to F the third, than of G the fourth, to H the fourth; I say that the same thing will follow, for (as hath been already shewn in three Magnitudes, there is greater reason of B to E, than of B, C, and G, to E, F and H. Therefore there will be yet greater reason of A to D. Then of B, C, and G, to E, F, and H, Therefore [k] alternately, there will be greater reason of A to B, C, and G, than of D to E, F, and H. And [l] in compounding, greater reason of A, B, C, and G, to B, C, and G, than of D, E, F, and H, to E, F, and H; [m] and alternately, greater reason of A, B, C, and G, to D, E, F, and H, than of B, C, and G, to E, F, and H. Which was in the first place proposed.

k) 27. 5.
l) 28. 5.
m) 27. 5.

Therefore having greater reason of the whole A, B, C, and G, to the whole D, E, F, and H, than of the part cut off B, C, and G; to the part cut off E, F, and H, [n] the remainder A, shall have greater reason to the remainder D, than the whole A, B, C, and G, to the whole D, E, F, and H. Which was in the second place proposed.

n) 33. 5.

But (as is demonstrated in three magnitudes,) forasmuch as there is greater reason of B, C, and G, to E, F, and H, than of G to H; and greater of A, B, C, and G, to D, E, F, and H, than of B, C, and G, to E, F, and H, as hath been shewn; there will be much greater reason of A, B, C, and G, to D, E, F, and H, than of the last G, to the last H. Which was in the third place proposed. The same may be shewn in five Magnitudes, by four, and in six by five, &c. as hath been shewn in four by three. Therefore, If, &c. Which was to be demonstrated.

The End of the Fifth Element of EUCLIDE.

THE

THE SIXTH ELEMENT OF EUCLIDE.

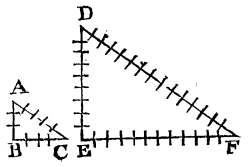
THE ARGUMENT.

THis Sixth Book treateth of the proportion of figures among themselves, of like and reciprocal figures, of proportional right lines, of the application of Parallelograms to right lines, which may either want or exceed by like Parallelograms, and how a terminated right line may be divided by extream and mean proportion, and of the proportions of circumferences and angles, also of Sectors in equal circles.

DEFINITIONS.

1 *Like right lined figures are those which have the angles equal each one to his correspondent, and the sides which are about the equal angles proportional.*

AS the triangles A B C, and D E F, shall be said to be alike, if they be equiangled; that is to say, that the angle A, be equal to the angle D, and the angle B to the angle E, and the angle C to the angle F; and in



in like manner, that the sides about the equal angles be proportional; to wit, as A B is to A C, so D E may be to D F, and as A B to B C, so D E to E F; and as A C to C B, so D F to E F.

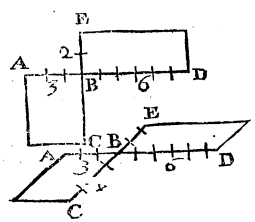
And if each of the angles of one of the figures be equal to each of the angles of the other. But the sides about the equal angles are not proportional; or contrarily, such figures shall not be said to be alike; as are the square and the oblong, or long square; for those figures have the angles equal, to wit, right angles, but the sides of the one, are not proportional to the sides of the other; the sides of the square having reason of equality; and those of the oblong about the right angle, reason of inequality; from whence it appears that all right lined figures equiangled, and equilateral, having the sides and the angles equal in number, are alike, although they be unequal.

2 *Reciprocal figures are such, when the reasons antecedents and consequents are in both the figures.*

Or thus,

The figures are Reciprocal, when the terms, antecedents and consequents of the reasons, are in both the figures.

FOr by these reasons, antecedents and consequents ought to be understood, the terms antecedent and consequent of the proportion; as if there be two figures rectangled, or not rectangled, A C and E D; and that as A B is to B D, so E B is to B C, those figures shall be termed reciprocal: Forasmuch as in the one is the term antecedent of the first reason, to wit, A B, and the consequent of the second reason, to wit, B C, and in the other is the term consequent of the first reason B D, and the antecedent of the second E B; such figures are also said to be equal to one another, as shall be hereafter demonstrated.



3 *A right line is said to be divided according to mean and extream reason, when as the whole is to the greatest segment, as the greatest segment is to the lesser.*

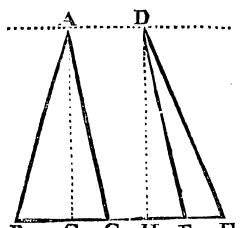
IF any right line, as A B, be divided, in such sort unequally in the point C, as that the whole A B, be to the greatest segment A C, as the greatest segment A C, is to the least segment C B, it shall be said to be divided according to mean and extream reason, which said division *EUCLIDE* teacheth at the thirtieth Proposition of this Book.



4 The

4 *The height of any figure is the perpendicular line drawn from the top to the base.*

IF from the vertex A, of the triangle A B C, there be drawn A G, perpendicular, to the base B C, the quantity of that perpendicular shall be the height or altitude of the triangle A B C. So also the perpendicular D H, drawn from the vertex D, of the triangle D E F, on the base E F, prolonged towards E, shall be the height of that triangle D E F.



Therefore if the perpendiculars of the two figures, drawn from their tops to their bases, (be the bases prolonged, or not,) are equal, such figures shall be said to have the same height. Now such perpendiculars shall be equal, when the bases of the figures and the tops shall be constituted between the same parallels, as are A G and D H.

For seeing that the interior angles A G H, and D H G, on the same part, are equal to two right angles, or to expresse it better, are right angles; [a] A G and D H shall be parallels, but A D and G H are also parallels; forasmuch as the triangles are proposed to be constituted between the same parallels; Therefore A D and H G, shall be a parallelogram, [b] and therefore the sides A G and D H, shall be equal: Therefore those triangles shall be said to have the same height the one as the other. And if in the same triangles, C and F be put for the tops, and A B and D E for the bases, those triangles shall not be said to have the same height; for the perpendicular drawn from F to the base D E, is not equal to the perpendicular drawn from from C, to the base A B, those triangles therefore cannot in any wise be constituted between the same parallels, as is manifest.

a) 28. 1.

b) 34. 1.

5 *A reason is said to be compounded of reasons, when the quantities of the reasons multiplyed in one another, do make some reason.*

FOrasmuch as the denominator of any reason whatsoever, doth expresse the quantity of the Magnitude antecedent, in respect of the consequent, (as the denominator of the reason quadruple; to wit 4, shewes that in all reason quadruple, the Magnitude antecedent, conteineth four times the consequent. But the denominator of the reason subquadruple, to wit $\frac{1}{4}$, demonstrateth that the Magnitude antecedent is the fourth part of the consequent, &c. (Geometricians use to term the denominator the quantity of the reason; so as that the quantity of the reason, and the denominator of the reason, is the same thing;) therefore this definition teacheth us that a reason is said to be compounded of two or more reasons, when the denominators or quantities of those reasons multiplyed in one another, shall make that reason, or (according to *Zambert*,) shall make the quantity or

denominator of that reason: As the reason duodecuple is said to be compounded of the double and the sextuple: Forasmuch as the denominator of the reason duodecuple, to wit 12, is produced of the multiplication of the denominator of the reason double, to wit of 2, by the denominator of the sextuple, that is to say, by 6. So the same reason duodecuple is said to be compounded of the triple and of the quadruple; For of the Multiplication of 3 by 4, is produced the denomina or 12, of the reason duodecuple, In like manner, the reason trigecuple is said to be compounded of the double, triple, and quintuple, for the denominators of that reason 2, 3, and 5, multiplyed in one another, produce 30, the denominator of that reason trigecuple. So the raason double is said to be compounded of the sesquialtera and sesquitertia, Forasmuch as the denominator of the sesquialtera, which is $1\frac{1}{2}$, multiplied by $1\frac{1}{3}$, denominator of the sesquitertia is produced of 2, the denominator of the double.

Again, the same reason double, is compounded of the sesquiseptuple, and the supertripartient fourths; for the denominator of those reasons $1\frac{1}{7}$, and $1\frac{1}{4}$, multiplyed in one another, do produce two denominators of the double: In like manner, the same reason double shall be said to be compounded of the subsesquiquarta, and of the double sesquialtera; forasmuch as their denominators $\frac{4}{5}$, and $2\frac{1}{2}$ multiplyed in one another, do produce in like manner the same denominator of the reason double, to wit 2, &c.

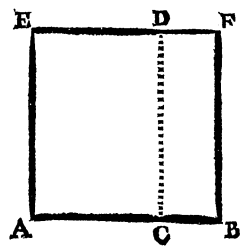
Therefore if you propose as many Magnitudes as you please in order, as A, B, C, and D, to wit, A to B in reason double, B to C triple, and C to D quintuple, the reason of the extreams A and D, shall be trigecuple, which shall be said to be compounded of the mean reasons, for if you multiply the denominator of A by B, which is 2, by that of B to C 3, you shall have 6 for the denominator of the reason of A to C.

A. B. C. D
60 30 10 2

Therefore A to C, hath reason sextuple. Again, multiplying 6 the denominator of the reason of A to C, by that of C to D 5, you shall have 30, for the denominator of the reason of A to D. Wherefore A shall be to D, in reason trigecuple, &c.

And thus much may be said of the five definitions of *EUCLIDE*, on this Sixth Book, to which shall be added this following definition; which will render as well the 27, 28, 29, and 30th. Propositions of this Book, as also divers others of the Tenth Book, more easie and intelligible, and is as followeth.

6 *A parallelogram being applyed according to some right line is said to want thereof by a parallelogram, when it occupieth not all that right line. But is said to exceed it, when it occupieth a greater line than that according to which it is applyed: In such sort nevertheless, that the parallelogram exceeding, or wanting, hath the same height as the parallelogram of which it wanteth, or which it exceedeth, and doth constitute therewith one only parallelogram.*



LEt the line AB, be the line on which the Parallelogram A D is to be constituted, not imploying the whole line A B, but leaving C B, having finished the parallelogram A F, the parallelogram A D, applyed according to the line A B, shall be said to want thereof by the parallelogram C F, so as that C F shall be called the want.

Again, Let A C be the line on which the parallelogram A F is to be constituted, having the side A B, greater than A C, to which let C D be drawn parallel to B F, the parallelogram A F, applyed according to A C, shall be said to exceed A C, by the parallelogram C F, in such sort as C F shall be termed the excesse.

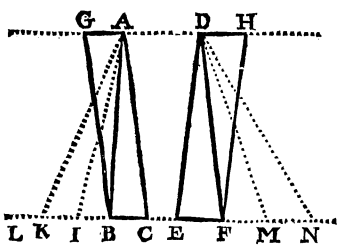
Now this want, or excesse in rectangles may be a square, or an oblong, and in parallelograms not rectangled, a Rhombe or Rhomboides, as is manifest.

PROPOSITIONS,

PROBLEMES, and THEOREMES.

PROPOSITION 1. THEOREM 1.

Triangles A B C *and* D E F, *and parallelograms* C G *and* E H, *which have the same height, are to one another, as their bases* B C *and* E F.



I Say that the triangle A B C, is to the triangle D, E, and F, and the parallelogram C G, to the parallelogram E A, as the base B C is to the base E F; that is to say, that if the base B C be taken for the first magnitude, and

and the base E F for the second. But the triangle A B C, or the parallelogram C G for the third, and the triangle D E F, or the parallelogram E H, for the fourth, the equimultiplices of the first and the third Magnitudes shall be both greater, or both lesser, or equal to the equimultiplices of the second and fourth, according to the meaning of the sixth definition of the fifth Book.

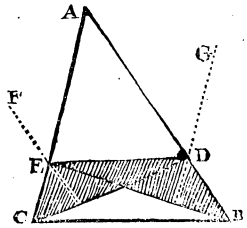
Demonstration. FOr let there be constituted as well the triangles, as the parallelograms, between the same parallels G H and L N, (then [a] they will have one and the same height, for that being constituted between the same parallels, the perpendiculars drawn from the tops, on the bases, shall be equal, as is said in the fourth definition,) and from B L take B I, I K, and K L, each equal to the base B C. [a] 4. def.

In like manner, from F N, take F M and M N, each equal to E F; and from the points A and D, draw the right lines A I, A K, A L, D M, and D N, [b] the triangles A B C, A I B, A K I and A L K, being constituted on equal bases, and between the same parallels, shall be equal to one another; by the same reason, the triangles D E F, D F M, and D M N, shall be also equal to one another. [b] 38. 1.

Now forasmuch as C L conteineth as many parts equal to B C, as the triangle A C L doth contein triangles equal to the triangle A B C, as hath been shewn, C L shall be as much Multiplex of C B, as the triangle A C L is of the triangle A B C. Likewise, forasmuch as E N doth contein as many parts equal to E F, as the triangle D E N conteineth triangles equal to the triangle D E F, as hath been also shewn, E N shall be as much Multiplex of E F, as the triangle D E N is of the triangle D E F. But forasmuch as if the base C L be equal to the base E N, [c] of necessity, the triangle A C L shall be equal to the triangle D E N, being between the same parallels, and therefore if C L be greater than E N, the triangle A C L shall be greater than the triangle D E N, and if lesse, lesse, as appears by the ninth Common Sentence. Therefore the right line C L, and the triangle A C L equimultiplices of the first Magnitude B C, and of the third A B C, will both want of the right line E N, and of the triangle D E N, equimultiplices of the second Magnitude E F, and of the fourth D E F, or shall be both equal, or both greater, the Magnitudes being so taken, as that they answer one another; [d] wherefore there will be greater reason of the first B C, to the second E F, than of the third A B C, to the fourth D E F, to wit, of the triangle to the triangle: Therefore as the base is to the base, so is the triangle to the triangle. Which was proposed. [c] 38. 5. [d] 6. def. 5.

For the second part; Seeing that as [e] the triangle A B C is to the triangle D E F, so the parallelogram C G [f] (which is double to the triangle A B C,) is to the parallelogram E H; (which [g] is double to the triangle D E F,) it is manifest that the parallelogram shall [h] be also to the parallelogram, as the base to the base, which may be proved by the same discourse which we have used in the Demonstration of triangles, if first of all, from the points I, K, and L, you draw parallels to B G, and also from M and N, to F and H: Therefore the triangles, &c. Which was to be demonstrated. [e] 15. 5. [f] 34. 1. [g] 34. 1. [h] 11. 5.

PROP. 2. THEOR. 2.



If to one of the sides B C, *of the triangle* A B C, *be drawn some parallel right line* D E, *it will divide the sides* A B *and* A C, *of the triangle* A B C, *proportionally; and if the sides* A B *and* A C *are divided proportionally, the right line* D E, *joyning the points of the sections shall be parallel to the other side* C B, *of the triangle* A B C.

Demonstration. FOr having drawn the right lines C D and B E, [a] the triangles D E B and D E C, constituted on the base D E, and between the same parallels D E and B C, shall be equal to one another: Therefore as the triangle A D E, is to the triangle D E B; so the same A D E shall be to the triangle D E C. But [b] as A D E is to D E B, so the base A D is to the base D B; (seeing that those triangles are of the same height, as appeareth, if by the point E, be drawn E F, parallel to A B,) and by the same reason, as the triangle A D E, is to the triangle D E C, so the base A E, to the base E C, (A E D and D E C, being between the same parallels A C and D G,) therefore [c] as A D is to D B, so A E is to E C, (seeing that these two reasons are the same to the reason of the triangle A D E, to the triangle D E B, and of the same triangle A D E, to the triangle D E C. Which was proposed.

a) 37. 1.
b) 1. 6.
c) 11. 5.

Secondly, Let the right line D E divide the sides A B and A C proportionally: I say that D E is parallel to the other side C B.

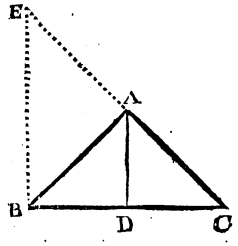
For again, having drawn B E and C D, as the [d] base A D shall be to the base D B, so the triangle A D E, to the triangle D E B; seeing they are both of one height; but as A D is to D B, so is A E to E C, by supposition. Therefore as the triangle A D E, shall be to the triangle D E B, so A E shall be to E C; but again, as the base A E is to the base E C, so the triangle A D E, to the triangle D E C, being of the same height. Therefore [e] as the triangle A D E, is to the triangle D E B, so the same triangle A D E, is to the triangle D E C; therefore [f] the triangles D E B and D E C shall be equal; and therefore seeing they are constituted on the same base, [g] they shall be also between the same parallels: Therefore D E is parallel to B C. Which was proposed. Therefore, If to one of the sides, &c. Which was to be demonstrated.

d) 16. 5.
e) 11. 5.
f) 9. 5.
g) 39. 2.

PROP.

PROP. 3. THEOR. 3.

If an angle B A C, *of a triangle* A B C, *be divided into two equal parts, and that the right line* A D, *which divideth the angle, doth also divide the base* B C, *the segments of the base* B D *and* D C, *shall have the same reason to one another, as the other sides of the triangle* B A *and* A C, *and if the segments of the base* B D *and* D C, *have the same reason to one another, as the other sides of the triangle* B A *and* A C, *the right line* A D, *drawn from the top* A, *to the point of section* D, *divideth the angle* B A C, *of the triangle* A B C, *into two equal parts.*



Demonstration. FOr, Let B E be drawn parallel to D A, meeting C A prolonged towards E. Now they will meet, seeing that the angles C and C B E, are lesse than two right angles, for [a] C and C D A, are lesse than two right angles, and [b] C D A is equal to C B E, the exterior angle to the interior, C and C E B, shall be likewise lesse than two right angles, and the angle E B A [c] shall be equal to its alternate angle E A D, and the angle E, equal to the exterior angle D A C; therefore seeing that the two angles B A D and D A C, are equal by supposition; the angles E B A and E, shall be equal to one another; [d] therefore [e] the lines A B and A E equal to one another.

a) 17. 1.
b) 29. 1.
c) 29. 1.
d) 6. 1.
e) 7. 5.

Therefore as E A shall be to A C, so B A shall be to the same A C: But [f] as E A is to A C, so B D is to D C, seeing that in the triangle B E C, the right line D A is parallel to the side B E. Therefore as B A shall be to A C, so B D shall be to D C. Which was proposed.

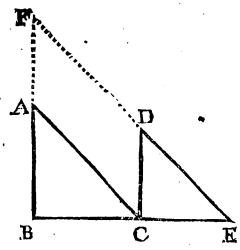
f) 2. 6.

Secondly, Let it be as B A is to A C, so B D to D C; I say that the right line A D doth divide the angle B A C, into two equal parts: For again, draw the right line B E, by the point B, parallel to D A, meeting C A prolonged in the point E; forasmuch therefore as B A is to A C, so B D is to D C, by supposition; but [g] as B D is to D C, so E A is to A C; (seeing that in the triangle B C E, the line A D is parallel to the side B E,) as B A [h] shall be to A C, so E A to the same A C. Therefore [i] B A and A E shall be equal to one another; [k] therefore the angles A B E and E, are equal: [l] Therefore seeing that the angle A B E is equal to its alternate angle B A D, and the angle E equal to its exterior angle D A C, the two angles B A D and D A C, shall be equal to one another. Which was proposed. Therefore, If the angle, &c. Which was to be demonstrated.

g) 2. 6.
h) 11. 5.
i) 9. 5.
k) 5. 1.
l) 29. 1.

PROP.

PROP. 4. THEOR. 4.



Of equiangled triangles ABC *and* DEC, *the sides* AB *and* BC, DC *and* CE, *which are about the equal angles* ABC *and* DCE, *as also* BC *and* CA, CE *and* ED, *which are about the equal angles* ACB *and* E, *and* AB *and* AC, *and* DC *and* DE, *which are about the equal angles* A *and* D, *are proportional, and the sides* AC *and* DE, *which subtend the equal angles* B *and* DCE, AB *and* DC *which subtend the equal angles* ACB *and* E, *as also* BC *and* CE, *which subtend the equal angles* A *and* D, *are homologal, or of the same reason.*

Demonstration COnstitute BC and CE, in a right line, in such sort as the exterior angle DCE may be equal to the interior angle ABC. Likewise the exterior angle ACB equal to the interior DEC; and forasmuch as the two angles ABC and ACB, are lesse than two right angles; but DEC is equal to ACB, the two angles B and E shall
a) 11. c. f. be lesse than two right angles. Wherefore [a] BA and ED, prolonged towards A and D, shall meet, let them then be prolonged, and they shall meet in the point F.

But forasmuch as the exterior angle DCE is equal to the interior and
b) 28. 1. opposite angle ABC, [b] CD and BF shall be parallels; and by the same reason CA and EF shall be also parallels, the exterior angle ACB being equal to the interior DEC. Therefore ACDF is a parallelogram: Therefore
c) 34. 1. [c] AF is equal to CD, and CA to DF. Therefore forasmuch as in the
d) 2. 6. angle BEF, the line AC is parallel to EF, [d] AB shall be to AF, that is to say to DC, its equal, as BC to CE: Therefore alternately, as AB to BC, so DC to CE.

Again, seeing that in the same triangle BEF, the line CD is parallel
c) 22. 5. to the side BF, as BC shall be to CE, so FD, that is to say CA, (which is equal to FD,) to ED. Therefore [c] alternately, as BC to CA, so CE to ED: Therefore seeing that as AB to BC, so DC to CE, and as BC to CA, so CE to ED: In equal reason, AB shall be to CA, as DC to ED. Which was proposed. Therefore the triangles, &c. Which was to be demonstrated.

COROLLARIE. I.

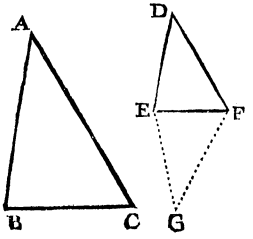
From hence it follows, that if in a triangle there be drawn a line parallel to one of the sides, it will take away a triangle resembling the whole, as in the triangle FBE, *where the line* CD *is drawn parallel to* BF. *I say that the triangle* DCE *is alike to the*
f) 29. 1. *triangle* FBE, *for they are equiangled;* [f] *seeing that the angles* EDC *and* ECD, *are equal to the exterior angles* F *and* B, *each to his correspondent angle, and the angle*

E *common; therefore* [g] *(as hath been demonstrated) they have the sides about the equal angles proportional; therefore according to the Definition they are proportional.* g) 4. 6.

COROLLARIE II.

It follows moreover, that if two triangles equiangled, that is to say, which have the sides proportional, are disposed according to one angle, in such sort, as that two sides of the same reason may be parallels, the other sides will directly meet with one another, as in the triangles before mentioned; where AB *and* DC, *and* AC *and* DE, *being parallels, the other sides* BC *and* CD, *will directly meet with one another, as is manifest by the fourteenth Proposition of the First Book; all the angles at the point* C *being shewn to be equal to the three angles of the triangle* ABC; *that is to say, to two right angles, by the* 32 *Proposition of the First Book; which* EUCLIDE *demonstrateth again in the* 32 *Proposition of this Book.*

PROP. 5. THEOR. 5.



If two triangles ABC, *and* DEF, *have the sides proportional; that is,* AB *to* BC, *as* DE *to* EF, *and as* BC *to* CA, *so* EF *to* FD, *and as* AB *to* AC, *so* DE *to* DF; *those triangles shall be equiangled, and shall have the angles equal, under which the sides of the same reason are subtended.*

Demonstration LEt the angle FEG be made equal to the angle B, and EFG equal to C, the two lines EG and FG, shall meet in the point G; and [a] the other angle G shall be equal to the other angle A. Therefore the triangles ABC and GEF are equiangled: Therefore [b] as AB to BC, so GE to EF; but as AB to BC, so DE to EF, [c] *suppositione*: Therefore [c] as GE to EF, so DE to the same EF. Wherefore GE and DE shall be equal. a) 32. 1. b) 4. 6. c) 11. 5.

Again, seeing that BC is to CA, as EF to FG; but [d] as BC is to CA, so EF to FD, by *suppositione*, as EF shall be to FG; so the same EF shall be to FD, therefore [e] FG and FD, shall be equal: Therefore seeing that the sides GE and GF are equal to the sides DE and DF, each to its correspondent; and the base EF [f] common, the angles G and D shall be equal. Therefore [g] the other angles GEF and GFE shall be equal to the other angles DEF and DFE, each to his correspondent. Wherefore seeing that the angle G is equal to the angle A, also D its equal, shall be equal to A, by the same reason, DEF equal to B, and DFE equal to C. Which was proposed. Therefore, If two triangles, &c. Which was to be demonstrated. d) 4. 6. e) 9. 5. f) 8. 1. g) 4. 1.

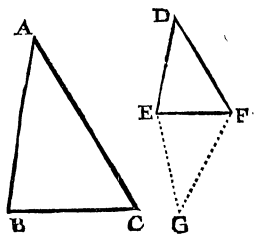
PROP. 6. THEOR. 6.

If two triangles ABC *and* DEF, *have an angle* B, *equal*

to

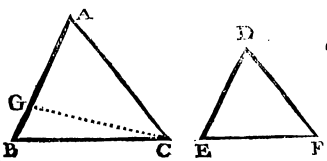
to an angle E, *and the ſides about thoſe equal angles proportional; that is, as* AB *to* BC, *ſo* DE *to* EF, *thoſe triangles ſhall be equiangled, and ſhall have the angles equal, under which the homologal ſides, or ſides of the ſame reaſon are ſubtended; that is, the angle* A *equal to the angle* D, *and the angle* C *to the angle* F.

Demonſtration. FOr, Let the angle FEG be made equal to B, and EFG equal to C, the triangle GEF ſhall be equiangled to the
a) 4. 6. triangle ABC, as is ſhewn in the precedent Propoſition. Therefore[a] as AB to BC, ſo GE to EF. But as AB to BC, ſo DE to EF by ſuppo-
b) 11. 5. ſition: Therefore[b] as DE to EF, ſo GE to the ſame EF; therefore DE and GE ſhall be equal: Where-



9. 5. fore ſeeing that the ſides DE and EF are equal to the ſides GE and EF, and the angles conteined of thoſe ſides alſo equal, (for the angle B, to which the angle GEF is made equal, is equal to DEF by ſuppoſition: therefore the two angles at the point E ſhall be e-
c) 4. 1. qual,) [c] the other angles D, and EFD ſhall be equal to the two others G, and EFG: Therefore ſeeing that the angle G is equal to the angle A, and EFG equal to C, the angles D and EFD, ſhall be alſo equal to the angles A and C; and therefore the triangles ABC and DEF, ſhall be equiangled. Which was propoſed. Therefore, If two triangles, &c. Which was to be demonſtrated.

PROP. 7. THEOR. 7.



If two triangles ABC, *and* DEF, *have an angle* A, *equal to an angle* D, *and about another angle* FACB, *the ſides proportional; that is,* AC *to* BC, *as* DF *to* EF. *But the other angles* B *and* E *together, the one and the other be leſſe or not leſſe than a right angle: The triangles* ABC *and* DEF *ſhall be equiangled, and ſhall have the angles equal, about which the ſides are proportional; that is to ſay, the angles* ACB *and* F, *and* B *and* E.

De-

Demonſtration FOr firſt of all, Let as well B as E, be leſſe than a right angle, that being, if the angles A C B and F are equal, the Propoſition is manifeſt: But if A C B and F are not equal; Let A C B be greater than F, and let A C G be made equal to F: Therefore ſeeing that A is put equal to D, [a] the other angle A G C ſhall be equal to E. Therefore the triangles A G C and D E F ſhall be equiangled. Wherefore [b] as A C ſhall be to C G, ſo D F to E F. But as D F to F E, ſo A C to C B, by ſuppoſition: Therefore [c] as A C to C G, ſo the ſame A C to C B; and therefore [d] C G and C B ſhall be equal; [e] and the angles C B G and C G B ſhall be equal.

a) 32. 1.
b) 4. 6.
c) 11. 5.
d) 9. 5.
e) 5. 1.

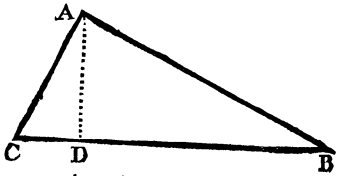
Therefore ſeeing that the angle B is put leſſe than a right angle, C G B ſhall be alſo leſſe than a right angle; and therefore A G C ſhall be greater than a right angle; ſeeing that [f] A G C and C G B are equal to two right angles. Now the angle A G C is ſhewn to be equal to the angle E. Therefore the angle E ſhall be alſo greater than a right angle. But it is leſſe than a right angle by ſuppoſition, which is impoſſible.

f) 13. 1.

Secondly, Let as well the angle B, as the angle E, not be leſſe than a right angle, as before, the angle B ſhall be equal to the angle C G B; and therefore C G B ſhall not be leſſe than a right angle; and ſo the angles C B G and C G B, in the triangle B C G, ſhall not be leſſe than two right angles, but greater, or equal to two right angles, which is abſurd; [g] for they are leſſe than two right angles. Therefore the angles A C B and F are not unequal, but equal; and therefore [h] the other angles B and E are equal. Which was propoſed. Therefore, If two triangles, &c. Which was to be demonſtrated.

g) 17. 1.
h) 32. 1.

PROP. 8. THEOR. 8.

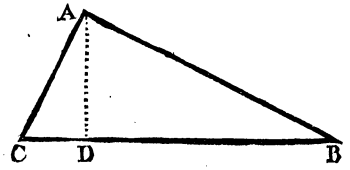


If in a rectangled triangle A B C, *there be drawn a perpendicular line* A D, *from the right angle* B A C, *on the baſe* C B, *the triangles* A D B, *and* A D C, *which are on both ſides of the perpendicular, are alike to the whole, and alike to one another.*

Demonſtration FOr ſeeing that in the triangles A B C and D B A, the angles B A C and A D B are right angles, and A D B are right angles, and the angle B common, [a] the other angles A C B and D A B ſhall be equal: Therefore the triangle D B A is equiangled to the triangle A B C; and [b] therefore they will have the ſides about the equal angles proportional, &c, that is to ſay, as C B ſhall be to B A, ſo B A to B D, and as B A to A C, ſo B D to D A, and as B C to C A, ſo B A to A D, for ſo the ſides having the ſame reaſon, are oppoſite to the equal angles, as by the fourth Propoſition of this Book. Wherefore the triangle A D B is alike to the whole triangle A B C, by the ſame reaſon it may be ſhewn

a) 32. 1.
b) 4. 6.

that the triangle A D C is alike to the same triangle A B C; for the angles B A C and A D C, are right angles, and the angle C common: Therefore [c] the other angles A B C and C A D are equal; [d] wherefore as B C to C A, so C A to C D, and as C A to A B, so C D to D A, and as C B to B A, so C A to A D; for so the sides having the same reason, are opposite to the equal angles, as by the fourth Proposition of this Book.



c) 32. 1.
d) 4. 6.

Even so it will be demonstrated that the triangles A D B and A D C are alike to one another, seeing that the angles A D B and A D C are right angles, and A B D and C A D are shewn to be equal, and also the angles B A D and A C D equal; and [c] therefore as B D to D A, so D A to D C; and as D A to A B, so D C to C A; and as A B to B D, so C A to A D. Therefore, If in a triangle, &c. Which was to be demonstrated.

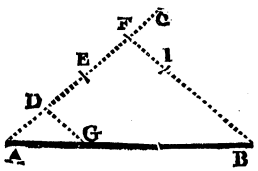
c) 4. 6.

COROLLARIE.

From this Proposition it is evident, that the perpendicular drawn from the right angle on the base, in a rectangle triangle, is a mean proportional between the two segments of the base. Likewise each of the sides which contein the right angle, is a mean proportional between the whole base and the segment of the base, which is adjacent, or toucheth the same side.

For it is demonstrated that as B D *is to* D A, *so* D A *is to* D C; *and therefore* D A *is a mean proportional between* B D *and* C D. *Likewise as* C B *is to* B A, *so* B A *to* B D; *and so* B A *is a mean proportional between* C B *and* B D: *Lastly, it is demonstrated that as* B C *is to* C A, *so* C A *is to* C D; *and therefore* C A *is a mean proportional between* B C *and* C D. *Which was proposed.*

PROP. 9. PROBL. 1.



From a given right line A B, *to take away a required part, as* A G.

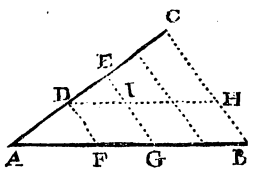
Construction. FRom A draw A C, making any angle whatsoever with A B, as C A B, and from A C take so many equal parts of such a Magnitude as you would that the part to be substracted shall denote; (for A C ought to be so great as need requires,) as in the example proposed, you ought to take three equal parts, A D, D E, and E F, then joyn F B, to which by the point D, draw D G, parallel to F B: I say that A G is the third part of A B required.

Demonstration. FOr seeing in the triangle A B F, D G is parallel to the side F B; [a] as F D to D A, so B G to G A; [b] therefore in compounding, as F A to D A, so B A to G A; but F A is triple to A D, by Construction, therefore B A shall be also triple to A G; and therefore A G shall be the required third part of A B. Therefore, From a given right line, &c. Which was to be done.

a) 2. 6.
b) 18. 5.

PROP.

PROP. 10. PROBL. 2.



To divide a given right line undivided A B, *alike to a given right line* A C, *which is divided, as in* D *and* E.

Construction. JOyne the two given lines in the point A, making any angle whatsoever, as B A C, and joyn the right line B C, then from D and E, draw D F and E G, parallel to B C. I say that A B is divided alike at F and G, as A C is divided in D and E, [a] that is to say, as A D to D E, so A F to F G. Therefore the parts A F and F G are proprtional to A D and D E, and if you draw D H parallel to F B, dividing E G in the point I.

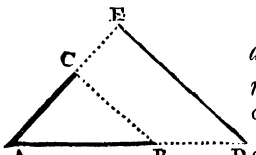
a) 2. 6.

Again, [b] as D E shall be to E C, so D I to I H; that is to say, so F G to G B [c].

b) 2. 6.
c) 34. 1.

Demonstration. FOrasmuch as F G is equal to D I, and G B to I H. Wherefore the parts F G and G B shall be also proportional to D E and E C. And so by the same reason if there were more parts. Therefore, We have divided a line, &c. Which was to be done.

PROP. 11. PROBL. 3.



To two given right lines A B *and* A C, *to find a third proportional line* C E.

Construction. DIspose the right lines A B and A C, so as to make any angle A, to which a third proportional ought to be found; that is to say, as A B is to A C, so A C is to a third.

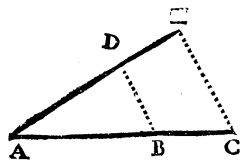
Prolong A B the antecedent line proposed to D, and assume B D, equal to A C the consequent or mean; then having drawn B C; by D draw D E, parallel to B C, meeting A C prolonged in E; I say that C E is the third proportional; that is to say, that as A B is to A C, so A C is to C E.

Demonstration. FOr seeing that in the triangle A D E, B C is parallel to D E, as [a] A B shall be to B D, so A C shall be to C E. But [b] as A B to B D, so the same A B to A C, equal to B D: Therefore as A B to A C, so A C to C E. Which was proposed. Therefore, To two given right lines, &c. Which was to be done.

a) 2. 6.
b) 7. 5.

PROP. 12. PROBL. 4.

To three given right right lines A B, B C, *and* A D, *to find a fourth proportional; that is, as* A B *to* B C, *so* A D *to a fourth* D E.

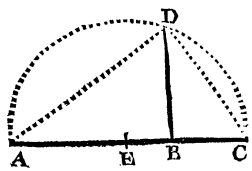


Construction Disposé the two first according to one right line, which let be A C; but that the third A D be so disposed as to make any angle with A B the first as A, then joyne B D, to which by C draw C E parallel thereto, meeting A D prolonged at E; I say that D E is the fourth proportional.

Demonstration Seeing that in the triangle A C E, B D is drawn parallel to the side C E; as [a] A B shall be to B C, so A D to D E: Therefore D E is the fourth proportional. Wherefore to three given right lines, &c. Which was to be done.

a) 2. 6.

PROP. 13. PROBL. 5.



To two given right lines A B *and* B C, *to finde a mean proportional.*

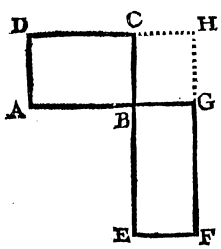
Construction Having disposed the right lines A B and B C, according to one right line A C, divide A C into two equal parts in the point E, and from the point E as a center, and the distance E A or E C, describe the semicircle A D C; then from the point B, in the line A C, raise the perpendicular B D, until it touch the circumference at D. I say that B D is a mean proportional between A B and B C.

Demonstration For having drawn A D and C D, [a] the angle A D C shall be a right angle in the semicircle; and seeing that from the right angle of the rectangle triangle A D C, the perpendicular D B, is drawn on the base A C, the same B D shall be a mean proportion [b] between A B and B C. Therefore, To two right lines, &c. Which was to be done.

a) 31. 1.

b) Cor. 8. 6.

PROP. 14. THEOR. 9.



The sides A B, B G, E B, *and* B C, *which are about the equal angles of equal parallelograms* A C *and* B F, *which have an angle* A B C, *equal to an angle* E B G, *are reciprocal; and the parallelograms which have an angle equal to an*

angle, and the sides which are about the equal angles reciprocal, are equal.

Demonstration For joyn those parallelograms according to the equal angles, in such sort as that A B and B G may make one right line E B and B C [a] shall also make a right line, as is evident; and let D C and F G be prolonged, untill they meet with one another in H.

a) 2. c. f. 13, 14. 1.

Forasmuch then as the parallelograms D B and B F are equal; as [b] D B shall be to B H, so B F to the same B H; but as D B to B H, so the base [c] A B to the base B G; forasmuch as the parallelograms are of the same height. Likewise as B F is to B H, so the base E B to the base B C: Therefore [d] as A B to B G, so E B to B C. Which was proposed.

b) 7. 5.

c) 1. 6.

d) 11. 5.

Contrarily, Let the sides about the said equal angles be reciprocal; to wit, as, A B to B G, so E B to B C: I say that the parallelograms D B and B F are equal.

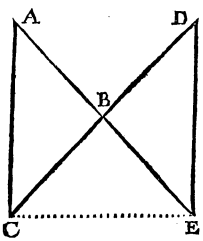
Use the same Construction, seeing that A B is to B G, as E B to B C But [e] as A B to B G, so D B to B H; and as E B to B C, so B F to the same B H. Likewise [f] D B shall be to B H, as B F to the same B H; and therefore [g] the parallelograms D B and B F shall be equal. Therefore, The sides, &c. Which was to be demonstrated.

e) 1. 6.

f) 11. 5.

g) 9. 5.

PROP. 15. THEOR. 10.



The sides which are about the equal angles of the equal triangles A B C *and* D B E, *which have an angle* B *equal to an angle* B, *are reciprocal; that is, as* A B *to* B E, *so* D B *to* B C. *And the triangles which have an angle equal to an angle, and the sides which are about the equal angles reciprocal, are equal.*

Demonstration Joyne those sides according to the equal angles, in such sort as that A B and B E may make one right line, that being D B, B C shall also make one right line; as is said in the precedent Proposition; and let C E be drawn; forasmuch as the triangles A B C and D B E are equal, as A B C shall be to B C E, so D B E shall be to the same B C E; but as the triangle A B C, to the triangle B C E; [a] so the base A B to the base B E, they being of one and the same height: And in like manner, as D B E to B C E, so is the base D B, to the base B C; therefore [b] as A B to B E, so is D B to B C. Which was proposed.

a) 1. 6.

b) 11. 5.

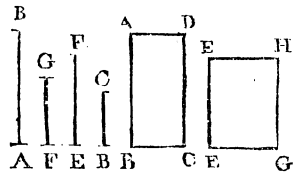
Now let the sides about the equal angles in the point B, be reciprocal; to wit, as A B to B E, so D B to B C; I say that the triangles A B C and D B E, are equal.

Let the same Construction be made, forasmuch as A B is to B E, as D B to B C; but as A B to B E, [c] so the triangle A B C to the triangle B C E;

c) 1. 6.

BCE, and as DB to BC, so the triangle DBE to the same triangle
d) 11. 5. BCE: Therefore d as ABC shall be to BCE, so DBE shall be to the same BCE; and therefore the triangles ABC and DBE shall be equal. Therefore the sides, &c. Which was to be demonstrated.

PROP. 16. THEOR. 11.



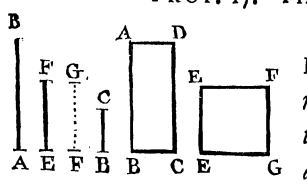
If four right lines AB, FG, EF, *and* BC, *are proportional; the rectangle* AC, *contained under the extreams* AB, *and* BC, *is equal to the rectangle* EG, *contained under the means* EF *and* FG, *and if rectangle contained under the extreams, be equal to the rectangle contained under the means, those four lines shall be proportional.*

Demonstration FOr seeing that the angles B and F are equal, being right angles, and that AB is to FG, as EF to BC, those sides
a) 14. 6. which are about the equal angles B and F, are reciprocal: Therefore a the Parallelograms AC and EG shall be equal. Which was proposed.

Contrarily, Let the rectangles AC and EG be equal; I say that the four lines AB, FG, EF and BC are proportional; to wit, as AB to FG, so EF to BC.

b) 14. 6. For seeing that the rectangles AC and EG are equal; having the angles B and F equal; to wit, right angles, b the sides which are about those angles shall be reciprocal; to wit, as AB to FG, so EF to BC: Therefore, If four right lines, &c. Which was to be demonstrated.

PROP. 17. THEOR. 12.



If three right lines AB, EF, *and* BC, *be proportional: The rectangle* AC *contained under the extreams* AB *and* BC, *is equal to the square* EG, *of the mean* EF *and if the rectangle contained under the extreams, be equal to the square of the mean: those three right lines shall be proportional.*

Demonstration FOr assume the line FG, equal to EF, the four right lines AB, EF, FG, and BC shall be proportional; to wit, as AB to EF, so FG to BC; and the square EG shall be contained under the

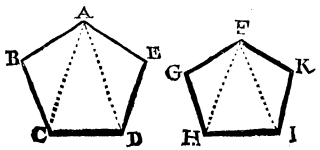
the means EF and FG; seeing that EF and FG are equal: Wherefore a the rectangle AC, contained under the extreams AB and BC, is equal to the square EG; that is to say, to the rectangle contained under the meanes EF and FG. Which was proposed. a) 16. 6.

But let the rectangle AC, and the square EG be equal; I say that as AB is to EF, so EF is to BC; for the rectangles AC and EG being equal; as AB shall be to EF, so FG to BC; but b as FG to BC, so EF c equal to the same FG, is to the same BC: Therefore as AB to EF, so EF to BC. Therefore, If three right lines, &c. Which was to be demonstrated. b) 16. 6. c) 7. 5.

COROLLARIE.

From the latter part of this Theorem it follows, that every right line is a mean proportional between two other, whatsoever right lines, which do contain a rectangle equal to the square of the same lines: For from that, that the right lines AB *and* BC, *do contain a rectangle equal to the square of the right line* EF, *it hath been shewn that as* AB *is to* EF, *so* EF *is to* BC. *Wherefore* EF *is a mean proportional between* AB *and* BC.

PROP. 18. PROBL. 6.



On a given right line CD, *to describe a rectiline figure* ABCDE, *alike, and alike posited to a given rectiline figure* FGHIK.

Construction FRom either of the angles, (as from F,) draw to each of the opposite angles, the right lines FH and FI, which lines do divide the rectiline figure into three triangles FGH, FHI, and FIK; then make the angle DCA equal to the angle IHF, and the angle CDA equal to HIF. Now AC and AD shall meet with one another at A, the angles ACD and ADC being lesse than two right angles, being made equal to FHI and FIH, (a which are lesse than two right angles,) and the other angle CAD shall be equal to the other angle HFI, and the triangle CAD equiangled to the triangle HFI. a) 17. 1. 32. 1.

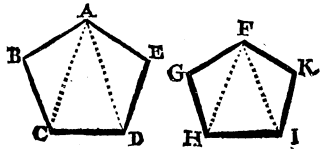
Again, Let the angle ADE be made equal to FIK, and DAE equal to IFK; forasmuch b as the two angles KIF and KFI, are lesse than two right angles, the two angles EAD and EDA, equal unto them, shall be in like manner lesse than two right angles; and therefore AE and DE shall meet in the point E, and the triangle AED shall be equiangled to the triangle FKI by the same reason. b) 17. 1.

Moreover, Make the triangle ABC equiangled to the triangle FGH, by the same reason, and so of the others, if there be any more triangles; I say that the rectiline figure ABCDE is alike, and alike posited to the rectiline figure FGHIK.

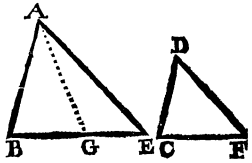
Demonstration FOr seeing that the angle ACD is made equal to FHI, and ACB to FHG, the whole BCD shall be equal to GHI; and by the same reason CDE is equal to HIK, and the other angles

angles equal to the other angles, as appears by the Conſtruction; ſeeing that each part of the one is made equal to each of the parts of the other. Therefore the rectiline [c] figure A B C D E ſhall be equiangled to F G H I K. Now foraſmuch as C D is to D A, as H I to I F, and as A D to D E, ſo I F is to I K, [d] in equal reaſon, C D ſhall be to D E, as H I to I K; therefore the ſides about the equal angles C D E and H I K are proportional; even ſo [e] the ſides about the equal angles E and K are proportional; becauſe of the equiangled triangles A E D and F K I. Again, [f] E A is to A D, as F K to F I, and A D to A C, as F I to F H, and A C to A B, as H F to F G: Therefore [g] in equal reaſon, E A ſhall be to A B, as K F to F G: Wherefore the ſides about the equal angles E A B and K F G are proportional, and ſo of the others; therefore ſeeing that the rectiline figures are equiangled, and have the ſides about the equal angles proportioual, they are alike, and alike deſcribed. Therefore, On a given, &c. Which was to be demonſtrated.

c) 4. 6.
d) 22. 5.
c) 4. 6.
f) 4. 6.
g) 22. 5.



PROP. 19. THEOR. 13.



Like triangles A B E *and* D C F, *are to one another in a duplicate reaſon of their ſides of the ſame reaſon.*

Conſtruction LEt the triangles A B E and D C F, have the angles B and C equal; alſo E and F, and as A B to B E, ſo D C to C F: I ſay they are to one another in a duplicate reaſon of their ſides of like reaſon B E and C F, or A B and D C, or A E and D F; that is to ſay, if you find the third proportional B G, the triangle A B E ſhall be to the triangle D C F, as B E is to B G; ſeeing that by the tenth Definition of the Fifth Book, the reaſon double is ſuch.

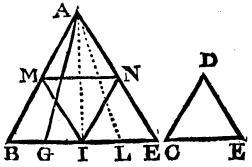
Demonſtration LEt therefore B G be a third proportional, and let A G be joyned to it; ſeeing that as A B is to B E, ſo D C to C F; alternately, as A B ſhall be to D C, ſo B E to C F; but as B E to C F, ſo C F to B G by Conſtruction: Therefore [a] as A B to D C, ſo C F to B G; therefore ſeeing that the triangles A B G and D C F have the ſides about the equal angles B and C reciprocal, [b] they ſhall be equal to one another: Therefore as the triangle A B E ſhallbe to the triangle D C F, ſo [c] the ſame triangle A B E ſhall be to the triangle A B G. But as the triangle A B E is to the triangle A B G, ſo [d] the baſe B E to the baſe B G, they being of the ſame height. Therefore as the triangle A B E is to the triangle D C F, ſo B E is to B G.

Now ſeeing that the three lines B E, C F, and B G, are continually proportional, the reaſon of the firſt B E to the third B G, ſhall be ſaid to be

a) 11. 5.
b) 15. 6.
c) 7. 5.
d) 1. 6.

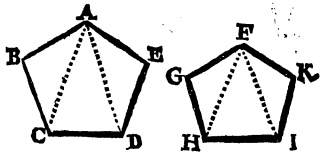
double to the reaſon of B E the firſt, to C F the ſecond; Therefore alſo the triangle A B E ſhall be to the triangle D C F, in reaſon double of the ſide B E to the ſide C F: Therefore, Like triangles, &c. Which was to be demonſtrated.

COROLLARIE.



Whence it follows that if there be three lines proportional, as the firſt is to the third, ſo the triangle deſcribed on the firſt, ſhall be to the like triangle, and alike deſcribed on the ſecond, &c. which is eaſie to be conceived by this laſt figure, where the third proportional line B G *being the quarter part of the baſe* B E; *or elſe* B E *being quadruple of the third proportional* B G, *the triangle* A B E *is alſo quadruple of the triangle* A B G, *as appeares by the eighteenth of the firſt*, G I, I L, *and* L E, *being equal to* B G; *and therefore alſo quadruple of the triangle* D C F, *as is ſhewn, and which is alſo manifeſt, the triangle* A B E *being divided into four other triangles* A M N, M B I, M I N, *and* I N E, *each of which is equal to the triangle* D C F, *which is eaſie to be underſtood by thoſe figures which I thought good here to adde, to facilitate the ſence and meaning of this Propoſition, which had need be well underſtood, to the end the next following and the others after, may be the better comprehended.*

PROP. 20. THEOR. 14.



Like Polygons divide themſelves into an equal number of like triangles, and proportional to their whole, and the Polygons are the one to the other in duplicate reaſon of their ſides of the ſame reaſon.

LEt the Polygons be A B C D E and F G H I K, having the angles B A E and G F K equal, alſo the angles B and G, and ſo following, and having alſo the ſides proportional about thoſe angles; to wit, as A B to B C, ſo F G to G H, and as B C to C D, ſo G H to H I, &c. I ſay firſt of all that thoſe Polygons may be divided into an equal number of like triangles.

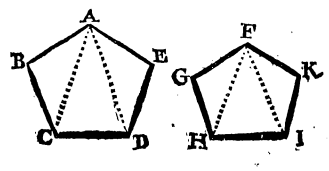
Demonſtration FOr from the angles A and F, draw A C, A D, F H, and F I, to the oppoſite angles C, D, H, and I, they ſhallbe divided into an equal number of triangles; and foraſmuch as the angles B and G are put equal, and the ſides about them proportional; [a] the triangles A B C and F G H ſhall be equiangled, having the angles B A C, G F H, B C A, and G H F, oppoſite to the equal Homologal ſides; and [b] therefore ſhall have the ſides about the equal angles proportioual, and ſo ſhall be alike; by the ſame reaſon, the triangles A E D and F K I ſhall be

a) 6. 6.
b) 4. 6.

be alike; having the angles E A D, K F I, A D E, and F I K equal. And furthermore, [c] seeing that as A C is to C B, so F H to H G, because of the similitude of the triangles A B C and F G H: But as B C to C D, so G H to H I, by supposition, by reason of the similitude of the Polygons [d] in equal reason, as A C to C D, so F H to H I; and forasmuch as the angle B C D is put equal to G H I; but the angle cut off B C A, is shewn to be equal to the angle G H F, the remainder A C D shall be equal to the remainder F H I. Therefore seeing that the triangles A C D and F H I have the angles A C D and F H I equal, and the sides about them proportional; they shall be equiangled, and therefore alike; and so of the others, if there were more.

c) 4. 6.
d) 22. 5.
e) 6. 6.

Secondly, I say that the triangles are proportional to their whole, that is to say, that each of the triangles in one of the Polygons, hath such reason to its triangle correspondent in the other Polygon, as the whole Polygon to the whole Polygon: Forasmuch as the triangles A B C and F G H are alike, they shall be to one another in double reason of their Homologal sides A C and F H, and by the same discourse the triangles A C D and F H I are in reason double of the same sides A C and F H. Wherefore as the triangle A B C is to the triangle F G H, so the triangle A C D shall be to the triangle F H I; seeing that the reason of the one and the other, is the duplicate reason of the side A C to F H. Likewise it may be concluded that the triangle A D E is to the triangle F I K, as the triangle A C D to the triangle F H I, and so following, if there were other triangles: Therefore the triangles of the one of the Polygons are proportional to the triangles of the other; in such sort as the triangles of the one are antecedents of the proportions, and the triangles of the other consequents. Now as one of the antecedents is to one of the consequents, [f] so all the antecedents are to all the consequents: Therefore as each of the triangles of one of the Polygons is to its correspondent triangle of the other Polygon, so the whole Polygon shall be to the whole Polygon: Therefore the triangles shall be proportional to their whole.



f) 12. 5.

Lastly, I say that the Polygons are to one another in a duplicate reason of their sides, of the same reason, that is to say, if to the Homologal sides (for Example A B and F G,) there be found a third proportional, that the Polygon A B C D E shall be to the Polygon F G H I K, as the line A B the first, is to third proportional found. For seeing that as the triangle A B C is to the triangle F G H, so the Polygon A B C D E, to the Poligon F G H I K. But the triangle A B C is to the triangle F G H in a duplicate reason [g] of the side A B to the side F G. Likewise [h] the Polygons shall be the one to the other in duplicate reason of the same sides A B and F G. Therefore, The Polygons, &c. Which was to be demonstrated.

g) 19. 6.
h) 11. 5.

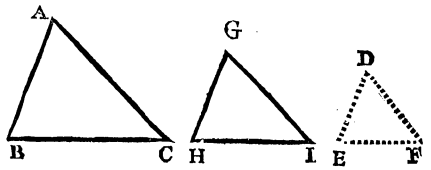
COROLLARIE.

From this Demonstration it is manifest that if there be three lines proportional, as the first shall be to the third, so the Polygon described on the first, shall be to the like

like Polygon, and alike described on the second, or else the Polygon described on the second, shall be to a like Polygon, and alike described on the third, as the first is to the third, which is evident, it having been demonstrated that the Polygon is to the Polygon, as the first A B is to the third proportional

PROP. 21. THEOR. 15.

Rectiline figures A B C *and* D E F, *alike to one and the same rectiline figure* G H I, *are also alike to one another.*



Demonstration FOr seeing that the angles of A B C, are equal to those of [a] G H I, because of their similitude, and those of D E F to those of G H I; for the same cause, [b] the angles of A B C, shall be equal to the angles of D E F.

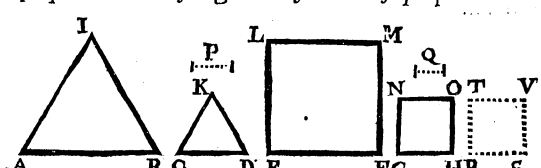
a) 1. def. 6.
b) 1. c. 5.

Again, because of the same similitude, the sides of A B C are proportional to those of G H I; to wit, those which are about the equal angles; also the sides of D E F are proportional to the sides of G H I; for the same cause, therefore [c] the sides of A B C shall be also proportional to the sides of D E F; to wit, those which environ equal angles. Therefore according to the Definition, A B C and D E F shall be alike: Therefore, Rectiline figures, &c. Which was to be demonstrated.

c) 11. 5.

PROP. 22. THEOR. 16.

If four right lines A B, C D, E F, *and* G H, *are proportional, the rectiline figures* A B I, C D K, E M *and* G O, *alike, and alike described on them shall be proportional, and if the rectiline figures alike, and alike described on such right lines are proportional, those right lines shall be also proportional.*



Demonstration FOr [a] to A B and C D, find the third proportional P, and to E F and G H, the third proportional Q: [b] In equal reason, as A B shall be to P, so E F to Q: But as A B is to P, so the

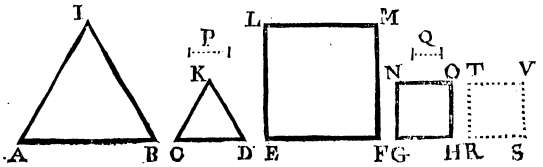
a) 11. 6.
b) 22. 5.

rectiline figure [c] ABI is to the rectiline figure CDK alike, and alike described; by the same reason, as EF is to Q, so the rectiline figure EM; is to the rectiline figure GO; therefore [d] as ABI is to CDK, so EM is to GO. Which was proposed.

c) Co. 20. 6.

d) 11. 5.

Secondly, Let the rectiline figures ABI, CDK, EM, and GO, be proportional; I say that the four right lines AB, CD, EF, and GH, are proportional; to wit, as AB to CD, so EF to GH.



For to the three lines AB, CD, and EF, let [e] there be found a fourth proportional RS, on which describe the rectiline figure RV, alike, and alike posited to the rectiline figure EM; and therefore [f] to the rectiline figure GO; forasmuch as AB is to CD, as EF is to RS: Likewise (as hath been already shewn,) as ABI shall be to CDK, so EM to RV; but as ABI to CDK, so EM to GO: Therefore [g] as EM to RV, so EM to GO; [h] therefore RV and GO are equal, which being alike, and alike posited, of necessity they shall be constituted on the equal right lines RS and GH: Therefore [i] as EF shall be to RS, so EF to GH; but EF is to RS, as AB to CD by supposition: Therefore [k] as AB to CD, so EF to GH: Therefore, If four lines, &c. Which was to be demonstrated.

e) 12. 6.

f) 21. 6.

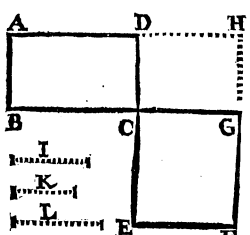
g) 11. 5.

h) 9. 5.

i) 7. 5.

k) 11. 5.

PROP. 23. THEOR. 17.



Equiangled Parallelograms AC *and* CF, *are the one to the other in a reason compounded of that of their sides*, (*that is*, *of* BC *of* CG, *and of* DC *to* CE.

THat is to say, if you take three lines I, K, and L, in such sort as I may be to K, as BC to CG, and K to L, as DC to CE, the Parallelogram AC shall be to the Parallelogram CF, as I to L, which is the reason compounded of I to K, and of K to L, by the fifth Definition of this Book.

Demonstration FOr, Dispose the Parallelograms according to the equal angles in the point C, in such sort as BC and CG may make one only right line, which being so, seeing that the angles BCD and ECG are equal, and EC and CD shall also make one right line, as hath been shewn in the fourteenth Proposition; prolong AD and FG to the point H, making the Parallelogram CH, and assume any right line, as

as I, and to BC, CG, and I, find a fourth proportional K; also to the three right lines DC, CE, and K, the fourth proportional L: Forasmuch as [a] BC is to CG, as AC to CH: But as BC to CG, so I to K, as hath been shewn; likewise [b] as AC to CH, so I to K.

a) 1. 6.

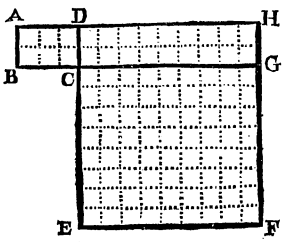
b) 11. 5.

By the same reason it shall be shewn, that as HC is to CF, so K is to L; for as DC to CE, so [c] HC to CF: Therefore seeing that K is put to L, as DC to CE; so likewise HC shall be to CF, as K to L; Therefore [d] in equal reason, AC shall be to CF, as I to L; but the reason of I to L, is compounded of the reasons of BC to CG, and DC to CE, by the fifth Definition of this Book: Therefore the reason of the Parallelogram AC to the Parallelogram CF, is compounded of the same reasons. Therefore, The Parallelograms, &c. Which was to be demonstrated.

c) 1. 6.

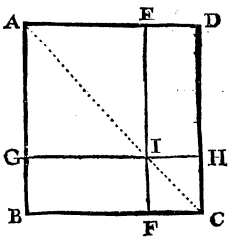
d) 22. 5.

SCHOLIUM.



This Proposition will be easily understood by the annexed figure, thus divided into equal parts, where the sides BC *is* 3, CG 9, *and* CE 8, *and in dividing* GC *by* BC, *to wit*, 9 *by* 3, *the quotient will give* 3, *for the denominator or quantity of the reason of* CG *to* CB; *and dividing also* CE 8 *by* DC 2, *the quotient will give* 4, *for the denominator or quantity of the reason of* CE *to* CD, *which denominators* 4 *and* 3, *multiplyed the one by the other, will produce* 12, *for the reason of the Parallelogram* CF, *to the Parallelogram* CA; *that is to say, that* CF *conteines* CA 12 *times, or else that* AC *is the twelfth part of* CF, *as is manifest, as well by the figure, as by the Demonstration.*

PROP. 24. THEOR. 18.



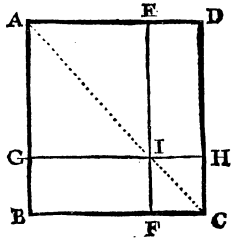
Parallelograms GE *and* FH, *which are about the Diameter* AC, *of every Parallelogram, are alike to their whole* BD, *and alike to one another.*

Demonstration FOr they are equiangled to the whole, forasmuch as the angle A is common to the Parallelogram GE, and to the whole BD; and [a] the exterior angle AEI equal to the interior angle D, and the exterior angle AGI equal to the interior angle B, and EIG the exterior angle, equal to the interior angle IFB; that is to say, to BCD, equal to the said IFB: Wherefore the Parallelogram EG is equiangled to BD, by the same reason FH is equiangled to the same BD.

a) 29. 1.

Now let EG and FH have also the sides above the equal angles proportional to the sides of the whole BD: It shall be thus demonstrated, seeing

seeing [b] that the triangle AGI is equiangled to ABC, and the triangle AEI equiangled to the triangle ADC, as AB shall be to BC, so AG to GI, and so the sides about the equal angles B and G shall be proportional; Again, [c] as BC to CA, so GI to IA: Likewise, as CA to CD, so IA to IE: Therefore [d] in equal reason, as BC to CD, so GI to IE; and therefore the sides about the equal angles BCD and GIE are proportional.

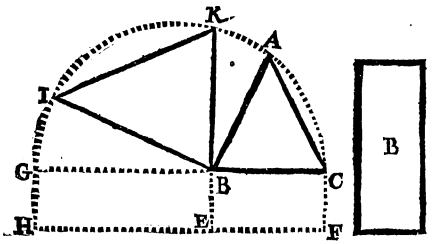


It may be likewise shewn that the sides about the other equal angles are proportional; Therefore by the first Definition of this Book, the Parallelogram EG shall be alike to the whole Parallelogram BD; and by the same reason, the Parallelogram FH alike to the same BD; and therefore also alike to one another: Therefore, Parallelograms, &c. Which was to be demonstrated.

b) 29. 1. or Cor. 4. 6.
c) 4. 6.
d) 22. 5.
c) 21. 6.

PROP. 25. PROBL. 7.

To describe a rectiline figure KIB, *alike to a given rectiline figure* ABC, *the which may be equal to another rectiline figure proposed* B.



Construction ON CB the side of the rectiline figure ABC, (to which another ought to be made alike,) constitute [a] the Parallelogram CE, in any angle whatsoever, equal to the rectiline ABC, and on BE make the Parallelogram BH, equal to B, having the angle EBG equal to BCF, and the lines CB and BG shall make one only right line, ABC, as by the 45th. Proposition of the first Book.

Now [b] finde the mean proportional BK, between CB and BG, on which constitute the rectiline figure BIK, alike, and alike posited to the rectiline I say that BIK is equal to the other rectiline figure B.

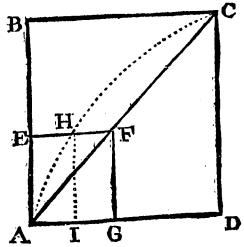
Demonstration FOr seeing that CB, BK, and BG, are proportional, as CB [c] the first is to BG the third, so the rectiline figure on the first CB, to the rectiline figure BIK, on the second BK; alike, and alike described; But [d] as CB to BG, so the Parallelogram CE, to the Parallelogram BH, of the same height: Therefore [e] as CE to BH, so ABC

a) 44, 45. 1.
b) 13. 6.
c) Cor. 19, or 20. 6.
d) 1. 6.
e) 11. 5.

ABC to BIK; But [f] as CE to BH, so ABC to B: Forasmuch as ABC and CE are equal, and BH and B also equal: Therefore [g] as ABC to B, so ABC to BIK: Therefore [h] B and IBK shall be equal: But KIB is alike, and alike posited to ABC, by Construction: Therefore, We have described, &c. Which was to be done.

f) 7. 5.
g) 11. 5.
h) 9. 5.

PROP. 26. THEOR. 19.



If from a Parallelogram BD, *there be cut off a Parallelogram alike to the whole, and alike posited* EG, *having an angle* EAG *common with the whole; it doth consist about one and the same diameter* AC, *with the whole* BD.

Demonstration DRaw AF and CF, which lines if they make one right line, it is evident that EG shall be about one and the same diameter with BD. But if they make not a right line, draw AC, the diameter of the whole Parallelogram BD, cutting the side EF in the point H, by which point draw HI, parallel to FG. Now forasmuch as the Parallelograms BD and EI are about one and the same diameter AHC, [a] they shall be alike, and alike posited; Therefore [b] as BA to AD, so EA to AI; but as BA to AD, so EA is to AG; forasmuch as BD and EG are alike by supposition, and alike posited: Therefore [c] as EA to AI, so EA to AG; and therefore AI and AG shall be equal, the part to the whole, which is absurd: Therefore AF and FC do make one only right line; that is to say, the Parallelograms BD and EF do consist about one and the same diameter. Which was to be demonstrated.

Now if it should be said that the right line AHC doth cut the side FG, there would happen the same absurdity.

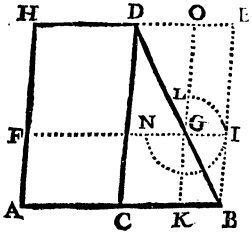
a) 24. 6.
b) 1. def. 6.
c) 11. 5.

PROP. 27. THEOR. 20.

Of all Parallelograms applyed according to one and the same right line AB, *and being deficient by some Parallelogram figures, alike, and alike posited to that which is described on the halfe* BC; *the greatest is that which is applyed on the other half, and alike to the deficient.*

DIvide the line AB into two equal parts in C, and on the half CB, constitute any Parallelogram whatsoever, as CE, whose diameter is BD: Therefore if the Parallelogram ABEH be finished, the Parallelogram AD constituted on the half AC, shall be applyed according to AB, and wanting of the Parallelogram CE, and alike to the deficient I say that the Parallelogram AD applyed to the half AC, and being defici-

deficient of the Parallelogram C E, is the greateſt of all thoſe which are applyed according to the line A B, and deficient by like Parallelograms, and alike poſited to C E.

Demonſtration FOr having aſſumed the point contingent G, in the diameter B D, by which point having drawn the right lines F G I and K G parallel to A B and B E, the Parallelogram F K ſhall be applyed
a) 24. 6. according to A B, and wanting by the Parallelogram K I, [a] which is alike to C E, and alike poſited; being about one
b) 43. 7. 

and the ſame diameter with C E. But [b] forc) 36. 1. much as the complements C G and G E are equal, if you adde the common Parallelogram K I, C I and K E ſhall be equal: But [c] C I is equal to C F, being on equal baſes A C and C B: Therefore C F and K E ſhall be alſo equal, and having added the common part C G, the Parallelogram A G, and the Gnomon L N ſhall be equal.

Therefore ſeeing that C E is greater than the Gnomon L N, (for it conteineth the Parallelogram G D more than the
d) 36. 1. Gnomon,) A D [d] equal to C E becauſe of the equal baſes A C and C B, ſhall be alſo greater than A G, by the ſame Parallelogram D G; In like manner, it may be ſhewn that A D is greater than all other Parallelograms which are applyed according to A B, in ſuch ſort as the point G be between the points B and D; that is to ſay, which imployeth or occupieth a line greater than the half A C; but have leſſer altitude than A D, provided that the deficients be alike to C E.

SCHOLIUM.

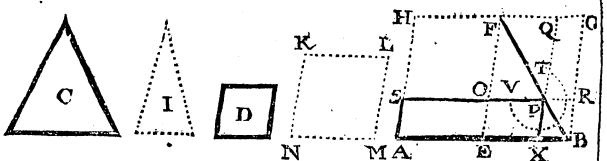
Otherwiſe it may be demonſtrated that A D *is greater than* A G, *in this manner;*
e) 36. 1. [e] *The Parallelograms* F D *and* D I *are equal, ſeeing that the baſes* H D *and* D E *are equal: But* D I *is greater than* G E, *that is to ſay, then the complement* C G, *(which is equal to* G E,*) by the Parallelogram* D G: *Therefore* F D *ſhall be alſo greater than* C G, *by the ſame parallelogram* D G; *and therefore adding the common* C F, A D *ſhall be greater than* A G, *by the ſame Parallelogram* D G.

PROP. 28. PROBL. 8.

To a given right line A B, *to apply a parallelagram* A P, *equal to a rectilire figure given* C, *wanting by a parallelogram figure* P B, *which is alike to another given parallelogram* D, *but it behoveth that the given rectiline figure to which an equal rectiline figure ought to be applyed, be not greater than that which is applyed to the half* A E *of the given line, the deficients being alike to that which is applyed on the halfe, and to that which ought to be deficient by a like parallelogram.*

Con-

Conſtruction THe line A B being divided into two equal parts in the point E, on the half E B [a] deſcribe the Parallelogram E G alike, and
a) 18. 6. alike poſited to D, and ſo finiſh the whole Parallelogram B H; If then A F be equal to C, being applyed to A B, deficient by the Parallelogram E G, which is made alike to D, you have your deſire. But if A F be greater than C, (for it ought not to be leſſe, ſeeing that by the precedent Propoſition it is the greateſt of all the applyed, the Deficiences being alike, you cannot apply any one to A B, equal to C, but they ſhall all be leſſe; Wherefore *EUCLIDE* hath added [but it behoveth that the given Rectiline, &c.] E G equal to the ſame, ſhall be alſo greater than C; let it then be greater by I, and conſtitute K M alike, and alike poſited to D, or to E G, but equal to the exceſſe I, in ſuch ſort as E G be equal to C, and to K M together; and therefore greater than K M: Therefore ſeeing that as G F to F E, ſo N K to K L, becauſe of their reſemblance, the ſides G F and F E ſhall be alſo greater than N K and K L; for if they were equal to them, or leſſe, E G ſhould be likewiſe equal to N L, or leſſe, as appears.



Having therefore cut off F O and F Q equal to K N and K L, and having made the Parallelogram O Q, it ſhall be equal to K M; and alike, and alike poſited thereto; and therefore to E G, and [b] alſo
b) 26. 6. about one and the ſame diameter with E G, which diameter is B F; and having prolonged Q P and O P, the Parallelogram A P ſhall be applyed to the line A B deficient of P B, [c] which is alike, and alike
c) 24. 6. poſited to E G; and therefore alſo to D: I ſay that A P is equal to the rectiline C.

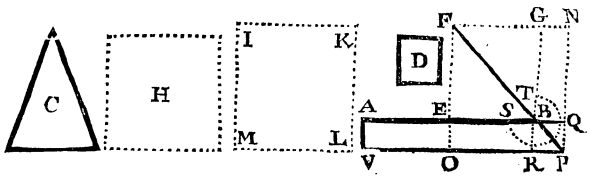
Demonſtration FOr [d] ſeeing that P G is equal to P E, the complement to the
d) 36. 1. complement, if you adde the common P B, the Parallelogram B Q ſhall be equal to E R; that is to ſay, to E S, which is equal to E R, the baſes A E and E B being equal: Wherefore if to the equal parallelogram A O and B Q, you adde the common part E P, A P ſhall be equal to the Gnomon T V: But the Gnomon T V is equal to the rectiline C; (for ſeeing that the Parallelogram E G is equal to C and N L together, if you take away the equal Q O and L N, the Gnomon T V ſhall remaine equal to C:) Therefore A P ſhall be alſo equal to the ſame C: Therefore, To a given right line A B, &c. Which was to be done.

PROP. 29. PROBL. 9.

To a given right line A B, *to apply a parallelogram* A P, *equal to a given rectiline figure* C, *exceeding the ſame given line* A B, *by a parallelogram* R Q, *alike to another parallelogram given* D,

Conſtruction DIvide A B into two equal parts in E, and on the halfe E B, deſcribe the Parallelogram E G alike to D, and alike poſited, then [a] make the ſquare H equal to the rectiline figure C, and Parallelogram E G, [b] to which make the Parallelogram I L equal, but alike to E G, and alike poſited: Wherefore I L ſhall be greater than E G, ſeeing that it is equal to the ſquare H, which is made equal to C, and to E G together; Therefore ſeeing that becauſe of the ſimilitude of M K and E G, as M I to I K, ſo E F to F G; the ſides M I and I K ſhall be alſo greater than E F and F G; for if I M and I K were equal to E F and F G, or leſſe, M K ſhould be equal to E G, or leſſe, as is manifeſt: Therefore having prolonged F E and F G, in ſuch ſort, as F O and F N may be equal to I M and I K, and having finiſhed the Parallelogram O N, the ſame ſhall be alike, and alike poſited to E G; ſeeing that it is equal to M K, and alike, and alike poſited: Wherefore [c] O N and E G are conſtituted about one and the ſame diameter F P; having prolonged A B and G B to Q and R, and P O until they meet with A V, parallel to F O, to the point V the Parallelogram A P ſhall be applyed to the line A B, exceeding it by the Parallelogram Q R, which [d] is alike to E G, and therefore to D.

a) 14. 2.
b) 25. 1.
c) 26. 6.
d) 24. 6.



Demonſtration I ſay then that A P is equal to the rectiline figure C, for [e] ſeeing that A O and E R are equal; and [f] E R is equal to the complement B N, A O ſhall be alſo equal to B N: Therefore adding the common part O Q, A P ſhall be equal to the Gnomon S T; but the ſaid Gnomon is equal to the rectiline C; (for ſeeing that M K, that is to ſay O N, is equal to C and E G together, if you take away the common part E G, there will remain the Gnomon S T, equal to C:) Therefore A P ſhall be alſo equal to the rectiline figure C: Therefore, To a given right line, &c. Which was to be done.

e) 36. 1.
f) 43. 1.

PROP. 30. PROBL. 10.

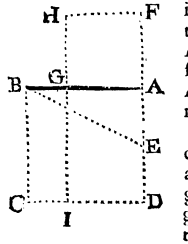
To divide a terminated right line propoſed A B, *according to extream and mean reaſon in* G.

Conſtruction HAving deſcribed on the propoſed right line A B, the ſquare A C, [a] to the ſide D A apply the rectangle D H, equal to the ſquare A C, exceeding D A by the Parallelogram A H, alike to the ſame ſquare, in ſuch ſort as A H ſhall be likewiſe a ſquare, conſidering that a ſquare only is alike to a ſquare: But let I H divide A B in the point G, I ſay that A B is divided according to extream and mean reaſon.

a) 29. 6.

Demon-

Demonſtration SEeing that the rectiline figure D H, and the ſquare A C, are equal, if you take away the common part A I, F G and G C will remain equal, the which having the angles A G H and B G I equal, to wit, right angles, [b] the ſide which are about thoſe equal angles ſhall be reciprocal, to wit I G, that is to ſay, A B its equal, is to G H, that is to ſay, to A G, equal to G H, as A G is to G B: Wherefore the whole A B being to the greateſt ſegment A G, as the ſaid ſegment A G is to the leſſer ſegment G B; A B is divided according to extream and mean reaſon, by the Definition.



Otherwiſe, we may demonſtrate that A B is divided in the point G, according to extream and mean reaſon; ſeeing that there are three lines given, A B, A G, and G B, and that the rectangle G C conteined under the firſt A B, and the third G B, is equal to the ſquare of the mean A G; [c] thoſe lines ſhall be proportional, to wit, as A B the firſt is to A G the ſecond, ſo A G the ſecond ſhall be to G B the third; Therefore by the Definition, A B is divided in the point G, according to extream and mean reaſon.

Yet otherwiſe, to finiſh the whole Problem; Let [d] A B be divided in the point G, in ſuch ſort as the rectangle conteined under the whole A B, and the ſegment G B be equal to the ſquare of the other ſegment A C: I ſay that A B is divided at G, according to mean and extream reaſon: For [e] again, as before, A B, A G, and G B, ſhall be proportional, and ſo the Propoſition manifeſt: Therefore, We have divided, &c. Which was to be done.

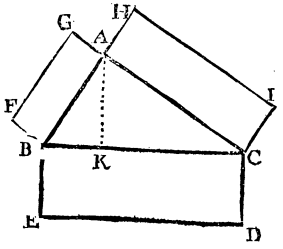
b) 14. 6.

c) 17. 6.

d) 11. 2.

e) 17. 6.

PROP. 31. THEOR. 21.



In rectangled triangles, as A B C, *the figure* B D, *deſcribed on the ſide* B C, *which ſubtendeth the right angle* B A C, *is equal to the two figures* A F *and* A I, *alike and alike deſcribed on the ſides* B A *and* A C, *which contein the right angle* B A C.

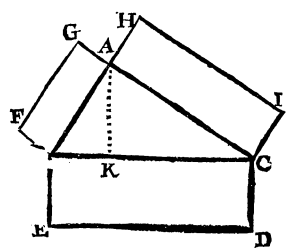
Demonſtration FOr having drawn the perpendicular A K, from the point A on B C, [a] as B C ſhall be to C A, ſo C A ſhall be to C K: Wherefore as B C the firſt, to C K the third, ſo the figure B D deſcribed on the firſt, ſhall be to the figure C H, alike, and alike deſcribed on the ſecond, [b] and by converſion of reaſon,

a) Cor. 8. 6.

b) Cor. 19, or 20. 6.

as C K to B C, so the figure C H to the figure B D: In like manner we shall shew that as B K is to B C, so the figure B G is to the figure B D; seeing that the three lines B C, B A, and B K are proportional.

Therefore seeing that as C K the first magnitude, is to B C the second, so C H the third, is to B D the fourth; also as B K the fifth magnitude
c) 24. 5. is to B C the second, so B G the sixth, is to B D the fourth: c As the first C K, with the fifth B K, shall be to the second B C; so the third C H, with the sixth B G, shall be to B D the fourth; but C K the first, and B K the fifth together, are equal to the second B C: Therefore the third C H, and the sixth B G together, shall be likewise equal to the
d) 8. 6. fourth B D. Which was proposed.



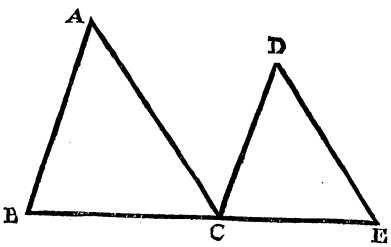
Otherwise, d seeing that the triangle K A C is alike to the triangle A B C, and that the sides B C and C A are Homologal; (for as B C to C A, in the triangle A B C, so A C to C K, in the triangle A C K;) e the triangle K A C
e) 19. 6. shall be to the triangle A B C in duplicate reason of C A to B C:
f) 19, 20. 6. But f the figure C H is to the figure B D, also in a duplicate
g) 11. 5. reason of C A to B C; Therefore g the triangle K A C shall be to the triangle A B C, as the figure C H to the figure B D, by the same reason it may be shewn that as the triangle K A B is to the triangle A B C, so the figure B G is to the figure B D.

Again, forasmuch then as K A C the first magnitude, is to A B C the second, so C H the third is to B D the fourth. Also as K B A the fifth, to
h) 24. 5. A B C the second, so B G the sixth, to B D the fourth; h as K A C the first, with K B A the fifth, shall be to the second A B C; so C H the third, and B G the sixth together, shall be to the fourth B D. But K A C and K B A, the first and fifth together, are equal to the second A B C: Therefore C H and B G, the third and sixth together, shall be likewise equal to B D the fourth. Which was proposed.

Otherwise, As the square of A C the first magnitude, is to the square of
i) 19, 20. 5. B C the second magnitude, so the figure C H the third magnitude, to the figure B D the fourth magnitude; seeing i that they are all in duplicate reason of their sides A C and B C; Likewise, as the square of A B the fifth magnitude, is to the square of B C the second magnitude, so the figure B G the sixth magnitude, is to the figure B D the fourth magni-
k) 24. 5. tude: Therefore k as the first magnitude with the fifth, to wit, the square of A C, with the square of A B, shall be to the second, that is to say, to the square of B C, so the third magnitude with the sixth, to wit, the figure C H, with the figure B G, is to the fourth, to wit, to the figure B D: But the squares of A C and A B together, are equal to the square of B C: Therefore the figures C H and B G together, shall be also equal to the figure B D. Which was proposed: Therefore, In rectangled triangles, &c. Which was to be demonstrated.

PROP.

PROP. 32. THEOR. 22.



If two triangles A B C *and* D C E, *which have two sides* A B *and* A C *proportional to two sides* D C *and* D E, *are disposed (compounded) according to an angle* A C D, *in such sort as their Homologal sides* A B *and* D C, *and* A C *and* D E *may be also parallels, the other sides* B C *and* C E *of those triangles shall be placed in one and the same right line.*

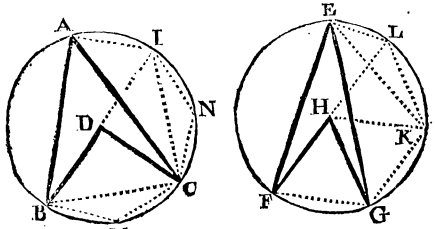
Demonstration FOr seeing that A B and D C are Parallels, a the angle A shall be equal to the alternate angle A C D, and by the
a) 29. 1. same reason, the angle D shall be equal to the same angle A C D: And therefore A and D shall be equal to one another; forasmuch then as the triangles A B C and D C E have the sides about the equal angles A and D proportional, they shall be equiangled, and shall have the angles B and D C E equal: Therefore adding the equal angles A and A C D, the two angles B and A shall be equal to the two angles D C E and A C D, that is to say, to the whole angle A C E: Again, adding the common angle A C B, the two angles A C E and A C B shall be equal to the three angles of the triangle A B C: But b these three angles are equal to two
b) 32. 1. right angles: Therefore the two angles A C E and A C B shall be equal to two right angles; and c therefore B C and C E shall make one right
c) 14. 1. line: Therefore, If two triangles, &c. Which was to be demonstrated.

PROP. 33. THEOR. 23.

In equal circles A B C *and* E F G, *the angles have the same reason to one another as the circumferenes* B C *and* F G, *on which they insist, whether they insist being constituted at the Centers* D *and* H, *or at the circumferences at* A *and* E; *And also the Sectors* B D C *and* F H G *are the same to one another, forasmuch as they are constituted at the Center.*

Demon-

Demonſtration FOr having drawn the right lines BC and FG, [a] apply in the circles the line CI, equal to BC, and GK and KL, each equal to FG, and draw the right lines ID, KH, and LH: Then foraſmuch as BC and CI are equal, [b] the arches BMC and CNI ſhall be likewiſe equal; and therefore [c] the angles BDC and CDI are equal; by the ſame reaſon, the arches FG, GK, and KL, and the angles FHG, GHK, and KHL, are equal: Therefore the angle BDI ſhall be as much Multiplex of the angle BDC, as the arch BCI is Multiplex of the arch BC; by the ſame reaſon, the angle FHL ſhall be as much Multiplex of the angle FHG, as the arch FGL is of the arch FG; ſeeing that the angles BDI and FHL are divided each into as many equal parts as the arches BI and FL, on which they inſiſt: Therefore, if the arch BCI be equal to the arch FGKL, [d] of neceſſity the angle BDI ſhall be equal to the angle FHL, and if the arch be greater than the arch, the angle ſhall be greater than the angle, and if leſſer, leſſer: Wherefore the arch BCI, and the angle BDI, equimultiplices of the firſt and third Magnitudes,



BC and BDC ſhall be together either greater, equal, or leſſer than the arch FGKL, and the angle FHL equimultiplices of the ſecond Magnitude FG, and of the fourth FHG; if they be taken ſo as to anſwer one another: Wherefore [e] as the arch BC the firſt Magnitude, is to the arch FG the ſecond Magnitude, ſo the angle BDC the third Magnitude, to the angle FHG the fourth Magnitude, the ſame may be ſhewn of the angles at the circumference.

Secondly, I ſay that as the circumference is to the circumference, ſo the ſector is to the ſector: Conſtitute in the ſegments BC and CI, the angles BMC and CNI, which [f] ſhall be equal, inſiſting on the equal arches BAC and CBAI: Wherefore the ſegments BMC and CNI ſhall be alike; [g] and therefore equal to one another, being conſtituted on the equal right lines BC and CI: Therefore adding the triangles BDC and CDI, [h] which are likewiſe equal, the ſectors BDC and CDI ſhall be made equal; wherefore the ſector BDI ſhall be as much Multiplex of the ſector BDC, as the arch BCI is of the arch BC: Likewiſe the ſector FHL ſhall be ſhewn to be as much Multiplex of the ſector FHG, as the arch FGKL is of the arch FG; but foraſmuch as if the arch BCI be equal to the arch FGKL, alſo the ſector BDI is equal to the ſector FHL, (as hath been ſhewn in the ſectors BDC and CDI,) and if greater, greater, and if leſſe, leſſe; therefore the arch BCI and the

a) 1. 4.
b) 28. 1.
c) 27. 1.
d) 27. 3.
e) 6. def. 5.
f) 27. 3.
g) 24. 3.
h) 4. 1.

the Sector BDI, equimultiplices of the firſt Magnitude BC, and of the third Magnitude BDC, ſhall be deficient together, of the arch FGKL, and of the ſector FHL, equimultiplices of the ſecond Magnitude FG, and of the fourth FHG, or together ſhall be equal, or ſhall exceed, if they be taken ſo as they anſwer to one another: Therefore [i] as the arch BC the firſt Magnitude, is to the arch FG, the ſecond Magnitude; ſo the ſector BDC, the third Magnitude, to the ſector FHG the fourth Magnitude: Therefore, If in equal Circles, &c. Which was to be demonſtrated.

i) 6. def. 6.

COROLLARIE. I.

It is manifeſt from this, that the Sector is to the Sector, as the angle is to the angle; for the reaſon of the one to the other, to wit, of the angle BDC, *to the angle* FHG, *and of the Sector* BDC *to the Sector* FHG, *is the ſame as the reaſon of the arch* BC *to the arch* FG: *Wherefore* [k] *they ſhall be the ſame to one another.*

k) 11 5.

COROLLARIE. II.

It is likewiſe manifeſt, that as the angle at the Center is to four right angles, [l] *ſo the arch which ſubtends the ſame angle, is to the whole Circumference; and contrarily, as four right angles are to the angle which is at the Center, ſo the whole Circumference is to the arch which ſubtends the ſaid angle.*

l) 33. 6.

For as the angle at the Center is to a right angle at the Center, ſo the arch which ſubtends the ſame angle, is to a quarter of the Circumference of the Circle, or to the Quadrant which ſubtends the ſaid right angle: Wherefore as the angle at the Center, ſhall be [m] *to the quadruple of the right angle; that is to ſay, to four right angles; ſo the arch ſubtended of the ſame angle, ſhall be to the quadruple of the Quadrant, that is to ſay, to the whole Circumference, by what hath been demonſtrated in the two and twentieth Propoſition of the Fifth Book. Which was firſt propoſed.*

m) 33. 6.

Foraſmuch then as the angle at the Center is to four right angles, ſo the arch which ſubtends that angle, is to the whole Circumference; by converſion of reaſon, as four right angles to the angle at the Center, ſo the whole Circumference ſhall be to the arch which ſubtends the ſaid angle at the Center. Which was in the ſecond place propoſed.

But it may alſo be thus demonſtrated, [n] *ſeeing that as the right angle at the Center is to the angle at the Center, ſo the Quadrant or fourth part of the Circumference, which ſubtends the ſame right angle, is to the arch which ſubtends the ſame angle at the Center not being a right angle. Likewiſe as the quadruple of the right angle, that is to ſay, four right angles, ſhall be to the angle at the Center, by what hath been ſhewn in the two and twentieth Propoſition of the Fifth Book; ſo the quadruple of the Quadrant, that is to ſay, the whole Circumference ſhall be to the arch which ſubtends the ſame angle. Which was propoſed.*

n) 33. 6.

The End of the Sixth Element of EUCLIDE.

THE SEVENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.



Hitherto, *Euclide* hath treated of the first part of *Geometry*, to wit, of that which concerneth *Planes*; there remaines the other part which treats of *Solids*, but first of all it hath been found necessary to treat of *Lines commensurable* and *incommensurable*, for as much as the knowledge of these *Lines* is requisite for the explication and demonstration of the proprieties of divers *Solid Bodies*, and principally of those which are named *Regulars*; for that, without the knowledge of those *Lines* the tract of *Solids* would be imperfect; Moreover, without those *Lines*, divers sides, as well *Planes* as *Solids* (if you will reduce *Geometry* to use and practice) cannot be expressed or understood, for divers of the sides are those *Lines* which the Greeks call ἄλογοι, that is to say, *irrational*, if they be not irrational, they are incommensurable in length amongst themselves, and therefore are not expressed by *Numbers*.

Now seeing that the explication and understanding of those *Lines*, is as it were wrapped up, and joyned with *Numbers*, in such sort as without them they cannot be known, it is found needful in the first place to treat of *Numbers*; Where-

fore, in this ſeventh, and the two following Books, *Euclide* unfoldeth the affections and proprieties of *Numbers*, in as much as they may ſerve in *Geometry*, to the end that in the Tenth Book he may make the Demonſtrations of *Lines commenſurable* and *incommenſurable*, more amply, and with greater facility.

DEFINITIONS.

1 UNITY *is that according to which every thing, of thoſe which are, is ſaid to be one.*

BEginning then, according to his Cuſtome, by the Principles, he defineth firſt of all Unity: For according to unity we uſe to call one body, one body: one animal, one animal: one ſtone, one ſtone: and ſo of other things. And finally, Unity receiveth no diviſion in Numbers, even as a point in Magnitudes.

2 NUMBER, *is a Multitude compoſed of Unities.*

THat is to ſay, if you take as many unities as you pleaſe together, the collection of them ſhall be called number, from whence it is manifeſt, that in every number there are ſo many parts, as there are unites that make it: So as unity is part of each number named by the ſame number of which it is a part, as the number 8, compounded of 8 unites, divideth it ſelf into ſo many parts, to wit in 8 unites, each of them being ſaid to be the 8th. part of the number 8, &c. from whence it follows, that all the numbers are commenſurable amongſt themſelves, ſeeing they are all meaſured by one and the ſame meaſure, to wit, by unity, as is before ſaid, which cannot agree with all Magnitudes, as ſhall be ſhewn in the Tenth Book.

3 *A Number is a part of another number; the leſſer of the greater, when the leſſer doth meaſure the greater.*

THis Definition reſembles that of the 5th. Book, by which *Euclide* defineth the part of a quantity continued, for as there, ſo here, he defineth the aliquot part, which is ſaid exactly to meaſure his whole. As 6 is ſaid to be part of 18, foraſmuch as 6 meaſureth 18 exactly, and leaves no remainder, in like manner, each of theſe nnmbers 3, 4, 6, 8, is a part of 576. Seeing that each of them doe meaſure it exactly, for every part taketh his name from the number by which it meaſureth another. As 6 is ſaid to be the ſeventh part of 42, becauſe 6 meaſureth 42 by 7, that is to ſay, is conteined in 42, 7 times exactly, and ſo of the reſt.

4 *But*

4 *But a number is ſaid to be parts of another greater number, when the leſſer doth not meaſure the greater.*

AS 4 is not ſaid to be part of 6, for as much as 4 the leſſer number doth not meaſure 6 preciſely, but ſhall be called the two parts of 6, to wit $\frac{2}{3}$, and 5 ſhall be called parts of 18, to wit $\frac{5}{18}$, for as much as unity which is their common meaſure, is conteined 5 times in 5, and 18 times in 18.

5 *A number is ſaid to be Multiplex of another leſſer number, when the leſſer doth meaſure the greater.*

EVen as a number which doth preciſely meaſure a greater number, is ſaid to be part of that greater number; ſo alſo the greater number which is meaſured by the leſſer number, is ſaid to be Multiplex of the leſſer number, and ſo alſo, as a number which doth not exactly meaſure a greater number, is not ſaid to be a part but parts of the greater number, alſo the greater ſhall not be ſaid to be Multiplex of the leſſer number by which it is not meaſured, as 4 is a part of 24. Seeing that 4 doth meaſure 24 preciſely, and in like manner, 24 is ſaid to be Multiplex of 4, for that it is meaſured of 4, and 30 is Multiplex of 6, foraſmuch as 6 doth meaſure 30, &c.

6 *An even number is that which may be divided in two equal parts.*

AS all the following numbers 4, 10, 40, 100, 1000, are ſaid to be even numbers, foraſmuch as each of them may be divided in two equal parts, whereof their halves are 2, 5, 20, 50, 500.

7 *But an odde number is that which cannot be divided into two equal parts, or which differs from an even number by an unite.*

AS all theſe numbers 5, 11, 15, 37, 101, are called odde numbers, becauſe they cannot be divided in two equal parts, or becauſe they differ by unity from the even numbers 4, 10, 14, 36, 100, &c.

8 *A number evenly even, is that which an even number doth meaſure by an even number.*

AS 32 is ſaid to be evenly even, becauſe 8 an even number doth meaſure it by 4, which is alſo an even number, 24 is alſo ſaid to be evenly even; ſeeing that 4 an even number doth meaſure it by 6, alſo an even number, &c.

9 *A number evenly odde, is that which an even number doth meaſure by an odde number.*

THat is to ſay, that if an even number meaſure an even number by an odde number, the nnmber meaſured ſhall be called evenly odde. As

30, which 2 an even number doth measure by 15 an odde number, and 6 an even number doth measure the same 30 by 5, an odde number, &c.

10 *But a number oddely odde, is that which an odde number doth measure by an odde number.*

AS 45, which is measured of 9 an odde number, by 5 also an odde number, is said to be a number oddely odde, and also 15 which is measured of 5 by 3, and these 9, 21, 25, 27, 33, 35, &c. are said to be oddely odde nnmbers.

11 *A Prime or First number, is that which only unity doth measure.*

THat is to say, that if a number cannot be measured by any other number, but onely by unity, so as it cannot be said to be evenly even, or evenly odde, or unevenly odde; that number shall be called a Prime number, as these numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, &c. for onely unity measureth them.

12 *Numbers, Primes to one another, are those which have onely unity for their common measure.*

THat is to say, that if two or more numbers are proposed, and that there cannot be found any number which doth precisely measure each of them, save onely unity, those numbers are said to be Primes amongst themselves, as 15 and 8, or 3 and 5, are Primes amongst themselves, for as much as there is onely unity that may be a common measure to them.

13 *A Compound number, is that which some number may measure.*

THe Geometricians do call that a Compound number which some other number doth measure besides unity, as 15, which is measured by 5 and 3. Now it is manifest that all even numbers excepting onely the Binary, are Compound numbers, being all measured by 2, from whence it follows, that all the Prime numbers save onely the Binary are odde, the Binary being the first of even numbers.

14 *But numbers compounded amongst themselves, are those which are measured by some number, as their common measure.*

TWo or more numbers, which are measured by some number besides unity, as by their common measure, are said to be compounded amongst themselves, although that each be not compounded of himself; As 15 and 24, are compounded amongst themselves, for as much as 3 doth measure both, as their common measure: also 7, 21, and 35 are compounded amongst themselves, for the first of them doth measure it self, and the two others, although it be a Prime of it self.

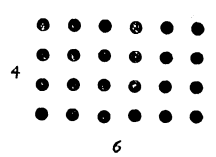
15 *A*

15 *A number is said to multiply another number, when there is produced another number, which is compounded as many times of the Multiplicator, as there are unites in the Multiplicand.*

AS 6 shall be said to multiply 8, when 8 shall be compounded as often times as there are unites in the Multiplicator 6, to wit, six times, and that there shall be produced another number, to wit 48, also by change, 8 shall be said to multiply 6, if you take 6 eight times, to wit, as often as there are unites in 8, and that it make the same 48; even so a number shall be said to be produced of two numbers, by multiplying the one in the other, as 63 of 7 and 9, &c.

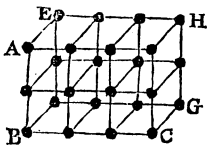
16 *But when two numbers multiplying one another doe produce a number, the number produced shall be called a Plaine, and the numbers which multiply one anonber shall be said to be sides of the Plaine.*

EVery number produced by the mutual Multiplication of two numbers is called a Plaine, forasmuch as his unites being disposed in length and breadth doe represent a rectangle Parallelogram, whose sides are the two numbers multiplying one another, as hath been said in the second Book, as 24 the product of 4 by 6, is said to be a Plaine number, whose sides are 4 and 6; Seeing that their unites disposed in length and breadth doe represent a rectangled Parallelogram.



17 *When three numbers multiplying one another doe produce a number, the number produced shall be called a Solid, and the numbers which multiply one another shall be called the sides of the Solid.*

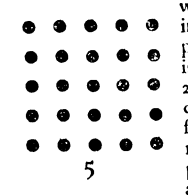
FOrasmuch as these three numbers 2, 3, 4, multiplying one another produce 24, for 2 by three produceth 6, and 6 by 4 produceth 24, otherwise 2 by 4 makes 8, and 8 by 3 makes 24, that number 24 shall be called a Solid, and the numbers 2, 3, 4, are the sides; for as much as the unites disposed in length, breadth and depth, doe represent a Solid figure, called Parallelepipedon, as shall be explained in the 11th. Book, whose three dimensions are expressed by the three numbers which are multiplied in one another; For if you multiply 2 by 4, you shall have 8 for the number of the Base BG of the Solid AG, whose length is 4 unites, and breadth 2, and if this Base be multiplied by



by 3, the whole number of the Solid, to wit 24, will be produced, being three unites high, &c.

18 *A Square number is that which is equally equal, or which is conteined under two equal numbers.*

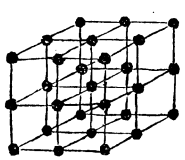
THat is to say, produced of two equal numbers multiplied by one another. This Plaine number, which is equally equal, that is to say, whose unites disposed in length and breadth, doe resemble a rectangle Parallelogram, whose Length and Breadth are equal, and in such sort as all the sides are equal, or that is produced of two equal numbers, and therefore is conteined under them, is called a square, as 25, conteined under 5 and 5, that is to say, is produced by the mutual multiplication of the two fives the one by the other, for his unites being ranged in form of a Plaine, doth represent a perfect Square, having five unites on each side, and therefore equally equal; But either of the sides of the Multiplication of which it is produced, is called the root of the Square.



19 *A Cube number is that which is equally equal equally, or which is conteined under three equal numbers.*

THat is to say, produced of three equal numbers multiplied by one another.

He calls this Solid a Cube, which is equally equal equally, that is to say, whose unites disposed in length, breadth, and depth, doe represent a Geometrical Cube, in such sort as that all his Dimensions, to wit, length, breadth and height, or depth, are equal; or which is produced of three equal numbers, as 27, which is conteined under these three numbers 3, 3, 3, for three times 3 makes 9, and three times 9 makes 27, for all his unites ranked in form of a Solid doe represent a Geometrical Cube, having 3 in every Dimension, and each of these three numbers is by the Geometricians called the Side of the Cube, and of divers Arithmeticians the Cubick Root.



20 *Numbers are proportional, when the first is as much Multiplex of the second, or the same part or parts thereof that the third is of the fourth.*

OR else when the first doth contein as many times the second, and over and above one and the same part or parts thereof, that the third doth contein of the fourth.

To the end you may the better comprehend all the numbers proportional are)

in all the kinds of proportion of inequality, (as all the unequal numbers are) we have added these words, as well because you may understand what *Euclide* saith hereon, as also to render this Definition more plain and intelligible; (or else, when the first conteineth as many times the second, and over and above one and the same part or parts thereof, as the third doth of the fourth.) For the vulgar definition of *Euclide*, (which *Clavius* thinks to be corrupted) comprehendeth onely the numbers proportional in the proportion multiplex and sub-multiplex, and the other proportions of lesser inequality. For in the proportion Multiplex, there are four numbers proportional, when the first is so much Multiplex of the second, as the third is of the fourth, and in the sub-multiplex when the first is the same part of the second, that the third is of the fourth; and in the other proportions of lesser inequality, when the first is the same parts of the second that the third is of the fourth, as the Definition of *Euclide* will have it, but by that it cannot be comprehended, which are the numbers proportional in reason *Super-particular*, *Super-partient*; Multiply *Super-particular* and Multiply *Super-partient*, for in all these the first number is not so Multiplex of the second, as the third is of the fourth, nor the same part or parts; but the first conteins the second, and the third the fourth equally, to wit, once or more, and over and above one and the same part thereof, or same parts; as is manifest by Definition the 4th. of the Fifth Book.

Wherefore these numbers 12, 4, 9, 3, are proportional, the first being so Multiplex of the second, as the third is of the fourth, to wit Triple, For 4 is such a part of 12, (to wit one third) as 3 is of 9: Againe, 6, 8, 9, 12, are proportional, forasmuch as 6 is the same parts of 8, as 9 is of 12, to wit, the three quarters; Lastly, 7, 6, 14, 12, and 7, 4, 14, 8, and 11, 5, 22, 10 and 12, 5, 24, 10, are numbers proportional; for as in the first example, the first number conteins once the second, and the third conteins once the fourth, and over and above, $\frac{1}{6}$ part, and in the second example, the first conteins once the second, and the third the fourth, and over and above $\frac{1}{4}$ parts, in the third example twice, and $\frac{1}{7}$ part, and in the last example the first conteins twice the second, and the third the fourth, and over and above $\frac{2}{5}$ parts, for otherwise the numbers should not be in any kind proportional: Therefore, whensoever 4 numbers are put proportional, you must accord that if the greater numbers are compared to the lesser, the first and the third shall be equi-multiplices of the second and the fourth; or else the first and the third shall equally contein the second and the fourth, and over and above the same part or parts, and contrariwise, if the first and the third be put equi-multiplices, or else that the first be said to contein so many times the second, as the third doth contein the fourth, and over and above the same part or parts, you may gather that the numbers are proportional, as if you compare the lesser with the greater, and that they are said to have the same proportion, you must confesse that the first is the same part of the second, as the third is of the fourth, or the same parts, and contrariwise, if the first be put the same part of the second as the third is of the fourth, or the same parts, you may conclude that those nnmbers are proportional.

But *Euclide* defineth onely those numbers proportional there, which have the same proportion of inequality: For if there be question of the proportion or reason of equality, it is manifest that the first ought to be equal to the second, and the third to the fourth, to the end they may be said to be proportional.

Now from this Definition may be gathered manifestly that the equal num-

numbers have the same reason to one and the same, and contrariwise one and the same to the equal, also that the numbers which have the same reason to one and the same, or to which one and the same number hath the same reason, are equal.

For seeing that equal numbers are equi-multiplices, or the same parts or the same parts; also seeing that one and the same number is equi-multiplex of the equal numbers, or the same part or parts, or conteineth them equally, and over & above the same part, or the same parts of them, it is manifest that equal numbers have the same reason to one and the same, or one and the same to equal numbers according to this Definition. Againe, forasmuch as the numbers which have the same reason to one and the same, are equi-multiplices thereof, or of the same part or parts, or doth equally contein it, and over and above the same part or parts thereof: And a number which hath the same reason to certain other numbers is equi-multiplex of them, or of the same part or parts, or conteineth it equally, and over and above the same part or parts thereof; according to this Definition, it is manifest that the numbers which have the same reason to one, or to which one and the same, hath the same reason, are equal amongst themselves.

In like manner, it may be said that there is greater reason of a greater number to one and the same, then of a lesser, and contrariwise, one and the same hath greater reason to a lesser, then to a greater number, also that number that hath the greatest reason to one and the same, is the greater number, and that number to which one and the same hath the greatest reason is the lesser number; which things are easie, if this Definition be well understood.

Clavius saith here, that this Definition agreeth also in broken numbers, whether there be whole numbers joyned with them or not: For Example, these four numbers are proportional, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, the first being so Multiplex of the second, as the third is of the fourth, to wit, double, which will appear, if you reduce the two first into the same Denomination, to wit to $\frac{4}{12}$ and $\frac{3}{12}$, and the two last to $\frac{2}{4}$ and $\frac{2}{3}$, in like manner these four are proportional $2\frac{1}{8}$, $4\frac{2}{15}$, $1\frac{1}{2}$, $2\frac{1}{15}$, seeing that the first is such a part of the second, as the third is of the fourth, to wit, the half, which is manifest, reducing them into one Denomination, to wit, to $\frac{15}{8}$, $\frac{62}{15}$, and $\frac{15}{8}$, $\frac{16}{8}$, in which *Clavius* is much mistaken, in not considering that to speak properly, there are no broken numbers or Fractions, Unity being indivisible, according to *Euclide*, and the Fraction making it self in the thing numbred, and not in the Numbers, to wit, when you divide any whole number into other equal parts, and of another name, which parts being applyed to numbers, or the numbers applyed to them, they are then absolute numbers, that is to say, whole numbers, as in the Examples above, where he saith, that $\frac{4}{8}$ are double to $\frac{2}{8}$, you must understand that these numbers 6 and 3, are taken absolutely, and as whole numbers, without regarding the name, for you ought onely to compare the number to the number, without comparing the name to the name, that is to say, to the Denominator, and so you ought to understand as touching the rest; for those things thus reduced are no more Fractions; Seeing that Fractions are so called onely in comparison to their whole, so divided into parts of another name: but these parts applyed to numbers, are whole numbers, and so *Euclide* doth understand.

21 *Plane*

21 *Like Plaine numbers and like Solid numbers, are such as have their sides proportional.*

TO the end that a Plaine number may resemble another Plaine number, it is not necessary that two sides of the one, (which you please,) be proportional to two sides of the other, (which you please also;) but it sufficeth that the one hath two sides, (which so ever they be) proportional to any two sides of the other, as the Plaine numbers 24 and 6 are alike; forasmuch as the sides of that of 6 and 4, are proportional to the sides of the other of 3 and 2, although that the other sides of the first 8 and 3, or 12 and 2, are not proportional to that of 3 and 2.

In like manner, it is not requisite that to the end that two Solid numbers be alike, that three of the sides of the one (which you please) be proportional to three of the other, (which you also please) but it sufficeth that three sides be proportional to any three sides, for so they are reduced into a Solid forme, according to their unites, and their Lengths, Breadths, and Heighths, shall be proportional, as the Solid numbers 192 and 24 are alike, the sides of the one 8, 6, 4, being proportional to the sides of the other, 4, 3, 2, although the other sides of the first, 12, 8, 2, or 16, 4, 3, are not proportional to the sides of the other, 4, 3, 2, &c.

22 *A perfect number, is that which is equal to all his aliquot parts.*

THat is to say, such a number to which his parts being put together are equal, (to wit all his aliquot parts,) and that number is called by the Mathematicians a perfect number, as 6, 28, 496, for the first conteineth these three aliquot parts 1, 2, 3, which together doe make 6, all these of the second are 1, 2, 4, 7, 14, whose summe do make 28, and those of the third 1, 2, 4, 8, 16, 31, 62, 124, 148, all which being added together do make 496, &c.

And if all the aliquot parts of a number taken together be greater then it, it shall be called abundant, if lesser, deficient.

Whence it appeares that this word part, is taken by *Euclide* onely for an aliquot part, for otherwise every number should be a perfect number, every number being equal to all his parts, if each lesser number may be said to be parts of a greater, whether it measure it or not.

To these Definitions of EUCLIDE we shall adde these which follow, according to some Interpreters.

23 *A number is said to measure a number by that number, which multiplying or being multiplied, produceth the same number.*

AS 4, shall be said to measure 12 by 3, forasmuch as 4 multiplied by 3 produceth 12; again, 3 shall be said to measure the same 12 by 4, seeing that 4 multiplying 3 produceth 12, &c.

24 *Proportion or Reason of numbers is an habitude of one number to another, according to what the one is Multiplex of the other, or part or parts, or else conteineth it once, or divers times, and over and above a certaine part or parts thereof.*

AS if you compare 20, with 4 according to what 20 is Multiplex of 4, to wit, quintuple, this comparison or habitude shall be called reason, as hath been said in the third Definition of the 5th. Book, where *Euclide* hath spoken of Reason in general.

25 *Two numbers are said to be terms or Roots of a proportion to which three cannot be found two lesse numbers, in the same reason or proportion.*

26 *When three numbers are proportional, the first is said to have to the third, the double reason of that of the first to the second: and when there are four, the first is said to be to the fourth in triple reason of the first to the second, and so in one and the same order, one over and above, until the proportion be finished.*

27 *If there be so many numbers as you please in order, the reason of the first to the last, is said to be composed of the reason of the second to the third, and of the third to the fourth, and so in order until the proportion be finished.*

PETITIONS.

It is required to be granted.

1 *That it is possible to take numbers equal or Multiplices to any number given.*

2 *That it is possible to take a greater number then any given number.*

ALthough that number cannot be infinitely diminished, but comes to unity indivisible, it may nevertheless be augmented infinitely by the

the continual addition of unity, therefore any number being proposed, a greater may be found, to wit, that which is produced by the addition of a unite, or of divers unites to the number given.

COMMON SENTENCES.

1 *Numbers equi-multiplices of equal numbers, or of one and the same number are equal to one another.*

2 *Numbers of which one and the same number is equi-multiplex, or of which equal numbers are equi-multiplices, are equal one to another.*

3 *Numbers that are the same part, or the same parts of equal numbers, or of one and the same number are equal to one another.*

4 *Numbers, of which one and the same number or equal numbers, are the same part, or the same parts, are equal to one another.*

5 *Unity measureth every number, by the unites that are therein, that is to say, by the same number.*

FOr the unite being taken so many times as there are unites in the number proposed, doth constitute the same number, and therefore it is measured by the unites that are therein, that is to say, by the same number made of his unites.

6 *Every number measureth it selfe by unity.*

SEeing that every number once taken is equal to it self, it is manifest that every number measureth it self by unity.

7 *If a number multiplying a number, produce another number, the Multiplicand shall measure the product by the Multiplicator: but the Multiplicator shall measure the same by the Multiplicand.*

FOr let B multiplying A produce C, I say, that A measureth C by B, and B measureth the same C by A, for (as by the 15 Defin.) seeing that B being

as many times compounded, as there are unites in A, doth make C: it is manifeſt that B doth meaſure C by A, and by the ſame reaſon, A doth meaſure the ſame by B; Seeing that B multiplying the ſame A doth produce the ſame C, as ſhall be demonſtrated in the 16 Propoſition of this Book.

A .... 4 B ... 3
C 12

8 *If a number meaſure another number, that number by which it meaſureth it, doth meaſure the ſame by the unites that are in the number meaſuring, that is to ſay, by the ſame number meaſuring.*

FOraſmuch as the number 6 doth meaſure 18 by 3, the ſame 3 ſhall meaſure 18, by 6, that is to ſay, by the unites that are in 6, for ſeeing that 6 doth meaſure 18 by 3, the number 18 ſhall be (according to the 23 Defin.) produced by the multiplication of 6 by 3, or of 3 by 6, therefore by the afore-going Axiome 3 ſhall meaſure 18 by 6.

9 *If a number meaſuring a number, multiply that number by which it meaſureth it, or is multiplied thereby, it will produce the number which it meaſureth.*

LEt the number A meaſure C by B, I ſay, that A multiplying B or multiplied of B produceth C, for (by the 25 Definition) A is ſaid to meaſure C by that number, the which multiplying or being multiplyed, produceth C, therefore A being put to meaſure C by B, it is manifeſt that A multiplying B, or being multiplyed thereby, produceth C.

A ....4 B ...3
C 12

10 *The number that meaſureth as many other numbers as you pleaſe, doth alſo meaſure the numbers compounded thereof.*

LEt the number A meaſure B C and C D; I ſay it doth alſo meaſure their compound B D; for ſeeing that A meaſureth B C and C D, as well

A 4 20
B ....4 E ....4 C ....4 F ....4 G ....4 D

B C as C D ſhall be Multiplex of A, dividing therefore B C according to the parts B E and E C, equal to A and C D, according to C F, F G and G D, equal to A; B D compounded of all the parts B E, E C, C F, F G and G D, equal to A ſhall be Multiplex of A; therefore A meaſureth B, which was propoſed.

11 *That*

11 *That number that meaſureth ſome other number, doth alſo meaſure every number which that number meaſureth.*

LEt the number A meaſure B, and B meaſure C D. I ſay, that A doth alſo meaſure C D. For ſeeing that B doth meaſure C D, the ſame C D ſhall be Multiplex of B, dividing therefore C D into C E and E D, equal to B, A ſhall meaſure C E and E D, ſeeing that A is put to meaſure B, a number equal as well to C E, as to E D, (as by the 10th. Axiome) therefore A ſhall alſo meaſure C D, compounded of C E and E D, which was propoſed.

A ...3
B 9
C .....9 E .....9 D

12 *That number that meaſureth the whole and the part cut off, meaſureth alſo the reſt.*

LEt A meaſure the whole B C, and the part cut off B D; I ſay it will alſo meaſure D C, the reſt; For ſeeing that A meaſureth B C and B D, as well B C as B D ſhall be Multiplex of A, or B D ſhall be equal to A; therefore having divided as well B C as B D, in parts equal to A, the reſt D C, ſhall be one onely part equal to A, or Multiplex thereof, wherefore A ſhall meaſure D C, which was propoſed.

A ...3
B .....9 D ...3 C
B .....9 D .....6 C
B ...3 D 6

All that hath been here above added, beſides the Definitions of *Euclide*, is done to render the Demonſtrations hereafter mentioned more clear and intelligible, they being all founded on what *Euclide* hath before ſet down in his former ſeveral Books.

PRO-

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

A.....10 E..2 G.1 B
C...3 F..2 D
H.....

If from two unequal numbers proposed A B, C D, *there be cut off ſtill alternately, the leaſt* C D, *from the greateſt* A B, *and that the remainder* E B *ſhall never meaſure his precedent* A B, *until the unite* G B *be taken; the numbers* A B, C D, *propoſed at the beginning, ſhall be Primes to one another.*

Demonſtration FOr if they be not Primes to one another, they ſhall be meaſured by ſome number, which ſuppoſe to be H, their common meaſure over and above unity; foraſmuch then as H meaſureth C D, and C D, A E, (C D being a part of A E, or equal thereto, ſeeing that it being taken from A B, there doth remaine E B) H [a] ſhall alſo meaſure A E. But H doth meaſure the whole A B, and [b] therefore ſhall meaſure the reſt E B. But E B doth meaſure C F, [c] therefore H doth meaſure the ſame C F: and therefore ſeeing that it meaſureth the whole C D, it [d] will alſo meaſure the reſt F D, and ſeeing F D meaſureth E G, H [e] ſhall alſo meaſure E G, but H meaſured the whole E B, therefore [f] H ſhall alſo meaſure the unite reſting G B, the whole his part, which is abſurd, therefore any number beſides unity, ſhall not meaſure A B and C D, and therefore they ſhall be Primes to one another which was to be demonſtrated.

a) 11. c. 5.
b) 12. c. 5.
c) 11. c. 5.
d) 12. c. 5.
e) 11. c. 5.
f) 12. c. 5.

PROP. 2. PROBL. 1.

A.....16 E.....6 B
C.....6 F..D
G.....

To find the greateſt common meaſure F D, *of two given numbers* A B, C D, *that are not Primes to one another.*

Conſtruction LEt the leſſer number C D, be taken from A B the greater number, as often as may be, and there will reſt E B, which being ſubtracted from C D leaveth F D, and ſo following, alwayes taking the leſſer from the greater by an alternate ſubtraction, by which means you ſhall arrive to a number which will meaſure the precedent; For if you ſhould proceed to unity, [a] A B and C D, ſhould be Primes to one another, which is contrary to the Hypotheſis; come we then to F D, the which being ſubtracted from the precedent, as often as may be, there will remaine nothing; I ſay then, that F D is the greateſt common meaſure of A B and C D.

a) 1. 7.

Demonſtration FOr that it meaſureth both of them, we ſhall thus demonſtrate, foraſmuch as F D meaſureth E B, and E B meaſureth C F, [b] F D, ſhall meaſure the ſame C F, and ſeeing it meaſureth it ſelf, [c] it ſhall alſo meaſure the whole C D, compounded of the two numbers C F and F D. But C D meaſureth A E, [d] therefore F D ſhall alſo meaſure A E, and ſeeing that F D doth alſo meaſure E B, it [e] ſhall meaſure the whole A B, compounded of A E and E B, therefore F D doth meaſure the two numbers A B and C D, and is alſo their greateſt common meaſure: For if it be not ſo, let it be another, greater then F D, ſuppoſe G; Foraſmuch then as G meaſureth A B and C D, being their common meaſure, and C D meaſureth A E, [f] the ſame G ſhall alſo meaſure A E, [g] and therefore E B: but E B meaſureth C F, [h] therefore G ſhall alſo meaſure C F, and [i] therefore it ſhall alſo meaſure the reſt, F D the greater, the leſſer, which is impoſſible; Therefore a greater then F D ſhall not meaſure A B and C D, and therefore F D is their greateſt common meaſure. But if the leſſer number C D doth meaſure the greater A B, in ſuch manner, as being ſubtracted there-from as often as ſhall be poſſible, there reſt nothing, C D ſhall be the greateſt common meaſure of the two; Seeing it meaſureth it ſelf, as appears by this figure. Therefore, &c. which was to be demonſtrated.

A.....10 B
C.....5 D

b) 11. c. 5.
c) 10. c. 5.
d) 11. c. 5.
e) 10. c. 5.
f) 11. c. 5.
g) 12. c. 5.
h) 11. c. 5.
i) 11. c. 5.

COROLARIE.

From this Demonſtration it is manifeſt that that number which meaſureth two others, meaſureth alſo their greateſt common meaſure; This is gathered from that part, by which it is ſhewn that F D *is the greateſt common meaſure of* A B *and* C D, *for it is ſhewn that if* G *meaſure* A B *and* C D, *it meaſureth alſo* F D, *their greateſt common meaſure.*

PROP. 3. PROBL. 2.

A.....16 D....4
B.....12 E..2
C.....6 F.....

To find the greateſt common meaſure D, *of three given numbers* A, B *and* C, *not Primes to one another.*

Construction LEt D be found, the greateſt common meaſure of A and B, if D doth alſo meaſure C, it is manifeſt that D is the greateſt common meaſure of the three, A, B, and C.

a) Cor. 2. 7. *Demonſtration* FOr if a greater number then D be ſaid to meaſure the three A, B, C, the ſame [a] ſhall alſo meaſure D the greateſt common meaſure of A and B, the greater, the leſſer, which is impoſſible. But if D doth not meaſure C, at leaſt D and C ſhall be compounded between themſelves, for A, B and C, being compounded, any common meaſure of theirs ſhall alſo meaſure D, the greateſt common meaſure of A and B; therefore, ſeeing that that common meaſure doth alſo meaſure C, D and C ſhall be compounded between themſelves, and E be their greateſt common meaſure; b) I ſay, that E is the greateſt common meaſure of the three, A, B and C, foraſmuch as E meaſureth C and D, but D meaſureth A and B; E meaſureth the ſame A and B, and therefore it ſhall meaſure the three A, B and C.

A.....16 D....4
B.....12 E..2
C.....6 F.....

b) 2.7.

Now that E is the greateſt common meaſure, it is evident: For ſuppoſe F to be their common meaſure, greater then E, foraſmuch as F meaſureth A and B, c) it ſhall alſo meaſure D, their greateſt common meaſure: But F alſo meaſureth C, therefore F meaſuring D and C, ſhall alſo in like manner, meaſure E, their greateſt common meaſure, the greater, the leſſer, which is impoſſible: Therefore, a greater number then E will not meaſure A, B and C, and therefore E is their greateſt common meaſure; Therefore, &c. which was to be demonſtrated.

c) Cor. 2. 7.

COROLARIE.

It is manifeſt from this Demonſtration, that one number that meaſureth three, will in like manner meaſure their greateſt common meaſure.

This is alſo gathered from the laſt part of the Demonſtration, where it is ſhewn that ſeeing that F *doth meaſure* A, B *and* C, *it meaſureth alſo* E, *their greateſt common meaſure: In like manner, if there be more then three numbers propoſed, not Primes to one another, this Probleme ſheweth how to find their greateſt common meaſure: For if there be four numbers, you muſt firſt find the greateſt common meaſure of three, and if five, of four, &c. and you muſt work, in the reſt, as hath been ſhewn of three numbers.*

PROP.

PROP. 4. THEOR. 2.

A.....7
B.....10

A.....5
B.....10

A..2 D..2 E..2 F
B.....10
C..2

Every leſſer number A, *is a part or parts of every greater number* B.

Demonſtration LEt A and B in the firſt place be Primes to one another, foraſmuch as each unite of A, is a part of B, it is manifeſt that A is parts of B, that is to ſay, ſo many parts as there are unites in A.

Secondly, let A and B not be Primes to one another, but compounded, and that A meaſureth B, that being [a] it appears that A is part of B.

a) 3. d. 7.

But if A doth not meaſure B, [b] find C their greateſt common meaſure; and let A be divided according to the parts A D, D E, and E F, each equal to C, foraſmuch as C is part of B, ſeeing that it meaſureth it, A D ſhall be alſo part of B, in like manner DE and EF; and therefore all the number A is parts of B, to wit, as many parts as C is conteined times in A E. Therefor every Leſſer, &c. Which was to be demonſtrated.

b) 2. 7.

c) 3 diff.

PROP. 5. THEOR. 3.

A.....6 D....4
B.....6 G.....6 CE....4 H....F

If one number be ſuch part of one number as another number is of another number, the one and the other together, ſhall be ſuch part of the one and the other together, as one ſingle number to a ſingle number.

THat is to ſay, if of four numbers, the firſt be ſuch a part of the ſecond, as the third is of the fourth, the firſt and third together, ſhall be ſuch part of the ſecond and the fourth together, as the firſt is of the ſecond.

Demonſtration FOr having divided the numbers B C and E F, according to the parts B G, G C, E H and H F, equal to A, to D, there will be in B C ſo many parts equal to A, as in E F, equal to D; ſeeing that A is ſuch part of B C, as D is of E F.

Foraſmuch then as A and B G are equal, if there be added to them the equal numbers D and E H, A and D together, ſhall be equal to B G and E H together. in like manner, A and D together, ſhall be equal to G C and H F together, and ſo on, if there were more parts in B C and E F: and therefore the one and the other, A and D together, ſhall be the ſame part of the one and the other, B C and E F together, as A is of B C, or D of E F; Therefore if one number, &c. Which was to be demonſtrated.

PROP. 6. THEOR. 4.



If one number A B, *be such parts of one number* C, *as another number* D E, *is of another number* F, *the one and the other* A B *and* D E, *together, shall be such parts of the one and the other* C *and* F *together, as one single number* A B, *of* C, *is to one single number* D E, *of* F.

THat is to say, if of four numbers, the first be such part of the second, as the third is of the fourth, the first and third together, shall be such parts of the second and fourth together, as the first is of the second.

Demonstration FOr having divided A B, and D E according to the parts A G, G B, D H, and H E, of the numbers C and F; A B shall contein as many parts of C, as D E of F; seeing that A B is such parts of C, as D E is of F, then forasmuch as A G is such part of C, as D H is of F, the one and the other A G, and D H, together, shall be such part of the one and the other C and F together, as A G is of C, or D H of F; By the same reason G B, and H E, together, shall be the same part of C and F together, as G B is of C, or H E of F, and so following, if there be more parts in A B, and D E; Therefore, the one and the other, A B and D E together, shall be the same parts of the one and the other, C and F together, as A B is of C, or D E of F: Therefore if one number, &c. Which was to be demonstrated. a) 5. 7.

PROP. 7. THEOR. 5.



If one number A B, *be such a part of one number* C D, *as the part cut off* A E, *is of the part cut off* C F, *the rest* E B, *shall be also such a part of the rest* F D, *as the whole* A B *is of the whole* C D.

Demonstration FOr let G C be taken, of which let E B be such a part as A E is of C F, or the whole A B, of the whole C D, forasmuch as A E is the same part of C F, as E B is of G C, the one and the other A E and E B, together, shall be such part of C F and G C, together, as A E is of C F, that is to say, as the whole A B, is to the whole C D: and therefore, A B being such a part, as well of F G, as of C D, the same F G, and C D, shall be equal to one another; then taking away the common C F, G C, and F D, will remaine equal: therefore, E B shall be such part of F D, as of G C, that is to say, as the whole A B, to the whole C D. Therefore, if one number, &c. Which was to be demonstrated. a) 5. 7.

PROP.

PROP. 8. THEOR. 6.

If one number A B *be such parts of another number* C D, *as the part cut off* A E, *to the part cut off* C F, *the rest* E B *shall be also such parts of the rest* F D, *as the whole* A B, *is of the whole* C D.

Demonstration FOr let G H be taken equal to A B; G H shall be then also the same parts of C D, as A B is of C D, that is to say, as A E of C F, having then divided G H according to G I and I H, parts of C D, and A E, according to A K and K E parts of C F, the number of the parts G I, and I H, shall be equal to the number of the parts A K and K E, and as well G I as I H, shall be such part of C D, as A K, or K E is of C F: and seeing that C D is greater then C F, as well G I as I H, part of C D, shall be greater then A K, or K E part of C F.

Having therefore taken G L and I M, equal to A K and K E; G L shall be the same part of C F, as A K, of the same C F, or as G I of C D, and therefore the whole G I, being the same part of the whole C D, that the part cut off G L is of the part cut off C F, the rest L I, shall be such part of the rest F D, as the whole G I, is of the whole C D. So M H shall be shewn to be the same part of F D, as the whole G I, or I H, is of the whole C D, therefore, seeing that as well G I as I H, is the same part of C D, as L I and M H, of F D, the one and the other, G I and I H together, shall be such parts of C D, as L I and M H together, of F D. But G H is the same parts of C D, as A B of the same C D, (A B, and G H, being equal,) therefore L I and M H together, shall be the same parts of F D, as A B of C D. a) 7. 7.

But forasmuch as if from the equal A B and G H, there be taken the equal A K, K E and G L, I M, the rest E B, shall be equal to the rest L I and M H, together: and therefore the rest E B, shall be the same parts of the rest F D, as the whole A B of the whole C D: to wit, as L I and M H, together, of the same F D. Therefore if one number, &c. which was to be demonstrated.

PROP. 9. THEOR. 7.



If one number A B, *be such part of one number* B C, *as another number* D, *is part of another number* E F, *also alternately, the first* A, *shall be such part or parts of the third* D, *as the second* B C, *is of the fourth* E F.

THat is to say, If of four numbers, the first be such part of the second, as the third is of the fourth, also alternately, the first shall be such part or parts of the third, as the second is of the fourth.

Demonstration FOr having divided B C, and E F, according to the parts B G, and G C, each equal to A, and E H, and H F, each equal to D, the number of the parts of B G, shall be equal to the number of the parts of E F. But B G, and G C, being equal to one another, and lesse then E H, and H F, also equal to one another; forasmuch as the whole B C, is lesse then the whole E F, by supposition, B G shall be the same part of E H, or the same parts, as G C of H F [a], and therefore B G, and G C, together, to wit, B C the second, shall be the same part or parts of E H, and H F, together, that is to say, E F the fourth, as B G is of E H, that is to say, as A the first is of D the third: Therefore, if one number, &c. Which was to be demonstrated.

A....4
B....4 G....4 C
D.....6
E.....6 H.....6 F

a) 5, or 6. 7.

PROP. 10. THEOR. 8.

A..2 G..2 B
C.....6
D.....5 H.....E
F.....15

If a number A B, *be such parts of a number* C, *as another* D E, *is parts of another* F, *also alternately, the first* A B *shall be such part or parts of the third* D E, *as the second* G, *is of the fourth* F.

Demonstration FOr having divided A B, and D E, according to A G, and G B, parts of C, and D H, and H E, parts of F, there will be as many parts in A B, as in D E, and as well A G, as G B, shall be such part of C as D H, or H E, of F, [a] therefore, alternately A G shall be the same part or parts of D H, and G B of H E, as C is of F, and therefore, A G shall be the same part or parts of D H, as G B is of H E: [b] therefore A G, and G B, together, that is to say, A B the first, shall be the same part or parts of D H, & H E together, that is to say, of D E the third, as A G is of D H, that is to say, as C the second of F the fourth: Therefore, if one number, &c. Which was to be demonstrated.

a) 9. 7.
b) 5, 6. 7.

PROP. 11. THEOR. 9.

A....4 E..2 B.
C.....8 F....4 D

If as the whole A B, *is to the whole* C D, *so the part cut off* A E, *is to the part cut off* C F, *also the rest* E B, *shall be to the rest* F D, *as the whole is to the whole.*

Demonstration FOr seeing that A B is to C D, as A E is to C F, [a] A B being lesse then C D, shall be the same part, or the same parts, of C D, as A E is of C F; therefore the rest E B, shall be the same part, or parts

a) 20. d. 7.

parts of the rest F D, as A B is of C D: [b] therefore, E B shall be to F D, as A B is to C D. Which was to be demonstrated.

b) 7, 8. 7.

PROP. 12. THEOR. 10.

A....4 C..2 E...3
B.....8 D....4 F.....6

If there be as many numbers as you please A, B, C, D, E, *and* F, *proportional, as one of the antecedents* A, *shall be to one of the consequents* B: *so all the antecedents* A, C, *and* E, *shall be to all the consequents* B, D, *and* F.

Demonstration FOrasmuch then as because of the same proportion, A is [a] such a part, or such parts (A, C, and E, being lesse) of B, as G is of D, and E of F, [b] A and C together; shall be such part, or parts of B and D together, as A is of B, or E of F.

Againe, forasmuch as A and C, together as one, is the same part or parts of B and D together as one, as E is of F, the two A and C together, as one and F together, shall be the same part, or the same parts of the two B and D together, as one, and F together, as A is of B; [c] Therefore, there will be the same proportion of A, C and E together, to B, D and F together, as of A to B.

a) 20. d.
b) 5, 6. 7.
c) 20. d. 7.

PROP. 13. THEOR. 11.

A.. 2
B....4
C.....6
D.....12

If four numbers A, B, C, *and* D, *be proportional, also alternately they shall be proportional.*

Demonstration FOr seeing that A is to B, as C to D, [a] A is the same part, or parts of B, as C is of D, therefore, alternately, A shall be the same part or parts of C, as B is of D, [b] and therefore, as A shall be to C, so B shall be to D, which was to be demonstrated.

a) 20, 10. 7.
b) 20. d.

PROP. 14. THEOR. 12.

A.....12 D..... 9
B.....8 E.....6
C....4 F..2

If there be as many numbers as you please of the one part; A, B, *and* C, *and as many of the other part,* D, E, *and* F, *the which being taken two and two, and in the same reason; also in equality they shall be in the same reason.*

Demon-

Demonstration FOr seeing that as A is to B, so D is to E, [a] alternately, as A shall be to D, so B shall be to E; in like manner, seeing that as B is to C, so E is to F, alternately, as B shall be to E, so C shall be to F: Therefore as A is to D, so C shall be to F, (for seeing that the one and the other reason of A to D, and C to F, is the same to the reason of B to E, as hath been demonstrated, they shall be also the same one to another) and [b] therefore, alternately, as A shall be to C, so D shall be to F. Which was to be demonstrated.

A.....12 D.....9
B.....8 E.....6
C....4 F..2

a) 13. 7.
b) 13. 7.

PROP. 15. THEOR. 13.

A.1
B.1 G.1 H.1C
D..2
E..2 I..2 K..2F

If unity A, *measureth some number* BC, *and that another number* D *doth measure as oftentimes another number* EF, *also alternately, unity* A *shall measure the third number* D, *as oftentimes as the second* BC *shall measure the fourth* EF.

Demonstration FOr having divided BC, according to the unites BG, GH, HC and EF, according to the parts EI, IK and KF, each equal to D, there will be in EF as many parts equal to D, as there are unites in BC: therefore BG shall be the same part of EI, as GH of IK, and HC of KF: [a] therefore as BG, is to EI, so GH to IK, and HC to KF, [b] therefore, as BG to EI, so is BG, GH and HC, to EI, IK, and KF, all the antecedents to all the consequents, that is to say, BC to EF, therefore BC shall be the same part of EF, as BG is of EI, that is to say, as A is to D: Therefore if unity, &c. Which was to be demonstrated.

a) 20. d. 7.
b) 12. 7.

PROP. 16. THEOR. 14.

Unity. 1
A...3 B....4
C.....12
D.....12

If two numbers A *and* B *multiplying one another, do produce other numbers* C *and* D, *the products* C *and* D, *shall be equal to one another.*

Demonstration FOrasmuch as A multiplying B produceth C [a], B shall be as often found in C, as unity in A, and therefore shall measure C, as often as the unite shall measure A [b], therefore, by permutation, A shall measure C, as often times as unity shall measure B: Again, seeing that B multiplying A produceth D, A shall be as often found in D as unity in

a) 15. d. 7.
b) 15. 7.
15. d.

in B, and therefore shall measure D, as oftentimes as unity shall measure B; But A shall measure C also, as oftentimes as unity shall measure B, therefore, A doth measure C and D equally, and therefore C and D are equal to one another. Therefore, if, &c. Which was to be demonstrated.

PROP. 17. THEOR. 15.

Unity. 1
A...3
B..2 C....4
D.....6 E.....12

If one number A, *by multiplying two others* B *and* C, *doth produce other numbers* D *and* E, *the products of those numbers shall have the same proportion to one another as the numbers multiplied.*

Demonstration FOr seeing that A multiplying B, maketh D; B shall be as oftentimes conteined in D as unity in A: In like manner, C shall be as oftentimes conteined in E, as unity in the same A, and therefore B shall measure D as oftentimes as C shall measure E. Therefore B shall be such part of D, as C of E [b], and therefore as B shall be to D, so C shall be to E [c], and alternately, as B is to C, so is D to E: Therefore if one number, &c. Which was to be demonstrated.

15. d.
b) 20. d.
c) 13. 7.

PROP. 18. THEOR. 16.

A..2 B....4
C...3
D.....6 E..... 12

If two numbers A *and* B, *multiplying some other number* C, *shall produce other numbers* D *and* E, *their products* D *and* E *shall have the same proportion as the numbers multiplying* A *and* B.

Demonstration FOr seeing that D is the product of C multiplyed by A [a], the same D shall be the product of A multiplyed by C: in like manner E being the product of C by B, the same E shall be the product of B by C: Forasmuch then, as the same number C multiplying A and B, makes D and E, as A shall be to B, so D shall be to E: Therefore, if two numbers, &c. Which was to be demonstrated.

a) 16. 7.
17. 7.

PROP.

PROP. 19. THEOR. 17.

A...3 B..2
C.....6 D....4
E.....12
F.....12
G.....18

If four numbers A, B, C, *and* D, *are proportional, the product of the first* A, *and the fourth* D, *shall be equal to the product of the second* B, *and the third number* C: *and if the number produced of the first* A, *and the fourth* D, *be equal to the product of the second* B, *and the third* C, *those four numbers shall be proportional.*

Demonstration AS A is to B, so C is to D, and as A the first, multiplying D the fourth, maketh E, and B the second, multiplying C the third, makes F, I say that E and F are equal to one another; for again, let A multiplying C make G; forasmuch as A multiplying C and D, makes G and E, as C shall be to D: that is to say, as A is to B, so G shall be to E.

17.7.

Again, forasmuch as A and B multiplying C, makes G and F, as [a] A shall be to B, so G shall be to F: therefore, as G shall be to E, so the same G shall be to F, therefore, E F shall be equal by what followes after the 20th. Definition.

a) 18.7.

Now let E the product of A the first, by D the fourth, be equal to F, the product of B the second, by C the third: I say, the four numbers A, B, C, and D, are proportional, to wit, as A is to B, so C is to D.

Againe, let G be produced of A multiplyed by C; forasmuch as A multiplying C and D, makes G and E; as C shall be to D, so G to E, or to F, equal to E, for G hath the same proportion to E, as to F, as is shewn at the 20th. Definition.

Again, forasmuch A and B multiplying C, makes G and F, as A is to B, so the same G is to F, therefore, the proportions of A to B, and C to D; being the same with the proportion of G to F, they shall be so to one another, and so as A shall be to B, so C shall be to D: Therefore, if four numbers, &c. Which was to be demostrated.

PROP. 20. THEOR. 18.

A.....9
B.....6 D.....6
C....4

If three numbers A, B, *and* C, *are proportional: to wit, as* A *to* B, *so* C *to* D, *the product of the extreames* A, *multiplyed by* C, *is equal to the product of the mean* B, *and if the product of the extreames* A *and* C, *be equal to the product of the mean, those three numbers shall be proportional.*

Demon-

Demonstration FOr let D be assumed equal to B, as A shall be to B, so D shall be to C [a], and therefore the product of A by C, shall be equal to the product of B by D; that is to say, of B multiplyed by it selfe.

a) 19.7.

Now let the product of A the first, by C the third, be equal to the product of B the mean multiplyed by it selfe; I say, A, B and C, are proportional.

For again, let D be equal to B; as A shall be to B, so D to C, and the product of A by C shall be equal to the product of B by D, equal to B, that is to say, of B multiplyed by himselfe: Therefore, the three numbers A, B, C, are proportional: Therefore, if three numbers, &c. Which was to be demonstrated.

PROP. 21. THEOR. 19.

A...3 G..2 B
C..2 H. 1 D
E.....10
F.....6

The least numbers AB, *and* CD, *of all those which have the same proportion with them* E *and* F, *do equally measure the numbers which have the same proportion with them: to wit, the greatest* AB, *the greatest* E, *and* CD *the lesser,* F *the lesser.*

Demonstration FOr seeing that AB is to CD, as E is to F [a], alternately, as AB to E, so CD to F, and therefore AB and CD, being lesse then E and F, AB [b] shall be such part or parts of E, as CD of F: But they cannot be parts, for (if possible) let AB, and CD, be divided in AG, and GB, and CH, and HD, parts of E and F; the parts AG, and GB, shall be equal in number to CH, and HD; and therefore, AG shall be the same part of E, as CH of F; therefore, as AG shall be to E, so CH shall be to F [c], and alternately, as AG shall be to CH, so E to F, or AB to CD: and therefore, AG, and GH lesse then AB, and CD, shall have the same proportion as AB, and CD, which is impossible: seeing that AB, and CD, are the least in their proportion by Supposition: Therefore, AB, and CD, shall not be the same parts of E and F, therefore, the same part. Therefore, AB, and CD, shall equally measure E and F. Therefore, the least numbers, &c. Which was to be demonstrated.

a) 13.7.

b) 20.d.

c) 20.d.

PROP. 22. THEOR. 20.

A....4 D.....12
B...3 E.....8
C..2 F.....6

If there be three numbers of the one part, A, B, and C, and as many of the other part, D, E, and F, the which being taken from two to two, and in the same reason, and that their proportion be disturbed: also by equality they shall be in the same reason.

Demonstration FOr seeing that as A is to B, so E is to F [a]; the product of A the first, by F the fourth, shall be equal to the product of B the second, by E the third; and seeing that B is to C, as D to E, the product of C by D, shall be equal to the same product of B by E; and therefore the product of A the first, by F the fourth, shall be equal to the product of C the second, by D the third [b]: Therefore, there shall be the same reason of A the first, to C the second, as of D the third, to F the fourth: Therefore, if there be three, &c. Which was to be demonstrated.

a) 19. 7.

b) 19. 7.

PROP. 23. THEOR. 21.

A.....7 B.....5
C.... D.... E..

The numbers that are primes to one another A and B, are the least of all those that have the same reason with them.

Demonstration FOr if it be denied, suppose C and D to be lesser in the same reason: Forasmuch, as C and D are lesser in the same reason of A and B [a], C shall measure A, and D shall measure B equally, and therefore according to one and the same number, which let be E, in such manner as C measureth as often times A and D, as often times B as Unity is conteined in E: Therefore, seeing that Unity measureth E and C, measureth A equally [b], alternately, Unity shall measure C and E, shall measure A equally: Again, seeing that Unity measureth E and D, the number B equally: also alternately Unity shall measure B equally; therefore, seeing that E measureth A and B the same E shall be their common measure: wherefore they shall not be primes to one another, but compounds, which is impossible, and against the Supposition; therefore, A and B are the least in the same reason: Therefore, the numbers, &c. Which was to be demonstrated.

a) 21. 7.

b) 15. 7.

PROP.

PROP. 24. THEOR. 22.

The leaſt numbers A *and* B *of all thoſe which have the ſame reaſon with them, are primes to one another.*

Demonſtration FOr if they be not Primes, they ſhall have one common meaſure, which ſuppoſe to be C; therefore, let C meaſure A as often times as Unity is in D, and B as many times as Unity is in E.

Foraſmuch then, as C is conteined as oftentimes in A, as Unity is in D, (Seeing that it meaſureth A according to D,) and as the ſame C is conteined as many times in B as Unity is in E [a]: D and E multiplying C, ſhall produce A and B: wherefore there ſhall be the ſame reaſon of A to B, as of D to E: therefore, ſeeing that D and E parts of A and B, are leſſer then the ſame A and B: A and B ſhall not be the leaſt of all thoſe which have the ſame reaſon with them, which is abſurd: Therefore, A and B are Primes to one another: and therefore the leaſt, &c. Which was to be demonſtrated. a) 9. c. ſ.

PROP. 25. THEOR. 23.

If there be two numbers A *and* B, *primes to one another, the number which meaſureth one of them, ſhall be prime to the other.*

Demonſtration FOr if B and C be not Primes to one another, they will have ſome number for their common meaſure, which ſuppoſe to be D.

Foraſmuch then, as D meaſureth C, and C meaſureth A, [a] D ſhall alſo meaſure A: But it alſo meaſures B: therefore, A and B are not Primes to one another, having D for common meaſure, which is impoſſible, and againſt the Suppoſition: Therefore, C ſhall be Prime to B: Therefore, If there be two numbers, &c. Which was to be demonſtrated. a) 11. c. ſ.

PROP. 26. THEOR. 24.

If two numbers A *and* B, *are primes to ſome other number* C, *their product* D, *ſhall be alſo prime to that other* C.

Demonſtration FOr if C and D are not Primes to one another, let E be their common meaſure, meaſuring D as often times as there are unites in F, forasmuch then, as E is as many times in D, as there are unites in F [a], F multiplying E ſhall produce D, and contrariwiſe, E multiplying F, ſhall produce the ſame D; But, D is the product of A by B, therefore, ſeeing that the number produced of E the firſt, by F the fourth, is the ſame as of A the ſecond, by B the third, there ſhall be the ſame reaſon of E the firſt, to A the ſecond, as of B the third, to F the fourth. But forasmuch as A and C are Primes to one another, and E is ſuppoſed to meaſure C, E ſhall be Prime to A, and therefore E and A being Primes to one another, ſhall be the leaſt of their reaſon; Therefore, they ſhall equally meaſure B and F, which have the ſame reaſon: to wit, E ſhall meaſure B, and A ſhall meaſure F; Therefore, ſeeing that E meaſureth both the one and the other, B and C: B and C ſhall not be Primes to one another; which is abſurd, and againſt the ſuppoſition; Therefore, D ſhall be Prime to C, Therefore, if two numbers, &c. Which was to be demonſtrated.

A.....7 B...3
C.....8
D.....21
E— F—

a) 9. c. ſ. 16. 7.
19. 7.
b) 25. 7.
23. 7.
21. 7.

PROP. 27. THEOR. 25.

A....4 B.....5
C.....16
D....4

If two numbers A *and* B, *are Primes to one another, the product of the one of them* A, *ſhall be Prime to the other.*

Demonſtration FOr let D be made equal to A, it ſhall be alſo Prime to B, therefore, A and D, being Primes to B [a], the product of A, multiplyed by D, this is to ſay, of A by himſelf; that is to ſay, C ſhall be Prime to the ſame B: ſo the product of B multiplyed by it ſelf, ſhall be ſhewn to be Prime to A, Therefore, if two numbers, &c. Which was to be demonſtrated.

a) 26. 7.

PROP. 28. THEOR. 26.

A.....5 B...3
E.....15
C....4 D..2
F.....8

If two numbers A *and* B, *are Primes to two other numbers* C *and* D, *the one and the other, to the one and the other, their Products ſhall be alſo Primes to one another.*

Demonſtration FOr ſeeing that as well A as B is Prime to C, E [a] their product ſhall be Prime to the ſame C.

Again, A and B, being Primes to D, E their product ſhall be alſo Prime to the ſame D; forasmuch then, as C and D are Primes to E [b], F their product ſhall be alſo Prime to E; Therefore, if two numbers, &c. Which was to be demonſtrated.

a) 26. 7.
b) 26. 7.

PROP. 29. THEOR. 27.

A...3 B..2
C.....9 D....4
E 27 F8
G 81 H 16

If two numbers A *and* B, *are Primes to one another, and the one and the other, multiplying it ſelfe, ſhall make a number, their products ſhall be Primes to one another, and if the numbers firſt propoſed, multiplying thoſe products do make a number, the ſame ſhall be alſo Primes to one another, and this will always happen about the extreams.*

Demonſtration FOr ſeeing that A and B are Primes to one another [a], C the product of A by himſelfe, ſhall be Prime to the other B, and in like manner, B and C being Primes to one another, D the product of B by himſelfe, ſhall be alſo prime to C: and therefore the two products C and D, Primes to one another. Again, forasmuch as A and B are Primes to one another [b], C product of A by it ſelfe, ſhall be Prime to B, and D the product of B by it ſelfe, prime to A: But C is alſo ſhewn to be Prime to B, therefore, the one and the other, A and C, ſhall be Prime to the one and the other, B and D [c], and therefore, E the product of A by C, ſhall be Prime to F, the product of B by D: And if again, G be the product of A by E, and H of B by F. Seeing that A and C are Primes to B [d], E their product ſhall be alſo Prime to B: by the ſame reaſon F ſhall be Prime to A; Forasmuch then, as the one and the other, A and E, is Prime to the one and the other, B and F, the number G product of A by E, ſhall be Prime to H, product of B by F, and ſo following, if there were more: Therefore, &c. Which was to be demonſtrated.

a) 27. 7.
27. 7.
b) 27. 7.
c) 28. 7.
d) 26. 2.
c) 28. 7.

PROP. 30. THEOR. 28.

A.....8 B....5 C
D...

If two numbers A B, *and* B C, *are primes to one another, the one and the other together,* A C *ſhall be prime to each of them: and if the one and the other together, be prime to one of them, the numbers firſt propoſed, ſhall be alſo primes to one another.*

Demonſtration FOr if A C and A B be not Primes to one another, they ſhall be meaſured by ſome number, beſides Unity, which ſuppoſe to be D: Therefore, ſeeing that D meaſureth the whole A C, and the

the part cut off A B, it shall also measure the rest B C; therefore A B and B C shall not be Primes to one another, seeing that D measureth them, which is absurd, and contrary to the supposition; therefore A C shall be Prime to A B, and to B C.

10. c. f. Now let A B and B C together, be Prime to one of them; to wit, to A B, I say A B and B C are Primes to one another, otherwise they should be measured by some number besides Unity, which let be D, therefore D measuring A B, and B C, shall also measure their Compound A C, and therefore A C, and A B are not Primes to one another, which is absurd, and against the supposition; therefore A B, and B C are Primes to one another, and in like manner, if A C be Prime to B C: Therefore, if two numbers, &c. Which was to be demonstrated.

A.....8 B.....5 C
D...

COROLLARIE.

It follows hence, that the number compounded of two others, and is Prime to one of them, is also Prime to the other, for if A C *be Prime to* A B, A B *and* B C *shall be Primes to one another, by the second part of this Proposition; therefore* A C *shall be also Prime to* B C, *by the first part of the same Proposition, which was proposed.*

PROP. 31. THEOR. 29.

A.....5 B.....8
C—

Every prime number A, *is prime to every other number which it measureth not.*

Demonstration FOr otherwise they should be measured by some number besides Unity, which suppose to be C; Therefore C shall not be the same as A, A being supposed not to measure B; therefore A being measured by another number C; A shall not be Prime, which is impossible, and contrary to the Supposition; Therefore, every first number, &c. Which was to be demonstrated.

PROP. 32. THEOR. 30.

A....4 B.....6
C.....24
D...3 E.....8

If two numbers A *and* B, *multiplying one another, do make some other* C, *and that some prime number* D, *doth measure their product; It shall also measure one of the numbers first proposed* A *or* B.

Demon-

Demonstration FOr as D doth not measure A, but it doth measure C as many times as there are unites in E, in such sort, as C may be produced of D by E, the which C is produced of A by B.

Forasmuch as C the product of the first, by E the fourth, is equal to the product of A, the second by B the third [a]; there will be the same reason of D the first, to A the second, as of B the third to A the fourth [b]: But D the first being prime to A, for that it measures it not [c], D and A shall be the least of their reason [d], therefore shall measure equally B and E: to wit, D the least, shall measure B the least, and A the greatest, E the greatest: Therefore, if D measure not A, it shall measure B, in like manner, it appears that if D measure not B, at least, it shall measure A: Therefore, &c. Which was to be demonstrated.

a) 19. 7.
b) 31. 7.
c) 23. 7.
d) 21. 7.

PROP. 33. THEOR. 31.

A..... 18
B.....6 C...

Every compound number A, *is measured by some prime number.*

Demonstration FOr being compounded, it [a] shall be measured by some number, which let be B: and if B be a Prime number, the Proposition is manifest: But if B be compounded, it shall be also measured by some number, which let be C, which shall be either prime or compounded: if prime, seeing that it measureth B, and B measureth A, the same [b] C prime, shall also measure A: therefore A shall be measured by a prime number: But if C be compounded, it shall be measured by another number.

Now forasmuch as one number cannot infinitely diminish it selfe, we shall arrive at last to a number, that no other number may measure, and therefore to a prime number measuring all the precedents, it [c] shall also measure the compound number A.

a) 13. d.
b) 11. c. f.
c) 11. c. f.

PROP. 34. THEOR. 32.

A.....9

Every number A *is a prime number, or else is measured by some prime number.*

Demonstration FOr seeing that every number is a prime or compound number, if A be a prime number, the Proposition is manifest, but if it be a compound number, it [a] shall be measured by some prime number: Therefore, every number is prime, or, &c, Which was to be demonstrated.

a) 33. 7.

PROP.

PROP. 35. PROBL. 3.

A.....6 B...4 C.....8
D..2
E...3 F..2 G....4
H — I — K —
L —

As many numbers as you pleaſe, A, B, *and* C, *being given, to find the leaſt numbers which have the ſame reaſon with them.*

Demonſtration FOr A, B, and C, are either primes to one another, or are not; if primes, [a] they ſhall be the leaſt of all thoſe which have the ſame reaſon; but if they be compounded, let D be found their greateſt common meaſure, which ſhall meaſure the ſame A, B, and C, by E, F, and G,: I ſay, that E, F, and G, are the leaſt of all thoſe that have the ſame reaſon of A, B, and C: foraſmuch as D meaſureth A, B, and C, by E, F, and G [b], D multiplying E, F, and G, ſhall produce A, B, and C: therefore E, F, and G, ſhall have the ſame reaſon as A, B, and C.

a) 23. 7.

b) 9. c. ſ.

I ſay alſo, that E, F, and G, are the leaſt: For if it be denied, let H, I, and K, be the leaſt in the ſame reaſon, the which ſhall meaſure A, B, and C, equally; Let them meaſure them then by L, that being ſo, [c] multiplying H, I, and K, will produce A, B, and A, and alternately, L ſhall meaſure A, B, and C, by H, I, and K; foraſmuch then, as E the firſt, multiplying D the fourth, and H the ſecond, multiplyiug L the third, doth produce the ſame A [d], as E the firſt, to H the ſecond; ſo L the third ſhall be to D the fourth: But E is greater then H, therefore, L ſhall be greater then D: and therefore, ſeeing that L doth meaſure A, B, and C; D ſhall not be the greateſt common meaſure of the numbers A, B, and C, which is contrary to the Hypotheſis: Therefore E, F, and G, are the leaſt in the reaſon of A, B, and C: Therefore, if, &c. Which was to be done.

21. 7.

c) 8. c. ſ.

d) 19. 7.

COROLLARIE.

It is manifeſt from this, that the greateſt common meaſure of as many numbers as you pleaſe, doth meaſure them by the leaſt numbers of all thoſe which have the ſame reaſon with them, for it hath been ſhewn that E, F, *and* G, *by the which* D, *the greateſt common meaſure of* A, B, *and* C, *meaſureth the ſame* A, B, *and* C, *are the leaſt in the reaſon of* A, B, *and* C, *and ſo of all others.*

PROP. 36. PROBL. 4.

A....4 B.....5
C.....20
D------
E---- F----

Two numbers A *and* B, *being given, to find the leaſt number that doth meaſure them.*

Con-

Conſtruction FIrſt of all, let A and B, be primes to one another, and multiplying one the other make C, I ſay, C is the number required.

Demonſtration NOw that it meaſureth them is manifeſt; For, C being the product of A by B, or of B by A, [a] A ſhall meaſure C by B, and B the ſame C by A; therefore, A and B do meaſure C, and if C be not the leaſt; let A and B meaſure another leſſer, to wit, D, (if poſſible) and as A meaſureth D by E, and B the ſame D by F [b]; ſo D ſhall be the product as well of A by E, as of B by F, and contrariwiſe; foraſmuch as the ſame D is produced of A the firſt, by E the fourth, and of D the ſecond, by F the third; as A the firſt, ſhall be to B the ſecond, ſo F the third, to E the fourth, therefore, A and B (being put primes to one another [c], and therefore the leaſt in their reaſon) they ſhall meaſure E and F the one as the other; to wit, A ſhall meaſure F: and B ſhall meaſure E.

a) 7. c. ſ. 9. c. ſ.

b) 19. 7.

c) 23. 7. 11. 7.

But, foraſmuch as A multiplying B and E, makes C and D; C ſhall be to D, as B to E: and therefore, ſeeing that B meaſureth E, as is ſhewn, C ſhall alſo meaſure D, the greateſt, the leaſt, which is impoſſible; therefore, C is the leaſt of all thoſe that do meaſure A and B.

Secondly, let A and B not be primes to one another; and [d] let C and D be found the leaſt in the ſame reaſon; to the end there be four numbers proportional; to wit, as A to B, ſo C to D, for ſo the product of [e] A the firſt, by D the fourth, ſhall be equal to the product of C the ſecond, by B the third, the which product let be E; I ſay, that E is the leaſt number which meaſureth A and B, and that it meaſureth them, is manifeſt; for E being the product as well of A in D, as B in C, [f] as well A as B, ſhall meaſure E, and if you deny it to be the leaſt, let A and B meaſure another, ſuppoſed leſſe then E; to wit F: Now as A meaſureth F by G, and B the ſame F by H, then [g] F ſhall be the product as well of A by G, as of B by H.

A.... B.....6
C..2 D...3
E.....12
F-----
G---- H---

d) 35. 7.

e) 19. 7.

f) 7. c. ſ.

g) 9. c. ſ.

Foraſmuch then, as the ſame F is made of A the firſt, in G the fourth, and of B the ſecond, in H the third: As A the firſt, to B the ſecond, ſo H the third, to G the fourth: therefore, C and D being the leaſt in the reaſon of A to B, or of H to G, it [h] meaſureth equally H and G, to wit, C ſhall meaſure H, and D ſhall meaſure G. But, foraſmuch as A multiplying D and G, makes E and F [i], E ſhall be to F, as D to G, and therefore D meaſuring G as is ſhewn, E ſhall alſo meaſure F, the greateſt the leaſt, which is impoſſible: Therefore, E is the leaſt: Therefore, Two numbers, &c. Which was to be done.

19. 7.

h) 21. 7.

i) 17. 7.

COROLLARIE.

From hence it follows, that if two numbers do multiply, the leaſt, having the ſame reaſon, the greateſt, the leaſt, and the leaſt the greateſt, the product ſhall be the leaſt number they ſhall

measure: For C *and* D *being ſuppoſed the leaſt in the reaſon of* A *to* B, *it hath been demonſtrated that* E *the product of* A *the leaſt, by* D *the greateſt, and of* B *the greateſt, by* C *the leaſt, is the leaſt, number meaſured by* A *and* B.

PROP. 37. THEOR. 33.

A..2 B...3
C------ F----D
E.....6

If two numbers A *and* B, *do meaſure ſome other number* C *and* D, *the leaſt it meaſureth* E, *ſhall alſo meaſure the ſame number.*

Demonſtration FOr if E doth not meaſure C D, having taken E from C D, as often as may be, there will remaine a number leſſe then E; let there remaine F D, leſſe then E, (if poſſible) in ſuch ſort as E may meaſure the part cut off C F: Foraſmuch then, as A, as well as B, doth meaſure E, and E meaſureth C F, [a] as well A as B ſhall alſo meaſure C F. Therefore, ſeeing that A and B meaſureth the whole C D, and the part cut off C F, [b] it ſhall alſo meaſure the reſidue F D. Now F D is leſſe then E: therefore, E is not the leaſt number that A and B do meaſure, which is contrary to Suppoſition: Therefore, if two numbers, &c. Which was to be demonſtrated.

a) 11. c. ſ.
b) 12. c. ſ.

PROP. 38. PROBL. 5.

A...3 B....4 C.....6
D.....12
E------

Three numbers A, B, *and* C, *being given, to find the leaſt number they meaſure.*

A..2 B...3 C....4
D.....6
E.....12
F-----

Conſtruction LEt [a] D be the leaſt number that A and B doth meaſure, C doth meaſure it, or doth not meaſure it: if it doth meaſure it: I ſay, that it is the leaſt that the three A, B, and C do meaſure: otherwiſe, ſuppoſe E to be leſſe then D meaſured by A, B, and C: therefore A and B meaſuring E, leſſe then D: D ſhall not be the leaſt meaſured by A and B, which is againſt the Suppoſition.

If C meaſure not D, ſuppoſe [b] E to be the leaſt meaſured by C and D; I ſay, that it ſhall be the leaſt that A, B, and C, do meaſure, for A B meaſuring D, and D meaſuring E, [c] A and B ſhall alſo meaſure E, but C in like manner, meaſureth E; therefore, the three A, B, and C, meaſure E; and if you ſay E is not the leaſt, let F leſſe then E, be meaſured by A, B, and

a) 36. 7.
b) 36. 7.
c) 11. c. ſ.

and C, (if poſſible. Seeing A and B do meaſure F, D the leaſt, meaſured by them, [d] ſhall alſo meaſure F and C, D meaſuring F: (for A, B, and C, meaſures it,) E the leaſt number meaſured by C D, ſhall meaſure in like manner, F the greateſt, the leaſt, which is abſurd: Therefore E is the leaſt that A, B, and C, meaſureth. Therefore, &c. which was to be done.

d) 37. 7.

COROLLARIE.

From hence follows, that if three numbers do meaſure a number that the leaſt number they meaſure, meaſureth alſo the ſame number: For in the laſt part of this Demonſtration, for that A, B, *and* C, *are put to meaſure* F, *it hath been demonſtrated that* E *which is the leaſt number meaſured by them, doth alſo meaſure* F.

PROP. 39. THEOR. 34.

A.....12
B....4 C...3

If one number B, *meaſure another number* A, *that which it meaſureth* A, *ſhall have a part denominated of the number meaſuring* B.

Demonſtration FOr let B meaſure A as many times as there are unites in C; foraſmuch then, as unity meaſureth C, and B meaſureth A equally: [a] alternately, unity ſhall meaſure B, and C ſhall meaſure A equally, and therefore unity ſhall be the ſame part of B, as C is of A [b], but unity is a part of B denominated of the ſame B: therefore, C ſhall be a part of A denominated by B. Therefore, if a number &c. Which was to be demonſtrated.

a) 15. 7.
b) 20. d. 7.

PROP. 40. THEOR. 35.

A.....15
B...3 C.....5

If a number A, *hath a part* B, *the number denominated of that part* C, *meaſureth it.*

Demonſtration FOr ſeeing that B part of A, is denominated of C, but unity is one part of C denominated of the ſame C, unity ſhall meaſure C, and B ſhall meaſure A equally [a]: Therefore, alternately, unity ſhall meaſure B; and C ſhall meaſure A equally. Therefore, if a number, &c. Which was to be demonſtrated.

a) 15. 7.

PROP. 41. PROBL. 6.

To find the leaſt number which hath the parts given A, B, *and* C.

D..2 A Second
E...3 B Third
F....4 C fourth
G.....12
H-----

Conſtruction LEt D, E, and F, be denominated of the parts A, B, and C, and let G [a] be the leaſt that D, E, and F, do meaſure; I ſay, that G is the leaſt, having the parts given, A, B, and C.

a) 38. 7.

Demonſtration FOr ſeeing that D, E, and F, do meaſure G, [b] G ſhall have the parts denominated by D, E, and F; that is to ſay, the parts A, B, and C, being denominated of D, E, and F; I ſay, that G is alſo the leaſt, having ſuch parts.

b) 39. 1.

40. 7. For if it be denied, let another leſſe then G have the ſame parts A, B, and C, which ſuppoſe to be H; Foraſmuch then as H hath the part A, B, and C, the numbers D, E, and F, denominated of the ſame parts A, B, and C, ſhall meaſure H; therefore, H being leſſe than G; G ſhall not be the leaſt number meaſured by D, E, and F, which is contrary to Suppoſition. Therefore, a leſſe number then G, ſhall not have the parts given, A, B, and C, but G ſhall be the leaſt: Therefore, &c. Which was to be demonſtrated.

The End of the Seventh Element of EUCLIDE.

THE EIGHTH ELEMENT OF EUCLIDE.

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

A.....8 D —
B.....12 E —
C.....18 F —

If there be as many numbers as you pleaſe continually proportional: A, B, *and* C, *and that the extreams* A *and* C, *be primes to one another, they ſhall be the leaſt of all thoſe which have the ſame rate with them.*

Demonſtration FOr if it be denied, ſuppoſe D, E, and F, leſſe then they in the ſame reaſon: Foraſmuch as there are as many numbers as you pleaſe of the one part A, B, and C, and as many of the other, D, E, and F, which taken two and two, are in the ſame rate [a]; in equal rate, as A ſhall be to C, ſo D to F [b]: But A and C are the leaſt of their rate, being primes to one another, by Suppoſition, [c] therefore as well A, as C, ſhall meaſure D and F equally, the greateſt, the leaſt, which is impoſſible: Therefore, A, B, and C, are the leaſt of their rate. Therefore, if, &c. Which was to be demonſtrated.

a) 14. 7.
b) 23. 7.
c) 21. 7.

PROP.

PROP. 2. PROBL. 1.

A 2 B 3
C 4 D 6 E 9
F 8 G 12 H 18 I 27

To find as many numbers as you please continually proportional, the least in a given rate.

Construction LEt A multiplying it self produce C, and multiplying B produce D, and B by it self produce E; I say, that C, D, and E, are the least in the rate of A to B, and are continually proportional.

Demonstration FOrasmuch as A multiplying A and B, makes C and D: as A is to B, so C is to D; again, seeing that B multiplying A, B makes D and E, [a] as A is to B, so D is to E: therefore, C, D, and E, are continually proportional in the rate of A to B, and are also the least.

a) 17. 7.

For seeing the extreams C and E, are products of A and B, multiplyed each by himself [b]; But A and B are primes to one another, being the least of their rate, [c] the extreams C and E shall be also primes to one another. Therefore C, D, and E, are the least in the rate of A, B, and C.

b) 24. 7.
c) 29. 7.

Secondly, let A multiplying the three numbers C, D, and E, produce F, G, and H, and B multiplying E make I; I say, that the four numbers F, G, H, and I, are the least in the same given rate of A to B, and continually proportional.

1. 8.

Demonstration FOr seeing that A multiplying C, D, and E, hath produced F, G, and H; F, G, and H, shall have the same rate as C, D, and E, that is to say, as A and B; again, seeing that A and B, multiplying E, have made H and I, [d] as A is to B, so H is to I, therefore F, G, H, and I, are continually proportional in the given rate of A and B, and are also the least.

d) 18. 7.

For A and B, being the least in their rate, [e] they shall be primes to one another: But C and E are products of A and B multiplyed by themselves, and F and I are also products of A and B, in C and E: to wit, F of A in C, and I of B in F, [f] F and I the extreams shall be also primes to one another: Therefore, F, G, H, and I, are the least of their rate, which is the rate of A to B, by the same reason we shall find 5 or 6, &c. Therefore, we have found, &c. which was to be done.

e) 24. 7.
f) 29. 7.
1. 8.

COROLLARIE. I.

It follows hence that, if three numbers are continually proportional, and the least of their rate, the extreams shall be squares: and if 4 be continually proportional, the extreames shall be Cubes, for C and E the extreams, are products of A and B, each by himself, and F and I, extreams of four proportionals are products of A and B, multiplyed each by his square C and D, &c.

CO-

COROLLARIE II.

It follows also that the extreams of the numbers continually proportional, found to be the least in the given rate, (by this Proposition) are Primes to one another, [a] it having been demonstrated that C E and F I, are Primes to one another.

a) 24, 29. 7.

COROLLARIE III.

It appears also that two numbers, the least in the given rate, do measure all the means whatsoever of the least in the same rate, being products of the multiplication of them by some other numbers, as D the mean, which is produced of A by B, and G H means of A, by D and E, or of B by C and D, &c.

PROP. 3. THEOR. 2.

A 8 B 12 C 18 D 27
E 2 F 3
G 4 H 6 I 9
K 8 L 12 M 18 N 27

If there be as many numbers as you please continually proportional A, B, C, and D, the least of all those that have the same rate with them, the extreams A and D, are primes to one another.

Demonstration FOr [a] let E and F be found the least in the rate of A to B, or B to C, or C to D, [b] E and F, shall be primes to one another: again, let the three G, H, and I, be found the least in the rate of E to F: and also the four K, L, M and N, and so following, until that K, L, M, and N, be equal in number to A, B, C, and D: Forasmuch, as A, B, C, and D, are the least of their rate, and K, L, M, and N, equal in number to them, being also the least in the same rate, they shall be equal to them, each to his correspondent; to wit, A to K, and D to N, [c] now K and N are primes to one another: Therefore, A and D are also primes to one another. Therefore, if there be, &c. Which was to be demonstrated.

a) 35. 7.
b) 2. 8.
c) 2 Cor. 1. 8.

PROP. 4. PROBL. 2.

A 6 B 5 C 4 D 3
H 4 F 24 E 20 G 15
I — K — L —

A 6 B 5 C 4 D 3 E 5 F 7
H 24 G 20 I 15 K 21
L — D — N — O —

A 6 B 5 C 4 D 3 E 2 F 7
H 24 G 20 I 15
M 48 L 40 K 30 N 105
O — P — Q — R —

There being given as many rates as you please A to B, and C to D, from their least numbers, to find as many numbers as you please continually proportional, the least according to the given rate.

Con-

Construction HAving found E the least number measured by B and C, 2 and 3; let F be found as oftentimes measured by A, as E by B, and G as oftentimes measured by D, as E by C: F, E, and G, shall be continually proportional, the least according to the given rate.

Demonstration FOrasmuch as A and B do equally measure F and E; that is to say, by one and the same number; let them measure it by H, that being, [b] A and B multiplying H, shall produce F and E; wherefore, as A is to B, so F is to E. In like manner, C and D, equally measuring E and G, as C to D, so E shall be to G; therefore, F, E, and G, are continually proportional, according to the rates of A to B, and C to D. I say, that they are also the least. For if it be denied, suppose I, K, and L, to be lesse in the same rates, each to his correspondent. Forasmuch then as A and B are the least in their rate, [c] the same shall equally measure I and K, being in the same rate; to wit, B consequent, shall measure K consequent: by the same reason C and D shall equally measure K and L; to wit, C antecedent, K antecedent: Therefore, B and C, measuring K and E, [d] the least, measured by B and C, shall also measure K, the greatest, the least, which is absurd, therefore F, E, and G, are the least.

A 6 B 5 C 4 D 3
H 4 F 24 E 20 G 15
I — K — L —

A 6 B 5 C 4 D 3 E 5 F 7
H 24 G 20 I 15 K 21
L — D — N — O —

A 6 B 5 C 4 D 3 E 2 F 7
H 24 G 20 I 15
M 48 L 40 K 30 N 105
O — P — Q — R —

Let there be three rates given to the least numbers of A to B, C to D, and of E to F, to find four numbers, the least continually proportional, according to the given rates, [e] having again found G the least that measureth B, and C 2 and 3; let H be found as often times measured by A, as G by B, and I as oftentimes measured by D, as G by C, that being so, E shall measure I, or shall not measure it. Let it measure it in the first place, and as often times as I is measured by E, let K be as often times measured by F. I say, that H, G, I, and K, are continually proportional, and the least, according to the given rates, as hath been shewn.

Now let E not measure I, and [f] having found K the least that E and I do measure, let L be as often times measured by G, and M by H, as K by I; and let also N be as often times measured by F, as K by E; I say, that M, L, K, and N, are the least according to the given rates.

Demonstration FOr, H, G, and I, measuring M, L, and K, equally, as is before shewn, M shall be to L, as H to G; and L to K, as G to I: but by the same reason, H is to G, as A to B, and G to I, as C to D; (A and B measuring H and G, and C and D, G and I, equally,) therefore, as A to B, so is M to L, and as C to D, so is L to K. But as E is to F, K is also to N: (E and F equally measuring K and N,) therefore, M, L, K, and N, are continually proportional, according to the given rates.

They are also the least; For if it be denied, let O, P, Q, and R, be supposed the least according to the same rates. Forasmuch as A and B are the least of their rate, [g] they shall equally measure O and P in the same rate; to wit, B consequent, shall measure P consequent, and by the same reason, C and D, shall measure P and Q equally; to wit, C antecedent, P antecedent: wherefore, B and C measuring P, [h] G the least number measured by B and C, shall measure the same P. Now

a) 36. 7.
b) 9. c. f. 18. 7.
c) 21. 7.
d) 37. 7.
e) 36. 7.
f) 36. 7.
g) 21. 7.
h) 37. 7.

Now having shewn, that as G is to I, so is L to K: that is to say, so is P to Q: alternately, as G to P, so I to Q: and therefore G measuring P, I shall measure Q: But E shall also measure Q, [i] seeing that E and F, the least of their rate, do equally measure Q and R, in the same rate; to wit, the antecedent E, the antecedent Q: therefore, I and E, measuring Q and K, the least number measured by I and E, shall also measure Q, the greatest, the least, which is absurd: Wherefore M, L, K, and N, are the least: and so following, if there be more rates given to the least numbers, &c. Therefore, being given, &c. Which was to be demonstrated.

i) 21. 7.

PROP. 5. THEOR. 3.

A 24 B 48
G 18
C 4 D 6 E 3 F 19

Plain numbers A *and* B, *are the one to the other in a rate compounded of their sides.*

Demonstration FOr D multiplying E, produce G, forasmuch as D multiplying C and E, hath produced A and G, A shall be to G, as C to E: and seeing that E multiplying D and F, hath produced G and B; [a] G shall be to B, as D to F: therefore, A, G, and B, are continually proportional, according to the rates of C to E, and of D to F the sides. But [b] the rate of A to B, is compounded of the rates of A to G, and of G to B: therefore, the same rate of A to B, shall be compounded of the rates of C to E, and of D to F, the sides. Therefore, the numbers, &c. Which was to be demonstrated.

17. 7.
a) 17. 7.
b) 27. d. 7.

PROP. 6. THEOR. 4.

A 16 B 24 C 36 D 54
F 4 G 9 H 9

If there be as many numbers as you please, continually proportional, A, B, C, *and* D, *and that the first* A, *measure not the second* B, *also neither of the other, shall measure any of the other.*

Demonstration FOr that neither of them doth measure his next following, is manifest, forasmuch as A, B, C, and D, are continually proportional, and as A measureth not B, [a] it shall be no part thereof, but parts: wherefore [b] B, which is the same parts of C, shall not also measure C, nor C and D: I say also, that neither of the other, shall measure any of the other, as A measureth not C. [c] For having found F, G, and H, the least in the rate of A to B, or of A, B, and C, in equal rate, as A to C, so is F to H: but seeing that as A is to B, so F to G, & as A measureth not B, F shall not measure G, wherefore F shall not be unity, otherwise F should measure G, unity measuring every number: therefore, [d] F and H, being primes to one another, & F being not unity, F shall not measure H, and therefore also A shall not measure

a) 3, 4. d. 7.
b) 20. d. 7.
c) 2. 8. 35. 7.
d) 3. 8.

C: for it hath been ſhewn that as A is to C, ſo F is to H; in like manner, is ſhewn that as B meaſureth not D, taking four numbers in the rate of A to B, and the leaſt, not any other ſhall meaſure any other. Therefore, &c. Which was to be demonſtrated.

PROP. 7. THEOR. 5.

A 3 B 6 C 12 D 24 E 48

If there be as many numbers as you pleaſe, A, B, C, D, *and* E, *continually proportional, and that the firſt* A, *meaſureth the laſt* E, *it ſhall alſo meaſure the ſecond* B.

Demonſtration FOr if A be not ſaid to meaſure B the ſecond, [a] alſo neither of the other ſhall meaſure either of the other: therefore A the firſt ſhall not meaſure E the laſt, which is abſurd, A being put to meaſure E. Therefore, if there be, &c. Which was to be demonſtrated.

a) 6. 8.

PROP. 8. THEOR. 6.

A 24 C 36 D 54 B 81
G 8 H 12 I 18 K 27
E 32 L 48 M 72 F 108

If between two numbers A *and* B, *there fall mean proportionals* C *and* D, *in continual proportion: as many as ſhall fall between thoſe (of means continually proportional) there ſhall fall as many mean proportionals between two others that ſhall have the ſame rate.*

Demonſtration FOr having taken G, H, I, and K, the leaſt in the rate of A to C, and equal in number to A, C, D, and B, in equal rate, as A ſhall be to B: and therefore, as E to F, ſo G to K [a]: wherefore, G and K, being primes to one another: ſeeing they are the extreams of the leaſt numbers, [b] and ſo the leaſt of their rate: [c] G ſhall meaſure E equally, and K ſhall meaſure F: therefore, as often times as G and K ſhall meaſure E and F, let there be two other numbers L and M, as many times meaſured by H and I, in ſuch ſort as G, H, I, and K, may meaſure E, L, M, and F, equally, each his correſpondent; [d] wherefore, G, H, I, and K, multiplying the number by the which they meaſure E, L, M, and F; they ſhall produce E, L, M, and F: therefore, E, L, M, and F, ſhall be in the ſame rate as G, H, I, and K, but G, H, I, and K, are continually proportional: therefore E, L, M, and F, ſhall be ſo alſo, and being equal in number to A, B, C, and D, there ſhall fall as many mean proportionals between E and F, as between A and B. Therefore if, &c. Which was to be demonſtrated.

2. 8.
a) 3. 8.
b) 23. 7.
c) 21. 7.
d) 9. c ſ.

PROP.

PROP. 9. THEOR. 7.

A 8 C 12 D 18 B 27
Unity 1
E 2 F 3
G 4 H 6 I 9
K 8 L 12 M 18 N 27

If two numbers A *and* B, *are primes to one another, and between them there fall mean numbers, continually proportional,* C, *and* D, *as many mean continual proportionals as ſhall fall between them, there ſhall alſo fall as many between each of them and unity.*

Demonſtration FOr having propoſed unity, let E and F be found the leaſt in the rate of A to C, and let the three G, H, and I, be taken in the ſame rate, and then the four K, L, M, and N, and ſo on until the numbers taken be as many as A, C, D, and B. Now A and B the extreams, being primes to one another, A, C, D, and B, ſhall be the leaſt in the rate of E to F: But K, L, M, and N, which are as many as A, C, D, and B, are alſo the leaſt in the ſame rate, by the conſtruction, therefore, K, L, M, and N, are equal to A, C, D, and B, each to his correſpondent, that is to ſay, K to A, and N to B, to the end there be none leſſer than the leaſt. 2. 8. 1. 8.

But, (as is manifeſt by the demonſtration of the ſecond Prop. 8.) E multiplying it ſelfe, hath produced G, and multiplying G hath produced K; E ſhall [a] meaſure G by E, and G ſhall alſo meaſure K by E: But [b] unity meaſures E by E, therefore unity meaſures E, and E meaſures G, and G meaſures K equally; and therefore, unity is the ſame part of E, and E of G, as G of K; therefore, unity and E, G, and K, are continually proportional: in like manner, unity and F, I, and N, are continually proportional: Therefore, as well E, G, and K, as F, I, and N, with unity, being equal in number to K, L, M, and N, there ſhall fall as many numbers continually proportional between unity and K, or his equal A; and between unity and N, or his equal B, as between A and B. Therefore, if two, &c. Which was to be demonſtrated. a) 7. c. ſ. b) 5. c. ſ. 20. d.

PROP. 10. THEOR. 8.

A 8 I 12 K 18 B 27
E 4 H 6 G 9
D 2 F f 3
C 1

If between two numbers A *and* B, *and unity* C, *there fall numbers continually proportional, as many as ſhall fall of continual proportionals between each of them and unity, there ſhall as many continual proportionals fall between them* A *and* B.

Demonſtration LEt A be the product of D in F, and I the product of D in H, and K the product of F in H, C being to D, as D to E, and

a) 5. c. f. b) 9. c. f. D to E, as E to A: [a] and C measuring D by the unites that are in D, D shall measure E, also by the unites which are in D, [b] therefore, D by it self shall produce E, and E by D shall produce A; in like manner, F by himself shall produce G, and G by F shall produce B.

c) 17. 7. A 8 I 12 K 18 B 27
E 4 H 6 G 9
D 2 F 3
C 1

Forasmuch then as D multiplying D and F, produceth E and H, [c] as D is to F, so E is to H: by the same reason, seeing that F multiplying F and D, hath made H and G, as D is to F, so H is to G: therefore, E, H, and G, are continually proportional.

d) 17. 7. Again, D multiplying E and A, hath made A and I; therefore, as E [d] shall be to H, so A to I: and seeing that D and F multiplying H, makes I and K, c 18. 7. [c] as D shall be to F, so I to K: and in f 17. 7. [f] like manner, K shall be to B, as H to G: therefore A, I, K, and B, are continually proportional: therefore between A and B, there doth fall two meanes continually proportional: to wit, as many as between A and unity, or B and the same unity. Therefore, &c. Which was to be demonstrated.

PROP. 11. THEOR. 9.

A 9 E 21 B 49
C 3 D 7

Between two square numbers A, *and* B, *there is a mean proportional number: and the square* A, *is to the square* B, *in a double rate of the side* C, *to the side* D.

Demonstration FOr let E be the product of C in D, or of D in C, forasmuch as C by himself hath made A [a] a square number, and a) 18. d. by D hath made E, [b] as C is to D, so A is to E: again, seeing that D multiplying C hath made E, and multiplying it self, hath made B a square b) 17. 7. number, as C shall be to D, so E to B: therefore, A, E, and B, shall be continually proportional in the rate of their sides C and D: Therefore, between A and B there falls a mean proportional number E. 18. d. 17. 7.

c) 6. d. 7. 10. p. 5. Secondly, seeing that A, E, and B, are continually proportional, [c] A the first, shall be to B the third in a double rate of A the first, to E the second, which is the same rate of the side C. Therefore, between, &c. Which was to be demonstrated.

PROP. 12. THEOR. 10.

A 27 H 36 I 48 B 64
E 6 G 12 F 16
C 3 D 4

Between two Cube numbers A *and* B, *there are two numbers mean proportional,* H *and* I, *and the Cube* A, *is to the Cube* B, *in a Triple rate of the side* C, *to the side* D.

Demon-

Demonstration FOr C multiplying it self, makes E; and D multiplying it self makes F; and C and D, multiplying one another makes G, and multiplying G makes H and I.

a) 17. 7. Forasmuch as C multiplying C and D, hath made E and G, [a] as C shall be to D, so E to G: In like manner, seeing that D multiplying D and C, hath made G and F, b 17. 7. [b] as C is to D, so G is to F: therefore, E, F, and G, are continually proportional in the rate of C to D.

19. d. c) 17. 7. Again, forasmuch as C multiplying E, hath made A, a Cube number, and multiplying G hath made H, by the construction, [c] A shall be to H, as E to G, that is to say, as C to D: in like manner, seeing that by the construction, D multiplying G, hath made I, and multiplying F hath made the Cube B [d], I shall be to B, as G to F [e], that is to say, as C to D. But, [f] H is also to I, as C to D: forasmuch as C and D multiplying G, have made H and I: Therefore, A, H, I, and B, are continually proportional in the rate of C to D: therefore, between the Cubes A and B, there do fall two mean proportionals H and I. d) 19. d. e) 17. 7. f) 18. 7.

Secondly, forasmuch as A, H, I, and B, are continually proportional, A the first shall be to B the fourth, in a Triple rate of A the first, to H the second, that is to say, of the side C, to the side D: for C is to D, as A to H. Therefore, between two Cubes, &c. Which was to be demonstrated.

PROP. 13. THEOR. 11.

A 2 B 4 C 8
D 4 N 8 E 16 O 32 F 64
G 8 P 16 Q 32 H 64 R 128 S 256 I 512

If there be as many numbers as you please continually proportional, A, B, *and* C, *and that each multiplying it self, make others, their products* D, E, *and* F, *shall be proportional, and if the numbers first taken* A, B, *and* C, *multiplying their products* D, E, *and* F, *do make others* G, H, *and* I, *those shall also be proportional, and alwayes this happens about the extreams.*

Demonstration FOr let A and B multiplying one another make N, and B and C make O: then let A multiplying N and E, make P and Q, and B multiplying O and F, make R and S.

a) 17. 7. Forasmuch as A multiplying A and B hath made D and N, [a] as A to B, so D to N, by the same reason, seeing that B multiplying A and B, hath made N and E, as A is to B, so N to E: therefore, D, N, and E, are continually proportional in the rate of A to B.

b) 17. 7. Again, seeing that B multiplying B and C, hath made E and O, [b] as B is C, so E is to O: in like manner, seeing that C multiplying B and C, hath made O and F, as B is to C, so O is to F; therefore, E, O, and F, are continually proportional in the rate of B to C, or of A to B: therefore, D, N, and

and E, being continually proportional in the same rate: in equal rate, as D to E, so E to F: therefore, D, E, and F, are continually proportional.

Moreover, seeing that A multiplying D, N, and E, hath made G, P, and Q; G, P, and Q shall be in the same rate as D, N, and E: that is to say, as A to B. And seeing that A and B multiplying E, have made Q and H, c) 18. 7. [c] as A is to B, so Q is to H; therefore, G, P, Q, and H, are proportional in the rate of A to B: in like manner, seeing that B multiplying E, O, and F, hath made H, R, and S, H, R, and S, shall be proportionals in the rate of E, O, and F, that is to say, of B to C, or of A to B.

d) 18. 7. Lastly, seeing that B and C, multiplying F, have made S and I, [d] S shall be to I, as B to C, or A to B: therefore, H, R, S, and I, are proportionals in the rate of A to B: therefore, seeing that G, P, Q, and H, are also in continual proportion in the rate of H, R, and I, in equal rate, as G shall be to H, so H to I: therefore G, H, and I, are continually proportional. Therefore, if there be, &c. Which was to be demonstrated.

PROP. 14. THEOR. 12.

A 4 E 12 B 36
C 2 D 6

If a square number A, *doth measure a square number* B, *also the side* C, *shall measure the side* D, *and if the side* C *doth measure the side* D, *also the square* A, *shall measure the square* B.

Demonstration FOr let C and D multiplying one another, make E, forasmuch as C multiplying C and D, hath made A and E, [a] as C shall be to D, so A to E: in like manner, seeing that D multiplying C a) 17. 7. and D, hath made E and B, E [b] shall be to B, as C to D: that is to say, as b) 17. 7. A to E: therefore, A, E, and B, are continually proportional in the rate of C to D. But A the first, measureth B the last: therefore, A the first, shall 7. 8. also measure E the second: therefore seeing that C is to D, as A to E, and A measureth E, therefore the side C, shall measure the side D.

Secondly, let the side C, measure the side D: I say, that the square A, measureth the square B: [c] (for as hath been demonstrated, A, E, and B, c) 11. 8. are continually proportional in the rate of C to D, but C measures D: therefore A shall measure E, and E shall measure B: [d] therefore also A shall d) 11 c. 5. measure B. Therefore, if one number, &c. Which was to be demonstrated.

PROP. 15. THEOR. 13.

A 8 H 24 I 22 B 216
E 4 G 12 F 36
C 2 D 6

If a cube number A, *measure a cube number* B, *also the side* C, *shall measure the side* D, *and if the side* C, *measure the side* D, *also the cube* A, *shall measure the cube* B.

Demon-

Demonstration FOr let C and D, each multiplying it self, make E and F, and multiplying one another make G, and lastly, multiplying G, make H and I: Forasmuch then, (as by the demonstration of the 12 Prop. 8.) as well E, G, and F, as A, H, I, and B, are continually proportional in the rate of C to D: But A the first, measureth B the last, [a] A the a) 7. 8. first shall measure also H the second, therefore A being to H, as C to D: C the side shall also measure D the side.

Now let the side C, measure the side D: I say, that the cube A shall measure the cube B: For by the reason above, as C is to D: so A is to H, A, H, I, and B, being continually proportional in the rate of C to D, as hath been shewn in the 12 Prop. 8. Wherefore C the side, measuring the side D, also A, shall measure H: but H measures I, and I measures the cube B: (being continually proportional:) [b] therefore A shall also measure the cube b) 11 c. 5. B. Therefore, if a number, &c. Which was to be demonstrated.

PROP. 16. THEOR. 14.

A 16 B 81
C 4 D 9

If a square number A, *doth not measure a square number* B, *also the side* C, *shall not measure the side* D; *and if the side measure not the side also, the square shall not measure the square.*

Demonstration FOr suppose the side C doth measure the side D: [a] therefore a) 14. 8. the square A shall measure the square B, which is absurd: for it is proposed not to measure it, wherefore C the side shall not measure the side D.

Now I say, that if the side C doth not measure the side D, that also the 14. 8. square A shall not measure the square B: for if A be said to measure B, the side C shall also measure the side D, which is absurd, for it is proposed not to measure it: Therefore, the square A shall not measure the square B. Therefore, if a square number, &c. Which was to be demonstrated.

PROP. 17. THEOR. 15.

A 8 B 27
C 2 D 3

If a cube number A, *measure not a cube number* B, *also the side* C, *shall not measure the side* D, *and if the side measure not the side, neither shall the cube measure the cube.*

Demonstration FOr suppose that the side C doth measure the side D, [a] therefore the cube A, shall measure the cube B, which is a) 15. 8. absurd: for it is proposed not to measure it: therefore, C shall not measure D.

Now

Now, as the side C measureth not the side D; I say, that the cube A, shall not measure the cube B. For if A be said to measure B, the side C b shall also measure the side D, which is absurd: For it is proposed not to measure it: Therefore the cube A, shall not measure the cube B. Wherefore if a cube number, &c. Which was to be demonstrated.

b) 15. 8.

PROP. 18. THEOR. 16.

A 13 G 18 B 27
C 6 D 2 E 9 F 3

Between two Plain numbers alike, A *and* B, *there is a mean proportional number* G, *and the Plain* A *is to the Plain* B, *in a double rate of the Homologal sides (or sides of like rate)* C E, *and* D F.

Demonstration FOr let D and E multiplying one another, make G; forasmuch then as C is to D, so E is to F, alternately, C shall be to E, as D to F: and seeing that D multiplying C and E, hath made A and G, a A shall be to G, as C to E: that is to say, as D to F: and seeing that E multiplying D and F hath made G and B, b G shall be to B, as D to F: therefore A, G, and B, are continually proportional in the rate of C to E: or of D to F: therefore between A and B, there is a mean proportional G. But forasmuch as A, G, and B, are continually proportional, c A is to B, in a double rate of A to G; that is to say, of C to E, or of D to F: Therefore, the Plain A, is to the Plain B, in a double rate of C to E, or of D to F, sides of one and the same rate: Therefore, between, &c. Which was to be demonstrated.

a) 17. 7.
b) 17. 7.
c) 26. d. 7.

PROP. 19. THEOR. 17.

Between two Solid numbers alike A *and* B, *there are two mean proportional numbers* M *and* N, *and the Solid* A, *is to the Solid* B, *in a triple rate of the Homologal side* C, *to the Homologal side* F.

A 30 M 60 N 120 B 240
I 6 L 12 K 24
C 2 D 3 E 5 F 4 G 6 H 10

Demonstration FOr let C and D, multiplying one another, make I; and F and G, make K; and D and F, make L: Lastly, E and H, multiplying L, make M and N: Forasmuch then as C, D, and E, are proportional to F, G, and H: alternately, they shall be also proportional; to wit, as C to F, so D to G, and E to H: and seeing that D multiplying C and F, hath made I and L, a as C is to F, so is I to L: in like manner, seeing that F multiplying D and G, hath made L and K; as D is to G, so is L to K; wherefore I, L, and K, are continually proportional in the rate of C to

a) 17. 7.

to F, D to G, or of E to H: And forasmuch as A solid, is produced from the mutual multiplication of the sides C, D, and E: and I is made of C in D, or of D in C; E multiplying I, shall make A; in like manner, B solid, being made of the mutual multiplication of F, G, and H; and K of F in G, or of G in F, H multiplying K, shall make B.

Wherefore, seeing that E multiplying I and L, hath made A and M, b A shall be to M, as I to L: that is to say, as C to F, or D to G, or E to H, by the same reason, seeing that H multiplying L and K, hath made N and B, c N shall be to B, as L to K: that is to say, as C to F, or D to G, or E to H: But d M is to N, as E to H, seeing that E and H multiplying L, have made M and N: therefore, A, M, N, and B, are continually proportional in the rate of C to F, or of D to G, or E to H: therefore between A and B, like solids, there falls two mean numbers continually proportional.

Secondly, seeing that A, M, N, and B, are continually proportional, c A is to B, in a triple rate of A to M: But A is to M, as C is to F, or D to G, or E to H: therefore A shall be to B in a triple rate of C to F, or of D to G, or of E to H, sides of like rate: Therefore, between two Solids, &c. Which was to be demonstrated.

b) 17. 7.
c) 17. 7.
d) 18. 7.
e) 10. d. 5. 26. d. 7.

PROP. 20. THEOR. 18.

A 18 C 24 B 32
D 3 E 4 F 6 G 8

If between two numbers A *and* B, *there fall a mean proportional number* C, *those numbers* A *and* B, *shall be like Plaines.*

Demonstration FOr having taken D and E, the least in the rate of A, C, and B; a D and E shall equally measure A and C: Let them measure them by F, they shall also equally measure C and B, in the same rate, which let be by G: therefore, b F multiplying D and E, shall produce A & C, and c G multiplying D & E, shall produce C and B: Forasmuch then as E multiplying F and G hath produced C and B, d as C shall be to B, so F to G, but as C is to B, so was D to E: therefore, D shall be to E, as F to G, and by permutation, as D to F, so E to G: But forasmuch as F multiplying D, hath made A, A shall be a Plain number, and the sides shall be D and F: in like manner, seeing that G multiplying E, hath made B, B shall be also a Plain whose sides shall be E and G, and those sides, being shewn to be proportional: to wit, as D to F; so E to G, e A and B shall be like Plaines: Therefore, if between, &c. Which was to be demonstrated.

a) 21. 7.
b) 9. c. f.
c) 9. c. f.
d) 17. 7.
c) 21. d. 7.

PROP. 21. THEOR. 19.

A 8 C 12 D 18 B 27
E 4 F 6 G 9
H 2 I 2 M 3 K 3 L 3 N 3

If between two numbers A *and* B, *there fall two mean proportionals* C *and* D, *they ſhall be like Solids* A *and* B.

Demonſtration LEt [a] E, F, and G, be the leaſt in the rate of A, C, D, and B; foraſmuch as between E and G, there falls a mean proportional F; [b] E and G ſhall be like Plaines. a) 2. 8. b) 20. 8.

Let H and I be ſides of E and K, and L the ſide of G, and foraſmuch as E, F, and G, doe meaſure A, C, and D, equally [c], being the leaſt in the ſame rate: Let them meaſure them by M, and [d] ſeeing that E, F, and G, do alſo meaſure C, D, and B; by the ſame reaſon, let it be by N: in ſuch manner, as M multiplying E, F, and G, may make A, C, and D; and N alſo multiplying E, F, and G, may make C, D, and B. Now E is produced of his ſides H and I, multiplyed by one another: and G is produced of the ſides K and L: therefore, A is the product of H, I, and M, multiplyed together, and B the product of K, L, and N: Therefore, [e] A is a ſolid number, having the ſides H, I, and M, and B alſo a ſolid number, having the ſides K, L, and N: But ſeeing that M and N, multiplying F, do make C and D, as is ſhewn, [f] C ſhall be to D, as M to N [g]; but C and D, are in the ſame rate as E and F: ſeeing that N multiplying E and F, hath made C and D, and alſo E and F, are in the ſame rate as H and K, or I and L: therefore, H ſhall be to K, and I to L, as M to N, and alternately, H ſhall be to I, as K to L, and I to M, as L to N: Therefore the ſides H, I, and M, are proportional to the ſides K, L, and M: [h] Therefore, the numbers A and B, are like Solids: Therefore, if between, &c. Which was to be demonſtrated. c) 21. d. 7. d) 21 7. e) 21. d. 7. f) 18. 7. g) 17. 7. h) 21. d. 7.

PROP. 22. THEOR. 20.

A 9 B 54 C 324

If three numbers A, B, *and* C, *be continually proportional, and that the firſt* A, *be a ſquare, the third* C, *ſhall alſo be a ſquare.*

Demonſtration FOr ſeeing that between A and D [a], there fals a mean proportional, A and C ſhall be like Plaines; therefore A being a ſquare, C reſembling it, ſhall be alſo a ſquare: Therefore, if there be three numbers, &c. Which was to be demonſtrated. a) 20. 8.

PROP.

PROP. 23. THEOR. 21.

A 27 B 45 C 75 D 125

If four numbers A, B, C, *and* D, *are proportional, and that the firſt* A, *be a Cube, the fourth* D, *ſhall alſo be a Cube.*

Demonſtration FOr ſeeing that between A and D, there falls two mean proportionals, B and C, [a] A and D ſhall be like Solids. Therefore, A being a cube, D reſembling him, ſhall be a cube alſo: Therefore, if four numbers, &c. Which was to be demonſtrated. a) 21. 8.

PROP. 24. THEOR. 22.

A 36 F 48 B 64
C 9 E 12 D 16

If two numbers A *and* B, *be in the ſame rate to one another, as a ſquare number* C, *to a ſquare number* D, *and if the firſt* A *be a ſquare, the ſecond* B, *ſhall alſo be a ſquare.*

Demonſtration FOraſmuch as C and D, are ſquares, [a] there ſhall fall between them a mean proportional: which let be E: [b] there will alſo fall one between A and B, which are in the ſame rate, which let be F: Foraſmuch as the three numbers A, F, and B, are continually proportional, and A the firſt is a ſquare, [c] B the third ſhall be alſo a ſquare. Therefore, if two numbers, &c. Which was to be demonſtrated. a) 11. 8. b) 8. 8. c) 22. 8.

COROLLARIE.

It is manifeſt from what is above demonſtrated that the proportion of every ſquare number, to any other number whatſoever that is not ſquare, cannot be exhibited in two ſquare numbers: for if they could, [d] *then the two firſt numbers which have the ſame rate as the ſquares of the proportion exhibited, ſhould be ſquares alſo: ſeeing that the firſt is propoſed a ſquare, which is abſurd, for the ſecond is propoſed no ſquare, whence it follows, that the numbers which are in a double rate, are not to one another as a ſquare number, to a ſquare number: for all theſe double numbers here* 4, 8, 16, 32, 64, 128, *&c. ſhould be ſquares, for the firſt number* 4 *being a ſquare,* [e] 8 *ſhould be alſo a ſquare, and alſo* 16, *and* 32, *&c. which is impoſſible.* d) 24. 8. e) 24. 8.

For between 4 *and* 8, *between* 8 *and* 16, *between* 16 *and* 32, *&c. there would fall a mean proportional, if they were ſquares; ſeeing that it hath been formerly ſhewn, that between any numbers that bear a double proportion to one another, there cannot fall a mean proportional.*

In like manner, the numbers which are in a quintuple rate to one another, ſhall not have the ſame rate as a ſquare number to ſquare number, for if it ſhould, f *there would fall between them mean proportional.*

f) 11. 8. 18. 8.

Therefore, there would alſo fall a mean proportional between 5 *and* 1, *which are the leaſt numbers in a quintuple rate: but we have ſhewn the contrary.*

PROP. 25. THEOR. 23.

A 8 G 12 H 18 B 27
C 64 E 96 F 144 D 216

If two numbers A *and* B, *have the ſame rate the one to the other, as one cube number* C, *to another cube number* D, *and that the firſt* A, *be a cube, the ſecond* B, *ſhall be a cube alſo.*

Demonſtration FOraſmuch as A is to B, as C is to D, but [a] between the cubes C and D, there falls two mean proportionals E and F: therefore between A and B, there ſhall alſo fall two mean proportionals G and H; therefore, foraſmuch as the four numbers A, G, H, and B, are continually proportional, and A the firſt is a cube, [b] B the fourth, ſhall be alſo a cube. Therefore, if two numbers, &c. Which was to be demonſtrated.

a) 12. 8. 8. 8.

b) 23. 8.

COROLLARIE.

It is manifeſt by what is above ſhewn, that the rate of any cube number whatſoever, to any other number whatſoever, not cube, cannot be found in two cube numbers: for if you ſuppoſe it could, the firſt numbers having the ſame rate as the cubes of the proportion found, ſhould be alſo cubes: ſeeing the firſt is propoſed a cube, which is abſurd, for the ſecond is propoſed no cube.

25. 8.

PROP.

PROP. 26. THEOR. 24.

A 20 C 30 B 45
D 4 E 6 F 9

Plain numbers A *and* B, *have the ſame rate to one another, as a ſquare number* D, *to a ſquare number* F.

Demonſtration FOr ſeeing that A and B are like Plaines, [a] their ſhall fall between them a mean proportional which ſhall be C.

a) 18. 8.

Therefore, having taken the three numbers D, E, and F, the leaſt in the rate of A, C, and B, the extreams D and F ſhall be ſquares: Therefore, ſeeing that in equal rate A is to B, as D to F: it is manifeſt, that A ſhall be to B, as a ſquare to a ſquare; to wit, as the ſquare number D, to the ſquare number F. Therefore the Plains, &c. Which was to be demonſtrated.

1 Cor. 2, 8

PROP. 27. THEOR. 25.

A 16 C 24 D 36 B 54
E 8 F 12 G 18 H 27

Like Solid numbers A *and* B, *have the ſame rate to one another, as a cube number* E, *to a cube number* H.

Demonſtration FOraſmuch as A and B are like Solids [a] there will fall beween them two mean proportionals C and D: And having taken the four numbers E, F, G, and H, the leaſt in the rate of A, C, D, and B, the extreams E and H ſhall be cubes: Therefore ſeeing that in equal rate A is to B, as E is to H, it appears that A ſhall be to B, as the cube number E, to another cube number; to wit, to H: Therefore the, &c. Which was to be demonſtrated.

a) 19. 8.

Cor. 1. 2, 8.

COROLLARIE. I.

If two numbers be in the ſame proportion the one to the other that a ſquare number is to a ſquare number: thoſe two numbers ſhall be like ſuperficial numbers. And if they be in the ſame proportion the one to the other, that a cube number is to a cube number, they ſhall be like ſolid numbers.

First, let the number A, have unto the number B, the ſame proportion, that the ſquare number C hath to the ſquare number D: Then I ſay, that A and B are like ſuperficial numbers. For, foraſmuch as between the ſquare numbers C and D, there falleth a mean proportional, [a] there ſhall alſo between A and B, (which have the ſame

B 72 D 64
A 18 C 16

a) 11 8.

same proportion with C and D) fall a mean proportional [b]. Wherefore A and B are like superficial numbers [c].

But if A be unto B, as the cube number C, is to the cube number D. Then are A and B like solid numbers. For, forasmuch as C and D are cube numbers, there falleth between them two mean proportional numbers [d]. And therefore, [e] between A and B, (which are in the same proportion that C is to D) there falleth also two mean proportional numbers. Wherefore [f] A and B, are like solid numbers.

B 16 C 8
A 56 D 27

b) 8. 8. c) 20. 8. d) 12. 8. e) 8. 8. f) 21. 8.

COROLLARIE. II.

If a number multiplying a square number, produce not a square number, the number multiplying shall be no square number.

For If it should be a square number, then should it and the number multiplyed, being like, superficial numbers (by reason they are square numbers) have a mean proportional [a]. And the number produced of the said mean, should be equal to the number contained under the extreams, which are [b] square numbers. Wherefore, the number produced of the extreams, being equal to the square number produced of the mean, should be a square number. But the said number by supposition, is no square number. Wherefore neither is the number multiplying the square number, a square number.

a) 18. 8. b) 20. 7.

The first pare of the first Corollarie is the converse of the 26 Proposition of this book, and hath some use in the tenth book. The second part of the same also, is the converse of the 27 Proposition of the same.

The End of the Eighth Element of EUCLIDE.

THE NINTH ELEMENT OF EUCLIDE.

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

A 6 B 54
D 36 E 108 C 324

If two numbers being like Plains A, *and* B, *multiplying one another, do produce some one* C, *the product* C *shall be a square.*

Demonstration. LEt D be the product of A by it selfe, D shall be a square number: Therefore, forasmuch as A multiplying A and B, hath produced D and C; [a] as A shall be to B, so D shall be to C: But between A and B, like Plains, [b] there falls a mean proportional, therefore also [c] there shall fall one between D and C, which shall be E, to the end that D, C, and E, may be continually proportional: But D the first, is a Square by the construction; therefore [d] C the third, shall be also a square: Therefore if two plain numbers, &c. Which was to be demonstrated.

a) 17. 7. b) 18. 8. c) 8. 8. d) 22. 8.

PROP. 2. THEOR. 2.

A 6 B 54
D 36 C 324

If two numbers multiplying one the other, A *and* B, *make a square* C, *they are like Plains.*

Demon-

Demonſtration FOr let D be the product of A by it ſelf, D ſhall be a ſquare, foraſmuch then as A multiplying A and B hath produced D and C, [a] as A is to B, ſo D is to C. But [b] between D and C ſquare numbers there fals a mean proportional, wherefore [c] between A and B there ſhall alſo fall a mean proportional, therefore [d] A and B are like Planes. Therefore, if two numbers &c. Which was to be demonſtrated.

A 6. B 54.
D 36. C 324.

a) 17. 7.
b) 11. 8.
c) 8. 8.
d) 20. 8.

PROP. 3. THEOR. 3.

A 8.
E 16. D 4.
F 32. C 2.
B 64. Unity.

If a Cube number A, *multiplying it ſelf, doth produce any one* B, *the product* B *ſhall be a Cube.*

Demonſtration LEt C be the ſide of the cube A, and let D be the product of C by it ſelf, and ſo the cube A ſhall be the product of C by D, foraſmuch then as C multiplying it ſelf hath made D [a] the ſame C, ſhall meaſure D by C: but [b] unity ſhall alſo meaſure C, by C, therefore, unity ſhall be the ſame part of C denominated of C, as C of D: and therefore, [c] as unity is to C, ſo is C to D.

Again, Foraſmuch as C, multiplying D, hath made A, [d] D ſhall meaſure A, by C: but C meaſures D by C, therefore C ſhall be the ſame part of D, as D is of A, therefore [e] as C is to D, ſo D is to A: but as C, is to D, ſo unity was to C, therefore as unity, is to C, ſo is, C to D, and D, to A, therefore between unity and A, there doth fall two mean proportionals C and D.

But [f] foraſmuch as A meaſure B by A, (for multiplying it ſelf makes B) and [g] unity meaſures A, by A, unity ſhall be the ſame part of A, as A of B, and therefore unity [h] ſhall be to A, as A to B, and therefore ſeeing that between unity and A, there falls two mean proportionals C and D, [i] there ſhall fall as many between A and B, which ſhall be E and F.

Therefore, ſeeing that the four numbers A, E, F, and B, are continually proportional, and A the firſt is a cube, B the fourth ſhall be in like manner a cube. Therefore, if a cube, &c. Which was to be demonſtrated.

a) 7. c. f.
b) 5. c. f.
c) 20. d.
d) 7. c. f.
e) 20 d.
f) 7. c. f.
g) 5. c. f.
h) 20 d.
i) 8. 8.
23. 8.

PROP. 4. THEOR. 4.

A 8. B 27.
D 64. C 216.

If a cube number A, *multiplying a cube, number* B, *makes ſome one* C, *the product* C *ſhall be a cube.*

Demonſtration LEt A multiplying it ſelf make D, D [a] ſhall be a cube, and foraſmuch as A multiplying A and B, hath made D and C, [b] as A ſhall be to B, ſo D ſhall be to C. But between the cubes A and B, there

a) 3 9.
b) 17. 7.

there fals two mean proportionals, [c] therefore, between D and C, there ſhall alſo fall two mean proportionals, therefore, [d] D being a cube, C ſhall be a cube alſo. Therefore if, &c. Which was to be demonſtrated.

c) 12. 8. 8. 8.
d) 23. 8.

PROP. 5. THEOR. 5.

A 8 B 27
D 64 C 216

If a cube number A *multiplying ſome number* B *doth produce a cube* C, *the number multiplyed* B, *ſhall be a cube alſo.*

Demonſtration FOr let D, be the product of A, multiplyed by it ſelf: [a] D ſhall be a cube.

Foraſmuch as A, multiplying A and B, hath made D and C, [b] as A, ſhall be to B, ſo ſhall D, be to C. [c] But between the cubes D and C, there falls 2 mean proportionals, [d] therefore, between A and B, there ſhall alſo fall 2 mean proportionals, [e] and therefore, A being a cube, B ſhall be alſo a cube. Therefore, if a cube number, &c. Which was to be demonſtrated.

a) 3 9.
b) 17. 7.
c) 12. 8.
d) 8. 8.
c) 22. 8.

PROP. 6. PROBL. 6.

A 8 B 64 C 512

If a number A, *multiplying it ſelf makes a cube* B, *the ſame* A, *ſhall be alſo a cube.*

Demonſtration FOr let C, be the product of B by A, C ſhall be a cube number, as is manifeſt. Foraſmuch then as B, cube multiplying another number, to wit, A hath produced C, a cube number, A ſhall be alſo a cube. Therefore, if a number, &c. Which was to be demonſtrated.

19. d.
5. 9.

PROP. 7. THEOR. 7.

A 6 B 11 C 66 D 2 E 3

If a compound number A, *multiplying, ſome number* B, *makes ſome other* C, *the product* C, *ſhall be a Solid.*

Demonſtration FOr ſeeing that A is compounded, ſome other number beſides unity ſhall meaſure it: Therefore, let D meaſure A, by E, that being done, [a] D multiplying E, ſhall produce A. Therefore, ſeeing that B, multiplying the ſame A hath made C; C ſhall be produced by the mutual multiplication of the three numbers D, E, and B, and

13. d.
a) 9. c. f.

17. d. therefore, C ſhall be a Solid, whoſe ſides are D, E, and B. Therefore, if a compound number, &c. Which was to be demonſtrated.

PROP. 8. THEOR. 8.

If from unity, as many numbers as you pleaſe A, B, C, D, E, F, G, H, I, K, L, *and* M, *are continually proportional, the third* B *from unity is a ſquare, and all the reſt which intermit, or leave out one* D, F, H, K, *and* M, *But the fourth is a cube* C, *and all the reſt in leaving two,* F, I, *and* M, *and the ſeventh,* F *is a cube, and a ſquare together, and all the reſt which leave five* M.

Unity. A 3 B 9 C 27 D 81 E 243 F 729.
G 2187 H 6561 I 19683 K 59049
L 177147 M 531441.

Demonſtration. FOraſmuch as unity, is to A, as A is to B, unity ſhall meaſure A, and A ſhall meaſure B equally. [a] But unity meaſures A, by A, therefore, A ſhall alſo meaſure B, by A: [b] therefore, A multiplying A, ſhall produce B, therefore, B is a ſquare.

a) 5. c. 5. b) 9. c. f.

Now, foraſmuch [c] as B, C, and D, are continually proportional, and B is a ſquare D ſhall be alſo a ſquare: in like manner ſeeing that D, E, and F, are continually proportional, and as D is a ſquare, F ſhall be alſo a ſquare, and all the others leaving one.

c) 22. 8.

Again, Foraſmuch as unity is to A, as B to C, unity ſhall meaſure A and B ſhall meaſure C equally. But [d] unity meaſureth A by A: therefore, B ſhall alſo meaſure C, by A, [e] and therefore, B multiplying, A ſhall produce C, therefore, ſeeing that A multiplying it ſelf makes B, and multiplying B makes C, [f] C ſhall be a cube.

d) 5. c. f. e) 9. c. f. f) 17. d. 7.

But foraſmuch as C, D, E, and F, are continually proportion, and as C the firſt is a cube, F the fourth [g] ſhall be alſo a cube, by the ſame reaſon ſeeing that F, G, H, and I, are continually proportional, and that F is a cube, I, ſhall be alſo a cube, and all the reſt leaving or intermitting two. Laſtly, foraſmuch as F the ſeventh, from unity, is ſhewn to be a ſquare and a cube; the ſame ſhall be a cube, and a ſquare together, and in the ſame manner, M the ſeventh from F, leaving G, H, I, K, and L, ſhall be a cube, and a ſquare together, and alſo all the others, which leave 5. Therefore, if from unity, &c. Which was to be demonſtrated.

g) 23. 8.

PRO-

PROP. 9. THEOR. 9.

If from Unity there be as many numbers as you pleaſe A, B, C, D, E, *and* F, *continually proportional; and if that number which follows unity* A, *be a ſquare, all the others alſo ſhall be ſquares, and if that which follows unity be a cube, alſo all the others ſhall be cubes.*

Unity A 4 B 16 C 64 D 256 E 1024 F 4096
Unity A 8 B 64 C 512 D 4096 E 32768 F 262144

Demonſtration. FOr ſeeing that A, B, C, D, E, and F, are continually proportional from unity, [a] B the third from unity, is a ſquare, and all the reſt leaving one: to wit, D and F, and foraſmuch as A, B, and C, are continually proportional and that A is a ſquare by the Suppoſition, [b] C the third ſhall be alſo a ſquare: in like manner, taking C, D, and E continually proportional, ſeeing that C the firſt is a ſquare, E the third, ſhall be alſo a ſquare: therefore, all the numbers A, B, C, D, E, and F, are ſquares, and ſo of the reſt.

a) 8. 9. b) 22. 8. 22. 8.

Now let A next to unity be a Cube; I ſay, that the others are Cubes.

Foraſmuch as from unity A, B, C, D, E, and F, are continually proportional, [c] C the fourth from unity is a cube, and all the others leaving two; to wit F, now that the others B, D, and E, are alſo cubes, we ſhall thus demonſtrate.

c) 8. 9.

Foraſmuch as unity is to A, as A to B, unity ſhall meaſure A, and A the number B, equally [d]; but unity ſhall meaſure A by the ſame A, therefore A ſhall meaſure B alſo by A [e]: therefore, A multiplying it ſelf, ſhall produce B: But, [f] A is a cube, therefore B ſhall be alſo a cube; and foraſmuch as A, B, C, and D, are continually proportional, and as A is a cube, ſo [g] D ſhall be alſo a cube: Moreover, ſeeing that B, C, D, and E, are continually proportional, and as B is a cube, ſo ſhall E [h] alſo be a cube: therefore A, B, C, D, E, and F, are all cubes, and ſo on, if there were more. Therefore, if from unity, &c. Which was to be demonſtrated.

d) 5. c. f. e) 9. c. f. f) 3. 9. g) 23. 8. h) 23. 8.

PROP. 10. THEOR. 10.

If from unity there be as many numbers as you pleaſe A, B, C, D, E, F, G, H, *and* I, *continually proportional; and if that number* A, *which follows after unity be no ſquare; alſo not any one of the others ſhall be a ſquare, except the third* B *from unity, and all the others who leave one* D, E, *and* F, *and if that which is after unity* A, *be not a Cube; alſo not any one of the others ſhall be a Cube beſides the fourth from unity* C, *and all the others who leave two.*

Unity A 2 B 4 C 8 D 16 E 32 F 64
G 128 H 256 I 512

Demonstration FOr if it be supposed that besides those there are other squares: Let E be supposed to be a square; forasmuch
a) 8. 9. then as [a] D is a square, and that as D is to E, or E to F, so A is to B, and by conversion, as E is to D, or F to E, so B to A; B shall be to A,
8. 9. as the square number E, to the square number D, or as the square F, to
b) 24. 8. the square E: But B the third from unity, is a square, therefore [b] A the first after unity, shall be a square, which is absurd, for by Supposition it is no square: therefore E shall not be a square, by the same reason we shall shew that not one other is a square, besides the numbers above mentioned.

Now let A next to unity be no cube; I say that not any other is a cube, besides C the fourth from unity, and all those that do leave two, to wit, F,
8. 9. and I, for if you say, that some other is a cube, suppose it to be D: therefore seeing that F is a cube, and as in equality as D is to F, so A is to C: seeing that D, E, F, and A, B, C, are in the same rate, and by conversion as F to D, so C to A; C shall be to A, as the cube number F, to the cube
c) 8. 9. number D; but [c] C the fourth from unity, is a cube, therefore [d] A the first
d) 25. 8. after unity, shall be also a cube, which is against the Supposition: Therefore D is no cube, by the same reasons may be shewn that none but the fore-mentioned can be cubes. Therefore, &c. Which was to be demonstrated.

PROP. 11. THEOR. 11.

If from Unity there be as many numbers as you please A, B, C, D, E, F, *and* G, *continually proportional, the least doth measure the greatest,* C *measures* G *by some one of those which are between the proportional numbers* A, B, C, D, E, F, *and* G.

Unity A 3 B 9 C 27 D 81 E 243 F 729 G 2187

Demonstration FOrasmuch as by equality, as C is to G, so unity is to D, (for C, D, E, F, and G, are in the same rate continued as unity,
a) 20. d. and A, B, C, and D) [a] unity shall measure D; and C the number G equally: But
b) 5. c. f. [b] unity measureth D, by the same D, therefore C shall also measure G by D: by the same reason E shall measure F by A: seeing that as E is to F, so unity is to A, &c. so A shall measure G by F: seeing that by equality as A is to G, so unity is to F, &c. and so on: Therefore, if, &c. Which was to be demonstrated.

PROP.

PROP. 12. THEOR. 12.

If from Unity there be as many numbers as you please A, B, C, *and* D, *all the prime numbers which measure the last number* D, *the same numbers shall also measure that which is next to Unity* A.

Unity A 6 B 36 C 216 D 1296 E 3

Demonstration FOr let E be a prime number, which shall measure the last D: I say, E shall also measure A: for if it shall not measure it, [a] the same E shall be prime to A: Forasmuch then as A and E are
a) 31. 7. primes to one another, and A multiplying it self, makes B, as appears, [b] B
b) 27. 7. shall be prime to E: therefore, seeing that A and B, are primes to E, and C is the product of A in B, [c] C shall be prime to E: again, seeing that A and C
c) 26. 7. are alike primes to E, and that D is the product of A in C, [d] D shall be
d) 26. 7. prime to E: therefore E measures not D, which is against the Supposition, therefore, E shall measure A: Therefore, &c. Which was to be demonstrated.

COROLLARIE.

Therefore, any prime number, greater then that next Unity, nor any lesser that measurtth not that which is next unity, shall not measure the last: for if it did measure it, it would also measure that which is next Unity, as hath been demonstrated, which is against the Supposition,

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PROP. 13. THEOR. 13.

Unity A 5 B 25 C 125 D 625
H---- G---- F---- E----

If from Unity there be as many numbers as you please continually proportional, A, B, C, *and* D; *and if that number next after Unity* A, *be prime, any other shall not measure the greatest* D, *beside those* A, B, *and* C, *that are between the proportionals.*

Demonstration FOr (if possible) let some other, as E, measure D, E shall be a prime, or a compound number; if a prime, and measuring 12. 9.
D the extream, it shall also measure A, the prime next to Unity, which is absurd, therefore, E is not prime, but compounded; and [b] therefore measured by some prime number, which cannot be any other then A: for E measuring b) 33. 7.

suring D, every prime number measuring E, shall also measure D. But
c) 12. 9. measuring D the greater, c it shall also measure A, the prime next to unity, which is absurd: Therefore, any other prime number then A, shall not measures E, for as A measures E by F: therefore A multiplying F, makes

Unity A 5 B 25 C 125 D 625
H---- G---- F---- E----

d) 17. 7. E and D: and therefore d E shall be to D as F to C, wherefore E measuring D, F shall measure C; and in like manner, it shall be shewn, that F is no Prime, but a compound number, which shall be demonstrated, as is before said to be only measured by the prime A.

Let A then measure F by G, therefore A multiplying G and B, produceth F and C; wherefore
e) 17. 7. e F shall be to C, as G to B; but F measureth C: therefore G doth also measure B.

f) 12. 9. Again, if G be a prime number measuring B, it shall f also measure the prime number A, which is absurd: therefore G is compounded, which shall be only measured by A, as hath been demonstrated of E: Let A then measure G by H; wherefore A multiplying A and H, shall make B and G: Therefore as G is to B, so H is to A; but G measures B; therefore H shall also measure A the first: wherefore H is equal to the same A.

g) 20. 7. But A is a mean proportional between H and G: seeing that g B the product of the extreams G and H, is equal to the same B, the square of A the mean, which is impossible: Therefore any other number besides A, B, and
h) 11. 9. C, shall not measure D the greatest number, of which the least h doth alwayes measure the greatest: Therefore, if from unity, &c. Which was to be demonstrated.

PROP. 14. THEOR. 14.

A 30 B 2 C 3 D 5
E---- F---

The least number A, *measured by certaine prime numbers* B, C, *and* D, *shall not be measured by any other prime number then by those* B, C, *and* D, *which measured it at first.*

Demonstration FOr suppose that E a prime number differing from B, C, and D, doth measure A by F: forasmuch as E measures A by F,
a) 9. c. f. E a multiplying F, shall make A, wherefore b each of the numbers B, C,
b) 32. 7. and D, should measure one of the two E and F, they shall not measure E prime, other then by these; therefore, F which is lesse then A, shall measure it, which is absurd; for A is proposed the least, which may be measured by the prime numbers B, C, and D: Therefore, the least, &c. Which was to be demonstrated.

PROP.

PROP. 15. THEOR. 15.

A 9 B 12 C 16
D 3 E 4

If three numbers A, B, *and* C, *be continually proportional, and the least of all those which have the same rate with them the number* B, *compounded of any two of them shall be prime to the number resting.*

Demonstration FOr having taken D and E, the least in the same rate, it appears by the demonstration of 2 Prop. 8. that A is square to D: and C square to E, and B the product of D in E; and the number compounded of D and E multiplyed by D, shall produce a number equal to the number compounded of A and B: but b D and E, being primes to one another, c the number compounded of them shall be prime to E: and therefore d also prime to C, the square of E, and by the same reason D shall be prime to C, and the number produced of the number compounded of D and E, multiplyed by D, shall be also prime to C, which product is shewn to be equal to the number compounded of A and B: therefore, the number compounded of A and B, is prime to C.

b) 24. 7.
c) 30. 7.
d) 27. 7. 26. 7.

Again, the number compounded of D and E, being prime as well to D, as to E, the product of the same compound number multiplyed by E, shall be also prime to D; but the same product is equal to C, the square of E, and to B together, the product of D in E: therefore, the numbers compounded of B and C, shall be also prime to D, and c therefore prime to A the square of D.

27. 7. 3. 9.
c) 27. 7.

Lastly, f seeing that D and E, are primes to one another, and g primes to their compound, B their product, h shall be also a prime to their compound; i and the product of the number compounded of D and E, multiplyed by it self, shall be prime to B: but the same product is equal to A and C, and to twice B together; therefore, the compound of A and C, and of twice B, is prime to B; then taking away twice B, the remainder of the compound of A and C, shall be prime to B: For if it were not so, they should be compounded to one another, and their common measure measuring the compound of A, C, and B, should also measure the compound of A, and C, and of twice B. Wherefore the compound of A, and C, and twice B, should not be prime to B, which is absurd, they having been demonstrated primes to one another: Therefore the compound of A, and C, is prime to B: Therefore, if three numbers, &c. Which was to be demonstrated.

f) 24. 7.
g) 30. 7.
h) 26. 7.
i) 27. 7.
10. c. f.

PROP. 16. THEOR. 16.

A 5 B 7 C----

If two numbers A *and* B, *are primes to one another: as the first* A, *is to the second* B, *so the second* B, *shall not be to any other.*

Demon-

Demonſtration FOr (if poſſible) as A is to B, ſo let B, be to another, to wit, C.

a) 23. 7. b) 21. 7. Forasmuch then, as A and B, are primes to one another, and [a] the leaſt that are in the ſame rate, [b] they ſhall equally meaſure B and C being in the ſame rate: to wit, A ſhall meaſure B, and B ſhall meaſure C: But A meaſureth alſo it ſelf, therefore A ſhall meaſure A and B, primes to one another, which is abſurd: Therefore, as A is to B, ſo B is not to C.

In like manner, as B is to A, ſo A ſhall not be to another: Therefore, if two numbers, &c. Which was to be demonſtrated.

PROP. 17. THEOR. 17.

If there be as many numbers as you pleaſe A, B, C, *and* D, *continually proportional; and that the extreams* A, *and* D, *are primes to one another, as the firſt* A, *ſhall be to the ſecond* B, *ſo the laſt* D, *ſhall not be to ſome other.*

A 8 B 12 C 18 D 27 E----

Demonſtration FOr if it be denied, ſuppoſe as A is to B, ſo D may be to ſome other; to wit, to E: Forasmuch then as A is to B, ſo D is to E, alternately, as A is to D, ſo B is to E: But A and D, being primes to one another, are [a] the leaſt in their rate; therefore [b] they ſhall meaſure B and E equally; to wit, A ſhall meaſure B, and D ſhall meaſure E: But as A is to B, ſo B is to C: therefore ſeeing that A meaſures B, alſo B ſhall meaſure C, and [c] therefore A ſhall meaſure C alſo, and ſeeing that as B is to C, ſo C is to D: But B meaſureth C, and C ſhall alſo meaſure D: wherefore A meaſuring C [d], ſhall in like manner meaſure D, and A meaſureth it ſelf; therefore A meaſureth A, and D, primes to one another, which is impoſſible, therefore as A is to B, ſo is not D, to ſome other number; to wit, to E: in like manner, as D is to C, ſo is not A to another: Therefore, if there be, &c. Which was to be demonſtrated.

a) 23. 7. b) 21. 7. c) 11. c. f. d) 11. c. f.

PROP. 18. PROBL. 1.

A 4 B 7
A 4 B 6 D 9 C 36
A 6 B 4 D 00 C 16

Two numbers A *and* B, *being given, to conſider if it be poſſible to find a third proportional to them.*

Demonſtration THe numbers A and B, are primes to one another, or not primes to one another, if primes, then [a] there can be no third proportional found; if they are not primes, let B multiplying it ſelf, make C. Now either A meaſureth C, or doth not meaſure it: if it meaſure it,

a) 16. 9.

it, let it be by D. I ſay, there may be a third proportional found, which is D, for ſeeing that A, meaſureth C, by D, [b] C ſhall be the product of A, by D: but the ſame is the product of B, multiplyed in himſelf: therefore, C, contained under the extreams A and D, is equal to the product of the mean B, multiplyed by it ſelf: therefore, [c] the three A, B, and C, are continually proportional, to wit, as A, is to B, So is B, to D: therefore, D ſhall be the third proportional.

b) 9. c. f. c) 20. 7.

But if A doth not meaſure C, I ſay, that there cannot be found a third proportional, ſeeing that as A, is to B: ſo B, is to D, the number produced of A and D, the extreams, ſhall be equal to the product of B, the mean multiplyed in it ſelf: that is to ſay, to C; therefore, ſeeing that C, is made of A, multiplyed by D, [d] A ſhall meaſure C, by D, which is abſurd, being propoſed not to meaſure it. Therefore, there cannot be found a third proportional to A and B, ſeeing that A, a Prime number meaſureth not C, the product of B, the ſecond multiplyed in it ſelf, &c. Therefore, two numbers, &c. Which was to be done.

d) 7. c. f.

PROP. 19. PROBL. 2.

Three numbers A, B, *and* C, *being given, to conſider if it be poſſible to find a fourth proportional to them.*

A 8 B 12 C 18 E 27 D 216.
A 4 B 8 C 9 E 18 D 72.

A 4 B 6 C 9 E---- D 54
A 3 B 14 C 10 E---- D 40

Conſtruction LEt B, multiplying C, make D, A ſhall meaſure D, or ſhall not meaſure it: ſuppoſe it to meaſure it by E, I ſay, that E, is the fourth proportional.

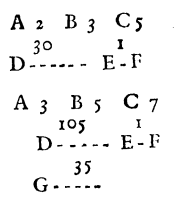
Demonſtration SEing that A, meaſures D, by E, [a] D ſhall be the product of A, by E: but D is alſo the product of B by C, by conſtruction, therefore, the product of the extreams A and E, is equal to the product of the means B, and C, therefore, [b] as A, is to B: So is C, to E.

a) 9. c. 5. b) 19. 7.

But if A, meaſures not D, I ſay, there cannot be found a fourth proportional, for otherwiſe ſuppoſe E, to be a fourth proportional. Forasmuch then as the four numbers A, B, C, and D, are continually proportional, [c] the number contained under the extreams A, and E, ſhall be equal to the number contained under the means B and C, to wit, to D. Therefore, ſeeing that D, is made of A, multiplyed by E, [d] the ſame A, ſhall meaſure D, by E, contrary to the ſuppoſition. In like manner we may examine, if as A, is to B, ſo C is to ſome other number. Therefore, three numbers, &c. Which was to be done.

c) 19. 7. d) 7. c. f.

PROP. 20. THEOR. 18.



Prime numbers are in a greater multitude then any multitude of prime numbers proposed. A, B, C.

THat is to say; That if you propose as many prime numbers as you please there will yet be found others.

Demonstration FOr [a] having taken D, the least that may measure A, B, and C, and adding unity E F, thereto, the whole D F, is a prime number, or not: if it be a prime number, then you have what is required, for A, B, C, and D F, are in greater number then the multitude proposed A, B, and C.

a) 38. 7.

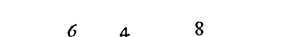
But if D F, be not a prime number, it [b] shall be measured by some, prime number, which let be G. I say, that G is not one of the numbers proposed A, B, C, for if it were one of them, seeing that A, B, and C, measureth D E, G should also measure D E; Therefore, measuring the whole D F, and the part cut off D E, it should also measure the remainder E F; to wit, a number should measure unity, which is impossible.

b) 33. 7.

11. c. 5.

Therefore the prime number G, is none of the numbers proposed A, B, C: Therefore, the prime numbers A, B, C, and G, are in a greater number then those proposed, A, B, and C, and so *ad infinitum* may be found other prime numbers: Therefore, The prime, &c. Which was to be demonstrated.

PROP. 21. THEOR. 19.



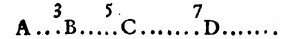
If as many even numbers as you please A B. B C. C D. *be added; the whole* A D. *shall be even.*

Demonstration FOrasmuch as [a] every even number may be divided in two equal parts, the halves of A B, B C, and C D, shall make the halfe of A D: but all the halves together, are equal to all the other halves together; therefore A D is an even number, &c. Which was to be demonstrated.

a) 6. 7.

PROP.

PROP. 22. THEOR. 20.



If as many odde numbers as you please A B, B C, C D, *and* D E, *be added, and that the number of them be even: the whole shall be even.*

Demonstration FOr seeing that A B, B C, C D, and D E, are odde, each of them shall differ from an even number, by unity, according to the Definition: Wherefore if from each you cut off unity, each of the remainders shall be even: wherefore [a] the number compounded of them shall be even: Therefore [b] the whole A and E compounded shall be in like manner even: Therefore, If as, &c. Which was to be demonstrated.

a) 21. 9.
b) 21. 9.

PROP. 23. THEOR. 21.

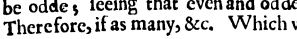


If as many odde numbers as you please A B, B C, *and* C D, *are added, and that the multitude of them be odde, the whole* A D, *shall be also odde.*

Demonstration FOr seeing that an odde number, differeth from an even number by unity; by the Definition: having cut off unity E D, from the odde number C D, the rest C E, shall be even: But [a] A C compounded of the odde number A B, and B C, even in multitude are even: therefore [b] also A E, compounded of the even numbers A C, and C E, shall be even, therefore if you adde unity E D, the whole A D, shall be odde; seeing that even and odde, differ by unity, by the Definition: Therefore, if as many, &c. Which was to be demonstrated.

a) 22. 9.
b) 21. 9.

PROP. 24. THEOR. 22.



If from an even number A B, *be cut off an even number* C B, *the remainder* A C, *shall be also an even number.*

Demonstration FOr if A C be not an even number, let there be cut off unity A D, the rest D C will remaine even, seeing that an even and odde number differeth onely by unity) therefore [a] the compounded number D B, shall be an even number: Therefore adding unity A D, to the compounded number D B, an odde number, the whole A B, shall be un-

a) 21. 9.

even; which is impoſſible, for it is even by Suppoſition: therefore A C, ſhall not be odde: Therefore if, &c. Which was to be demonſtrated.

PROP. 25. THEOR. 23.

If from an even number A B, *there be cut off an odde number* C B, *the reſt* A C, *ſhall be alſo odde.*

Demonſtration FOr from C B, having cut off unity C D, the reſt D B, ſhall be even. Therefore, foraſmuch as the whole A B, is put even, the reſt [a] A D, ſhall be even; having therefore taken away unity C D, the reſt A C, ſhall be odde: Therefore, if from a number, &c. Which was to be demonſtrated. a) 24. 9.

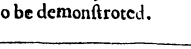
PROP. 26. THEOR. 24.



If from an odde number A B, *be cut off an odde number* C B, *the reſt* A C, *ſhall be even.*

Demonſtration FOr having cut off unity from the odde numbers A B, and C B, the remainders A D, and C D, ſhall be even numbers: foraſmuch then as from the even number A D, the even number C D, is cut off, [a] the reſt A C, ſhall be an even number: Therefore if from, &c. Which was to be demonſtroted. a) 24. 9.

PROP. 27 PROP. 25.

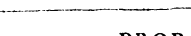


If from an odde number A B, *be cut off an even number* C B, *the reſt* A C, *ſhall be odde.*

Demonſtration FOr, from the odde number A B, let unity A D, be cut, the reſt D B ſhall be even: from which the even number C B, being cut, the [a] reſt D C, ſhall be alſo even: Therefore adding unity A D, A C ſhall be odde: Therefore, if from an, &c. Which was to be demonſtrated. a) 24. 9.

PROP.

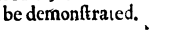
PROP. 28. THEOR. 26.



If an odde number A, *multiplying an even number* B, *produce ſome one* C, *the product* C, *ſhall be even.*

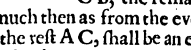
Demonſtration FOr ſeeing that C is the product of A multiplyed by B: the ſame C ſhall be compounded of ſo many numbers equal to B, as there are unites in A: but B is an even number, therefore C ſhall be compounded of as many even numbers equal to B, as there are unites in A; and therefore [a] C ſhall be an even number: Therefore, if an, &c. Which was to be demonſtrated. a) 21. 9.

COROLLARIE.



It follows alſo that an even number multiplyed by it ſelf, produceth an even number, as is manifeſt if you put the even numbers A *and* B, *equal.*

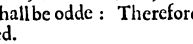
PROP. 29. THEOR. 27.



If an odde number A, *multiplying an odde number* B, *produce one* C, *the product* C, *ſhall be odde.*

Demonſtration FOr ſeeing that C is produced of A in B, the ſame C ſhall be compounded of ſo many numbers equal to B, as there are unites in A: Therefore ſeeing that as well A as B, is an odde number; C ſhall be compounded of as many odde numbers, equal to B, as there are unites in A, the odde number: Wherefore the multitude of them ſhall be odde, and therefore [a] C ſhall be an odde number. Therefore, if an, &c. Which was to be demonſtrated. a) 23. 9.

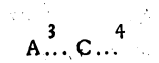
COROLLARIE.



It follows from this that an odde number multiplyed by it ſelf, produceth an odde number, as is manifeſt if the number A *and* B, *are propoſed to be equal.*

PROP.

PROP. 30. THEOR. 28.



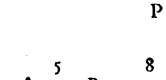
If an odde number A, *measureth an even number* B, *it ſhall alſo meaſure its halfe.*

Demonſtration FOr let A meaſure B by C, the number C ſhall be even; and therefore ſhall have a half: Wherefore ſeeing that A meaſureth B by C, [a] C ſhall alſo meaſure B by A: and ſo C ſhall be a part of B, denominated of A, as is ſhewn in the third Definition of the Seventh Book, but ſeeing that as C is to his half, ſo is B to his half, and alternately, as C is to B, ſo the half of C, is to the half of B. But C is a part of B, denominated of A, as is ſhewn; alſo the half of C, ſhall be a part of the half of B, denominated of A: therefore [b] A ſhall meaſure the half of B: Therefore, if an, &c. Which was to be demonſtrated.

a) 8. c. ſ.

b) 40. 7.

PROP. 31. THEOR. 29.



If an odde number A, *be prime to ſome number* B, *it ſhall be prime to its double* C.

Demonſtration FOr if A and C, are not primes to one another, they ſhall be meaſured by ſome number, which let be D, which of neceſſity ſhall be odde: For if it be even, meaſuring the odde number A: [a] A the product of D an even number, multiplyed by the number by which it meaſures it, ſhall be an even number, which is abſurd; for A is propoſed odde: Therefore D an odde number, meaſuring C an even number, (for C is an even number, ſeeing it hath an half B,) [b] ſhall alſo meaſure B his half: But it ſhall alſo meaſure A, therefore D ſhall meaſure A and B, primes to one another, which is abſurd; Therefore, A is a prime to C: Therefore, if a number, &c. Which was to be demonſtrated.

a) 28. 9.

b) 30. 9.

PROP. 32. THEOR. 30.

Of all the numbers B, C, D, *and* E, *which follow the binary* A, *in a double Progreſſion, each of them is onely evenly even.*

Unity A 2 B 4 C 8 D 16 E 32

Demonſtration THat it is ſo, is manifeſt; For having propoſed unity, ſeeing that A is a binary, and B, C, D, and E, doubles from the binary, they ſhall be continually proportional from unity; to wit, in double proportion: Therefore [a] A ſhall meaſure each of them, B, C, D, and E, and

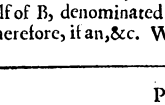
a) 11. 9.

and each leſſer ſhall meaſure the greateſt following, by ſome, one of theſe, A, B, C, D, and E, which being all even numbers, to wit, double from the Binary, an even number ſhall meaſure each of theſe B, C, D, and E, by an even number; therefore each of them, ſhall be evenly even, according to the Definition.

Now that they are onely evenly even is manifeſt: For ſeeing that A, B, C, and D, are continually proportional from unity, and that A next to unity, is a prime number, to wit, a binary, [b] not any other number, ſhall meaſure each of them, except thoſe A, B, C, D, and E, which being all even, an even number ſhall meaſure each of them only by an even number: Therefore, of all, &c. Which was to be demonſtrated.

b) 13. 9.

PROP. 33. THEOR. 31.



If a number A, *hath its half* B, *odde, the ſame* A, *is evenly odde.*

Demonſtration FOraſmuch as B an odde number, is the half of A, the binary C ſhall meaſure A an even number, by the ſame half B, an odde number; therefore [a] A is evenly odde: I ſay alſo, it is onely ſo: for let B be the half of A, and C a binary, if A be not evenly odde onely, it ſhall be alſo evenly even; therefore ſome even number ſhall meaſure it by an even number: Let D an even number, meaſure it by E, alſo an even number, and ſo [b] A ſhall be the product of D by E: but the ſame A is the product of C the binary, by B the half of A; therefore the number produced of C, the firſt, by B the fourth, is equal to that which is produced of D the ſecond, by E the third: therefore [c] as C is to D, ſo E to B, but C the binary, meaſures D an even number; therefore E an even number, ſhall meaſure alſo B an odde number, which is abſurd; therefore A is not evenly even, but is onely evenly odde. Therefore, if, &c. Which was to be demonſtrated.

a) 9. d.

b) 9. c. ſ.

c) 19. 7.

PROP. 34. THEOR. 32.

If an even number A, *be none of thoſe that are double from unity, nor hath its half odde, it is evenly even, and evenly odde.*

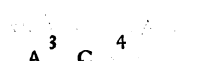
Demonſtration THat it is evenly even, is evident: For ſeeing that its half is even, the binary which is even, ſhall meaſure it by the ſame half, therefore it ſhall be evenly even.

9. d.

It is alſo evenly odde, for the ſame A being divided into two equal parts, and its half again divided in two, and ſo on, you ſhall at laſt come to an odde number (before you come to the binary, ſeeing that it is none of thoſe that are double from unity) which ſhall meaſure A by an even number. For if it ſhould meaſure it by an odde number, (ſeeing [a] that an odde number multiplying

a) 29. 9.

PROP. 30. THEOR. 28.



If an odde number A, *measureth an even number* B, *it shall also measure its halfe.*

Demonstration FOr let A measure B by C, the number C shall be even; and therefore shall have a half: Wherefore seeing that A measureth B by C, [a] C shall also measure B by A: and so C shall be a part of B, denominated of A, as is shewn in the third Definition of the Seventh Book, but seeing that as C is to his half, so is B to his half, and alternately, as C is to B, so the half of C, is to the half of B. But C is a part of B, denominated of A, as is shewn; also the half of C, shall be a part of the half of B, denominated of A: therefore [b] A shall measure the half of B: Therefore, if an, &c. Which was to be demonstrated.

a) 8. c. s.

b) 40. 7.

PROP. 31. THEOR. 29.



If an odde number A, *be prime to some number* B, *it shall be prime to its double* C.

Demonstration FOr if A and C, are not primes to one another, they shall be measured by some number, which let be D, which of necessity shall be odde: For if it be even, measuring the odde number A: [a] A the product of D an even number, multiplyed by the number by which it measures it, shall be an even number, which is absurd; for A is proposed odde: Therefore D an odde number, measuring C an even number, (for C is an even number, seeing it hath an half B,) [b] shall also measure B his half: But it shall also measure A, therefore D shall measure A and B, primes to one another, which is absurd; Therefore, A is a prime to C: Therefore, if a number, &c. Which was to be demonstrated.

a) 28. 9.

b) 30. 9.

PROP. 32. THEOR. 30.

Of all the numbers B, C, D, *and* E, *which follow the binary* A, *in a double Progression, each of them is onely evenly even.*

Unity A 2 B 4 C 8 D 16 E 32

Demonstration THat it is so, is manifest; For having proposed unity, seeing that A is a binary, and B, C, D, and E, doubles from the binary, they shall be continually proportional from unity; to wit, in double proportion: Therefore [a] A shall measure each of them, B, C, D, and E, and

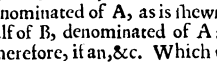
a) 11. 9.

and each lesser shall measure the greatest following, by some, one of these, A, B, C, D, and E, which being all even numbers, to wit, double from the Binary, an even number shall measure each of these B, C, D, and E, by an even number; therefore each of them, shall be evenly even, according to the Definition.

Now that they are onely evenly even is manifest: For seeing that A, B, C, and D, are continually proportional from unity, and that A next to unity, is a prime number, to wit, a binary, [b] not any other number, shall measure each of them, except those A, B, C, D, and E, which being all even, an even number shall measure each of them only by an even number: Therefore, of all, &c. Which was to be demonstrated.

b) 13. 9.

PROP. 33. THEOR. 31.



If a number A, *hath its half* B, *odde, the same* A, *is evenly odde.*

Demonstration FOrasmuch as B an odde number, is the half of A, the binary C shall measure A an even number, by the same half B, an odde number; therefore [a] A is evenly odde: I say also, it is onely so: for let B be the half of A, and C a binary, if A be not evenly odde onely, it shall be also evenly even; therefore some even number shall measure it by an even number: Let D an even number, measure it by E, also an even number, and so [b] A shall be the product of D by E: but the same A is the product of C the binary, by B the half of A; therefore the number produced of C, the first, by B the fourth, is equal to that which is produced of D the second, by E the third: therefore [c] as C is to D, so E to B, but C the binary, measures D an even number; therefore E an even number, shall measure also B an odde number, which is absurd; therefore A is not evenly even, but is onely evenly odde. Therefore, if, &c. Which was to be demonstrated.

a) 9. d.

b) 9. c. s.

c) 19. 7.

PROP. 34. THEOR. 32.



If an even number A, *be none of those that are double from unity, nor hath its half odde, it is evenly even, and evenly odde.*

Demonstration THat it is evenly even, is evident: For seeing that its half is even, the binary which is even, shall measure it by the same half, therefore it shall be evenly even.

9. d.

It is also evenly odde, for the same A being divided into two equal parts, and its half again divided in two, and so on, you shall at last come to an odde number (before you come to the binary, seeing that it is none of those that are double from unity) which shall measure A by an even number. For if it should measure it by an odde number, (seeing [a] that an odde number multiplying

a) 29. 9.

plying an odde number, makes an odde number) A ſhould be odde, which is impoſſible, ſeeing that it is propoſed even: therefore ſeeing that an odde number meaſureth A by an even number, and that an even number doth alſo meaſure it by an even number, A ſhall be evenly even, and evenly odde. Therefore, if, &c. Which was to be demonſtrated.

PROP. 35. THEOR. 33.

If there be as many numbers as you pleaſe, continually proportional, A, B C, D, *and* E F, *and if there be cut off a number* C G, *from the ſecond* B C, *as alſo from the laſt* EF, *a number* H F, *equal to the firſt, as the exceſſe* B G, *of the ſecond* B C, *ſhall be to the firſt* A, *ſo the exceſſe* E H, *of the laſt* E F, *ſhall be to all the antecedents* A, B C, *and* D.

A.....8

4 8

B.... G.....C

D.....18

9 6 4 8

E.....K.....I.... H.....F

Demonſtration FOr let F I be cut off equal to B C, and F K equal to D; foraſmuch as F I is equal to BC, and the part cut off F H, is equal to the part cut off C G, the reſt I H ſhall be equal to the reſt B G; but foraſmuch as A is to B C, ſo B C is to D, and D to E F: and by converſion, as E F is to D, ſo D is to B, and B C to A: But K F is equal to D C, & I F, to B C, and H F to A: in like manner, as E F ſhall be to K F, ſo K F to I F, and I F to H F; therefore by dividing, as E K, to K F, ſo K I, to I F, and I H to H F: and a) 12. 7. [a] therefore alſo all the numbers E K, K I, and I H, ſhall be to all the numbers K F, I F, and H F, that is to ſay, the whole E H, to D, B C, and A, together, (to which K F, I F, and H F, have been taken equal) as I H, to H F; that is to ſay, as B G to A, for B G and A, are equal to I H and H F: Therefore, if, &c. Which was to be demonſtrated.

PROP. 36. THEOR. 34.

If from unity be taken as many numbers as you pleaſe, A, B, C, *and* D, *continually proportional in a double proportion, until the whole compound* E, *be made a prime number, and that whole* E *multiplyed by the laſt* D, *maketh ſome one* F, *the product* F, *ſhall be a perfect number.*

De mo-

Unity A..2 B....4 C....8 D.....16
E.31 G-----62 H---124 I---248 F----496
K 31 M 31 L 31
O---- P---- N 465

Demonſtration FOr, Let there be taken E, G, H, and I, in a double rate, from E, and equal in multitude to A, B, C, and D, foraſmuch then as A, B, C, and D, and E, G, H, and I, are equal in multitude, and being taken two and two, are in the ſame rate, being all in a double proportion in equal rate, as [a] A ſhall be to D, ſo ſhall E be to I, and therefore, a) 14. 7.
[b] the product of A the firſt, by I, the fourth, ſhall be equal to the product of D, the ſecond, by E the third: But F is the product of D in E: therefore, b) 19. 7.
F ſhall be alſo the product of A in I: therefore [c] I ſhall meaſure F by A the binary, wherefore F ſhall be double to I, and therefore E, G, H, I, and F, are continually proportional in a double rate. c) 7. c. ſ.

Let there be cut off from G the ſecond, and F the laſt, K and L, equal to E the firſt, whoſe exceſſes are M and N, as [d] M ſhall be to E, ſo N ſhall d) 35. 9.
be to all the antecedents E, G, H, and I, together: But M is equal to E, being the half of G double to E: therefore N ſhall be equal to E, G, H, and I, together, But L is equal to unity, and A, B, C, and D, together, being propoſed equal to E or K, therefore the whole L N, that is to ſay, F is equal to unity, and to the numbers A, B, C, D, E, F, G, H, and I, together: wherefore [e] ſeeing that unity and each of the numbers A, B, C, D, E, F, G, H, and I, c) 7. c. ſ.
do meaſure F, (for ſeeing F is made of E in D, [f] D ſhall meaſure F, and f) 11. c. ſ.
therefore unity and A, B, and C, which meaſureth D, ſhall alſo meaſure F: and again, ſeeing that I, as hath been ſhewn, doth meaſure F, E, G, and H, which meaſureth I: [g] becauſe of the double proportion ſhall alſo meaſure g) 11. c. ſ.
F, and no other ſhall meaſure F, as we ſhall preſently ſhew: Unity, and A, B, C, D, E, G, H, and I, ſhall be all the parts that F can have, to which F being ſhewn to be equal, [h] the ſame F ſhall be a perfect number.

Suppoſe ſome other number to meaſure F, beſides the aforegoing; to wit, O by P, [i] that being, F ſhall be the product of O by P: but the ſame F is alſo the product of E by D, therefore the number produced of E the firſt, by i) 9. c. ſ.
D the fourth, is the ſame, with the product of P the ſecond, by O the third; therefore [k] as E is to P, ſo O is to D. k) 19. 7.

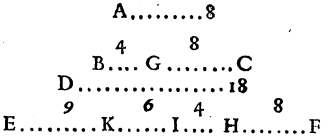
But, [l] ſeeing that A, B, C, and D, are continually proportional from Unity, and that A next to unity, is a prime number, no other number ſhall meaſure D the laſt, beſides thoſe of the proportion A, B, and C. Now O is different from A B and C, by the Suppoſition; therefore O ſhall not meaſure D; but as O is to D, ſo E is to P, therefore alſo neither ſhall E meaſure P: therefore [m] E being a Prime: E and P, ſhall be Primes to one another; m) 31. 7.
and ſo [n] the leaſt of their rate Wherefore E ſhall meaſure O, and P ſhall n) 23. 7.
meaſure D equally; being in the ſame rate: But, [o] no number doth meaſure D, except A, B, and C, therefore P ſhall be the ſame with one of o) 21. 7.
thoſe A, B, and C. d) 13. 9.

Suppoſe it the ſame as B, and let E, G, and H, be taken double from E, in ſuch multitude, as B, C, D, [p] in equal rate, as B to D, ſo E to H; [q] but p) 14. 7.
the product of B in H, is equal to the product of D in B, and the product of q) 19. 7.
D in E, hath been ſhewn equal to the product of P in O: therefore the

plying an odde number, makes an odde number) A ſhould be odde, which is impoſſible, ſeeing that it is propoſed even: therefore ſeeing that an odde number meaſureth **A** by an even number, and that an even number doth alſo meaſure it by an even number, A ſhall be evenly even, and evenly odde. Therefore, if, &c. Which was to be demonſtrated.

PROP. 35. THEOR. 33.

If there be as many numbers as you pleaſe, continually proportional, A, B C, D, *and* E F, *and if there be cut off a number* C G, *from the ſecond* B C, *as alſo from the laſt* EF, *a number* H F, *equal to the firſt, as the exceſſe* B G, *of the ſecond* B C, *ſhall be to the firſt* A, *ſo the exceſſe* E H, *of the laſt* E F, *ſhall be to all the antecedents* A, B C, *and* D.



Demonſtration FOr let F I be cut off equal to B C, and F K equal to D; foraſmuch as F I is equal to BC, and the part cut off F H, is equal to the part cut off C G, the reſt I H ſhall be equal to the reſt B G; but foraſmuch as A is to B C, ſo B C is to D, and D to E F: and by converſion, as E F is to D, ſo D is to B, and B C to A: But K F is equal to D C, & I F, to B C, and H F to A: in like manner, as E F ſhall be to K F, ſo K F to I F, and I F to H F; therefore by dividing, as E K, to K F, ſo K I, to I F, and I H to H F: and
a) 12. 7. [a] therefore alſo all the numbers E K, K I, and I H, ſhall be to all the numbers K F, I F, and H F, that is to ſay, the whole E H, to D, B C, and A, together, (to which K F, I F, and H F, have been taken equal) as I H, to H F; that is to ſay, as B G to A, for B G and **A**, are equal to I H and H F: Therefore, if, &c. Which was to be demonſtrated.

PROP. 36. THEOR. 34.

If from unity be taken as many numbers as you pleaſe, A, B, C, *and* D, *continually proportional in a double proportion, until the whole compound* E, *be made a prime number, and that whole* E *multiplyed by the laſt* D, *maketh ſome one* F, *the product* F, *ſhall be a perfect number.*

De mo-

Unity A..2 B....4 C....8 D.....16
E.31 G-----62 H---124 I---248 F----496
K 31 M 31 L 31
O---- P---- N 465

Demonſtration FOr, Let there be taken E, G, H, and I, in a double rate, from E, and equal in multitude to A, B, C, and D, foraſmuch then as A, B, C, and D, and E, G, H, and I, are equal in multitude, and being taken two and two, are in the ſame rate, being all in a double proportion in equal rate, as [a] A ſhall be to D, ſo ſhall E be to I, and therefore, a) 14. 7.
[b] the product of A the firſt, by I, the fourth, ſhall be equal to the product b) 19. 7.
of D, the ſecond, by E the third: But F is the product of D in E: therefore, F ſhall be alſo the product of A in I: therefore [c] I ſhall meaſure F by A c) 7. c. ſ.
the binary, wherefore F ſhall be double to I, and therefore E, G, H, I, and F, are continually proportional in a double rate.

Let there be cut off from G the ſecond, and F the laſt, K and L, equal to E the firſt, whoſe exceſſes are M and N, as [d] M ſhall be to E, ſo N ſhall d) 35. 9.
be to all the antecedents E, G, H, and I, together: But M is equal to E, being the half of G double to E: therefore N ſhall be equal to E, G, H, and I, together. But L is equal to unity, and A, B, C, and D, together, being propoſed equal to E or K, therefore the whole L N, that is to ſay, F is equal to unity, and to the numbers A, B, C, D, E, F, G, H, and I, together: wherefore [e] ſeeing that unity and each of the numbers A, B, C, D, E, F, G, H, and I, e) 7. c. ſ.
do meaſure F, (for ſeeing F is made of E in D, [f] D ſhall meaſure F, and f) 11. c. ſ.
therefore unity and A, B, and C, which meaſureth D, ſhall alſo meaſure F: and again, ſeeing that I, as hath been ſhewn, doth meaſure F, E, G, and H, which meaſureth I: [g] becauſe of the double proportion ſhall alſo meaſure F, and no other ſhall meaſure F, as we ſhall preſently ſhew: Unity, and g) 11. c. ſ.
A, B, C, D, E, G, H, and I, ſhall be all the parts that F can have, to which F being ſhewn to be equal, [h] the ſame F ſhall be a perfect number.

Suppoſe ſome other number to meaſure F, beſides the aforegoing, to wit, O by P, [i] that being, F ſhall be the product of O by P: but the ſame F is alſo the product of E by D, therefore the number produced of E the firſt, by i) 9. c. ſ.
D the fourth, is the ſame, with the product of P the ſecond, by O the third; therefore [k] as E is to P, ſo O is to D. k) 19. 7.

But, [l] ſeeing that A, B, C, and D, are continually proportional from Unity, and that A next to unity, is a prime number, no other number ſhall meaſure D the laſt, beſides thoſe of the proportion A, B, and C. Now O is different from A B and C, by the Suppoſition; therefore O ſhall not meaſure D; but as O is to D, ſo E is to P, therefore alſo neither ſhall E meaſure P: therefore [m] E being a Prime: E and P, ſhall be Primes to one another; m) 31. 7.
and ſo [n] the leaſt of their rate Wherefore E ſhall meaſure O, and P ſhall n) 23. 7.
meaſure D equally; being in the ſame rate: But, [o] no number doth meaſure D, except A, B, and C, therefore P ſhall be the ſame with one of 21. 7. o) 13. 9.
thoſe A, B, and C.

Suppoſe it the ſame as B, and let E, G, and H, [p] be taken double from E, in ſuch multitude, as B, C, D, [q] in equal rate, as B to D, ſo E to H; but p) 14. 7.
the product of B in H, is equal to the product of D in B, and the product of q) 19. 7.
D in E, hath been ſhewn equal to the product of P in O: therefore the

r) 19. 7. number made of P by O, is the same as that which is produced of B by H, therefore r as P is to B, so is H to O; but P was the same with B, therefore H shall be the same with O, which is absurd; for O was put different from any of the numbers A, B, C, D, E, G, H, or I; Therefore, A, B, C, D, E, G, H, and I, do only measure the number F, the which they do compose: Therefore, if from Unity, &c. Which was to be demonstrated.

SCHOLIUM.

Euclide *in these three fore-going Books intends onely Arithmetical Numbers and not continued quantity in which there happens Fractions; but in numbers there happens none, but only in relation to the things numbered when they are divided into other parts, which are called Fractions, insomuch as they are compared to their whole divided into certain number of equal parts, the which are yet neverthelesse explained by whole numbers, that is to say, by the numbers of which* Euclide *speakes in these three fore-going Books; and then when such continued quantities are reduced to the same equal parts, they are no more Fractions, and numbers compared to such things, are compared the one to the other as whole numbers, and are taken absolutely, without referring them to any intire quantity, and all the demonstrations and operations, which may be performed by vulgar Fractions of common Arithmetick, are all performed by Arithmetical and whole numbers, concerning which* Euclide *speaks in these Seventh, Eighth, and Ninth Books.*

The End of the Ninth Element of EUCLIDE.

THE

TENTH ELEMENT

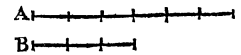
OF

EUCLIDE.

THE ARGUMENT.

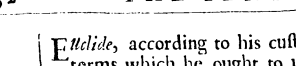
Euclide having finished those things which concern numbers in the three fore-going Books, so far forth as was needful for the understanding of Geometrical things: Now in this Tenth Book, he attempts the disputation of commensurable and incommensurable Lines, for whose cause, as we have said, those Books of numbers were undertaken by him. For without the knowledge of these Lines, divers Magnitudes, as well Solids, as Plaines, can neither be perfectly understood, nor brought into practice when it is required, because divers of their sides are incommensurable, which may be said likewise of Plains, and Solids themselves, for they also are often times incommensurable, as we shall demonstrate at the end of this Book.

DEFINITIONS.



I *Commensurable magnitudes* A *and* B, *are such as are measured by one and the same common measure* C.

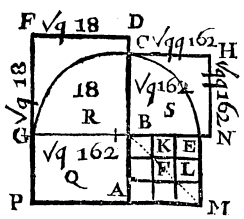
Euclide, according to his cuſtom, begins here by the explication of the terms which he ought to uſe: and declareth in the firſt place what Magnitudes are ſaid to be commenſurable, and which incommenſurable, defining that thoſe are ſaid to be commenſurable, which are meaſured by one and the ſame common meaſure, or Magnitude: As the Magnitudes A and B; the which one and the ſame meaſure, as C doth meaſure: (for being repeated ſix times, it meaſureth A, and being repeated three times, it meaſureth B) ſhall be ſaid to be commenſurable.



After the ſame manner, a line of 20 palmes, and one of 13, are ſaid to be commenſurable; ſeeing that as well the line of one palme, as that of half a palme, &c. meaſureth both of them: In like manner, thoſe Superficies are ſaid to be commenſurable that are meaſured by one and the ſame Superficies, and ſolid bodies are alſo ſaid to be commenſurable; when they are meaſured by one and the ſame Solid.

2 *But Magnitudes incommenſurable, are thoſe which have no common meaſure.*

SUch Magnitudes are the Diameter of any ſquare, and the ſide of the ſame ſquare whereof it is the Diameter: forasmuch as there is no common Magnitude that can meaſure them both, as ſhall be demonſtrated in the laſt Propoſition of this Book: There are alſo divers other lines incommenſurable, divers of which are unfolded in this Book; and the manner how to find them, is ſhewn: Again, Superficies and Solids, are ſaid to be incommenſurable, the which admit of no common meaſure.



3 *Right lines* A B *and* B D, *are commenſurable in power, when their ſquares* B M, *and* B F, *are meaſured by one and the ſame Space or Superficies* E.

OF Right-lines, ſome are commenſurable in Length; to wit, thoſe which are meaſured by one and the ſame line, and are ſaid ſimply or abſolutely commenſurable, ſome others are ſaid to be incommenſurable; that is to ſay, ſuch as have no common meaſure, as hath been expounded in the firſt and ſecond Definitions.

Again, of Lines which are incommenſurable in Length, ſome of them are ſo, in ſuch manner as that their ſquares are commenſurable; as a line of two feet, and another, which ſhall be √ q. 8. (that is to ſay, of the which the ſquare ſhall contein 8 feet) are ſaid to be commenſurable in power, the ſquares 4, and 8, being meaſured by one and the ſame Superficies. So a line of three feet, and a line of 6 feet, ſhall be alſo commenſurable in power, ſeeing their ſquares 9, and 36, are meaſured by one and the ſame Superficies; to wit, by 3, or by 6. Again,

Again, a line which ſhall be √qq of 20, and one that ſhall be √qq 125, are commenſurable in power: for their q q √ q 20, and √ q 125, are commenſurable, being as 5, to 2, and √ q 5, ſhall be in √ q 20, twice, and in √ q 125 5 times, which will be more eaſily underſtood by the figure, where the line A B is 3, and his q B M 9, whoſe Diameter is alſo B M, to which A B is incommenſurable, as hath been ſaid in the 2 Definition; and having drawn B D equal to the Diameter B M, and deſcribed the ſquare B F, the ſame ſhall be 18; it [a] being double the ſquare of B A, and his ſide ſhall be √ q 18 commenſurable in power to B A: for B meaſureth both the one and the other ſquares, to wit, B M 9 times, and B F 18 times, E tripled meaſureth alſo B F 6 times, and B M 3 times. [a] 47. 1.

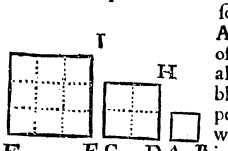
Now if you will have the ſquare E to be √ q 6, the q of the line A B, make √ q 486, and the q of B D, ſhall make √ q 1944, and √ q 6, ſhall meaſure both, & √ q 54, the triple of √ q 6, ſhall alſo meaſure thoſe two quantities.

4 *But Right Lines are incommenſurable in power, when the ſquares made of them cannot be meaſured by any common meaſure.*

THe other lines incommenſurable, whoſe ſquares are alſo incommenſurable, are ſaid to be incommenſurable in power, as a line of 2 feet, and a line which ſhall be √ √ 6, ſhall be incommenſurable in power, as not having any common ſpace that may meaſure their ſquares 4, and √ 6 1. So √ q 8, and √ qq 12, are alſo incommenſurable in power, forasmuch as no Superficies can meaſure their Squares 8 and √ q 12.

Again, as in the figure of the third Definition, the rectangled Superficies P B, is propoſed √ q 162, to the which [a] let the ſquare Superficies B H, be made equal, the line G B, or B D, his equal; ſhall be incommenſurable in power to B C, or B N; for B C is a mean proportional between G B, and B A, or B E, equal to B A: therefore G B, B C, and B E, being continually proportional, [b] G D the ſquare of G B, is to B H the ſquare of B C, as G B the firſt, to B E the third: But G B is incommenſurable to B E, as hath been ſaid in the ſecond Definition: therefore [c] the ſquare of G B, which is G D, ſhall be incommenſurable to the ſquare of B C; that is to ſay, to B H: therefore G B, and B C, or B N, his equal, are incommenſurable in power; that is to ſay, their ſquares are incommenſurable: and this may not be thought ſtrange, that this Demonſtration be built on the Propoſitions hereafter mentioned; for that it is not neceſſary for the Demonſtration of any of them. [a] 14. 1. [b] 20. 6. [c] 10.

Now all the right lines commenſurable in Length, are ſo alſo in power, ſeeing that the ſquare of their common meaſure, doth meaſure their ſquares, as appears by this figure: For as A B meaſureth C D, and E F, ſo the ſquare of A B, doth meaſure C H, and E I, not that all the right lines which are commenſurable in power, are ſo alſo in length, as appears by the figure of the third Definition, where the lines A B, and B D, are ſhewn to be incommenſurable in length, the which are ſhewn commenſurable in power, and as it ſhall be alſo demonſtrated in this Book.



5 *Theſe*

5 *These things being so, it is shewn, (that is to say, manifest,) that to every right line proposed, an infinite number of right lines are commensurable, and an infinite number are incommensurable, some in length and power, others onely in power; Now let that right line proposed, be called rational.*

IF there be proposed some right line, of a known, and determinated Magnitude, of all the lines compared with it, some shall be commensurable in length and power thereto, others in power onely, also some shall be incommensurable in length thereto, and some in length and power, as shall be shewn in this Book: But this line proposed, by reason of which, the others are said to be commensurable, or incommensurable, is named by the Greeks ῥητὴ, that is to say, rational; for that it is alwayes proposed of a known Magnitude, and that according thereto it is argued: But the others compared therewith, are not alwayes known: although according to the same lines taken apart, they may be known, each of them being divided into such equal parts as you please.

6 *And the right lines commensurable to this rational line, either in length and power, or in power alone, may be also called rationals.*

Euclide shews that the other lines commensurable, after what manner soever, to the line called *rational*, are also rationals, not by supposition, as the line it selfe: but as being compared therewith, they will be found commensurable thereto in length and power, or in power only.

7 *And the lines incommensurable to this rational line, may be called irrationals.*

SEeing that in the precedent Definition *Euclide* hath called the lines commensurable in length and power, or in power onely to the rational line proposed, Rationals, it is manifest by that Definition, that those lines which are incommensurable to the Rational proposed in length and power, and not in length onely, are Irrationals.

8 *And the square described of the rational line proposed, may be called rational.*

EVen as the line proposed to be known and determinated, is said to be rational: even so the square described thereof, is called rational, forasmuch as that also is known and certaine, and in comparison thereof, the other Superficies are said to be commensurables, or incommensurables.

9 *And*

9 *And the figures commensurable to this square rational, may be called rationals.*

ALL the Plain Superficies commensurable to the square of the rational proposed, are said to be rationals, yet not by position as the same, but as being compared to the same square, they are found to be commensurable thereto, or incommensurable, even so also as the lines in what manner soever commensurable to the line proposed rational, are called rationals.

10 *But the figures incommensurable to the rational square, may be called irrationals, or Surds.*

EVen so as the lines incommensurable to the line proposed rational, are said to be irrational, so also the plain Superficies incommensurable to the square of the rational line, are called irrationals.

11 *And the right lines which are in power as the same irrational figures, are said to be irrationals, and Surds; to wit, if they be squares, their sides may be irrational; but if they be other rectilines whatsoever, the lines which are in power, as the squares equal to those irrational figures may be irrational.*

HE calls the lines which are equal in power to the spaces, incommensurable to the square of the rational line, irrationals, in such sort, that if those spaces irrational are squares, their sides shall be said to be irrational, if they be not squares, but are other rectilines; the right lines which describe the squares, equal to these spaces incommensurable, shall be said to be irrational: for a right line is said to be equal in power to a figure, when the square described of the same line is equal to the same figure.

PETITIONS.

That any Magnitude whatsoever, may be multiplyed so many times, as that it may exceed any Magnitude whatsoever proposed of the same kind.

FOr two Magnitudes unequal, but of the same kind being proposed, the greatest being not infinite, and the least in power to be augmented infinitely, it is manifest, that the least may be multiplyed so many times, as that it may exceed the greatest, which appeares also by the fifth definition of the fifth Book.

COM-

COMMON SENTENCES.

1 *A magnitude measuring as many magnitudes as you please, doth also measure the magnitude compounded of them.*

2 *A magnitude which measureth any magnitude whatsoever, doth in like manner, measure every other magnitude, which that magnitude doth measure.*

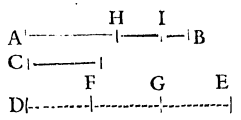
3 *A magnitude measuring a whole magnitude, and the part cut off there-from, doth also measure the rest.*

PRO-

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

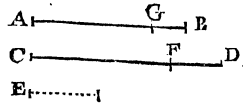


Two unequal magnitudes being proposed, A B, *and* C, *if from the greatest* A B; *there be cut off more then the halfe, and of the rest again, more than the halfe; and if that be done alwayes so, in continuing, there will remain in the end some magnitude lesse then the least magnitude* C *proposed.*

Demonstration FOr let C be multiplyed so many times, as that the product may exceed A B, and let D E be the product, so as that D E may be multiplex of C, and next greater then A B; and let D E be divided according to the parts D F, F G, G E, each equal to C, and from A B, let there be cut off more then the half A H, and of the rest H B, also more then the half H I, and so alwayes until the parts of A B, may be equal in number to those of D E; Let the parts then A H, H I, I B, be as many in number, as D F, F G, G E; Forasmuch as D E is greater then A B, and that from D E there be taken D F, lesse then the halfe, (or the halfe, if D E were onely double to C,) and from A B more then the halfe A H, the rest F E shall be greater then the rest H B.

Again, F E being greater then H B, if from F E be taken the halfe F G, (or lesse then the halfe, if F E were greater then the double of C:) but from H B, more then the half H I, the rest G E shall be in like manner greater then the rest I B; But G E is equal to C, therefore C shall be also greater then I B, the last part of A B: Therefore, of A B there will remaine a magnitude lesse then C. The same will happen, if you take away the halfe of A H, and from the rest H B, the halfe H I, for in like manner, I B the rest shall be lesse than G E, that is to say, then C. Therefore, Two, &c. Which was to be demonstrated.

PROP. 2. THEOR. 2.



Two magnitudes unequal A B *and* C D, *being propoſed, if you cut off ſtill alternately, the leaſt* A B, *from the greateſt* C D, *and that the magnitude remaining, ſhall never meaſure his precedent magnitude, thoſe magnitudes* A B *and* C D, *ſhall be incommenſurable.*

Demonſtration FOr otherwiſe they ſhould be meaſured by ſome common meaſure, which ſuppoſe to be E: Therefore, E ſhall be equal to A B, or leſſe, and having taken A B from C D, as many times as may be, let there reſt F D leſſe then A B, in ſuch ſort as A B meaſures C F.

Again, having taken F D from A B, let there be left G B, leſſe then the ſame F D, in ſuch ſort as that F D meaſure A G, and let this cutting off be ſtill continued, till at laſt [a] there ſhall remaine of C D or A B ſome magnitude leſſe then E.

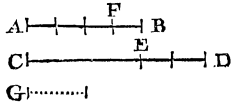
a) 1. 10.

Foraſmuch as A B taken from C D leaves F D, leſſe then it ſelfe, C F cut off ſhall be greater then the half C D: for if C F were the halfe, or leſſe then the halfe of C D, A B might yet be cut off from C D: in like manner, A G, cut off from A B, ſhall be greater then the halfe of A B: and ſo it ſhall ſtill happen, cutting off ſtill therefore more then the halfe, there will remaine in the end ſome magnitude leſſe then E. Let then G B the reſt be leſſe then E.

Foraſmuch as E meaſures A B, and A B meaſures C F, [b] E ſhall alſo meaſure C F: but E meaſures the whole C D: therefore [c] E ſhall alſo meaſure the reſt F D, and F D ſhall meaſure A G: therefore [d] E ſhall alſo meaſure A G: but E meaſures the whole A B; therefore [e] E ſhall alſo meaſure the reſt G B the greateſt, the leaſt, which is abſurd: Therefore no magnitude ſhall meaſure A B and C D; Therefore they are incommenſurables: Therefore, If, &c. Which was to be demonſtrated.

b) 2. c. f.
c) 3. c. f.
d) 2. c. f.
e) 3. c. f.

PROP. 3. PROBL. 1.



Two magnitudes commenſurable A B, *and* C D, *being given, to find their greateſt common meaſure.*

Conſtruction FOraſmuch as A B and C D are commenſurable from the greateſt C D: Let there be taken A B the leaſt, as often as you can: and that there remaine E D, and from A B let there be taken E D alſo as often as you may, and there will reſt F B, and doing ſo ſtill

ſtill alternatively, there will at laſt remaine a magnitude, which ſhall meaſure his precedent, otherwiſe A B and C D ſhould be incommenſurable, contrary to the Suppoſition. Let F B then meaſure E D: I ſay, that F B is the greateſt common meaſure of A B and C D.

Demonſtration FOr ſeeing that F B meaſures E D, and E D meaſures A F [a] F B ſhall meaſure A F: but F B meaſures it ſelfe: therefore, it ſhall alſo meaſure the whole A B: therefore [b] F B ſhall alſo meaſure C E, which is meaſured of A B: therefore ſeeing that F B meaſures alſo E D, [c] the ſame F B ſhall meaſure the whole C D, it ſhall therefore meaſure A B, and C D therefore is their common meaſure.

a) 2. c. f. 1. c. f.
b) 2. c. f.
c) 1. c. f.

For if it be denied, ſuppoſe G to be the greateſt common meaſure: Foraſmuch then as G meaſures A B and C D, and that A B meaſures C E, [d] G ſhall alſo meaſure C E: but G meaſures C D, therefore [e] G ſhall alſo meaſure E D the reſt; (ſeeing that G meaſures the whole C D, and the part cut off C E:) but E D ſhall meaſure A F: therefore [f] G ſhall alſo meaſure A F, and meaſuring the whole A B, it ſhall alſo meaſure the reſt F B, leſſe then it ſelfe, which is abſurd; therefore F B is the greateſt common meaſure of A B and C D.

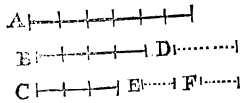
d) 2. c. f.
e) 3. c. f.
f) 2. c. f.
3. c. f.

Now if the leaſt A B, meaſure the greateſt, in ſuch manner, as being cut off from C D, there ſhall remaine nothing, A B ſhall be the greateſt common meaſure of the two, ſeeing that A B meaſures it ſelfe: Therefore, Two magnitudes, &c. Which was to be done.

COROLLARIE.

From this Demonſtration it is manifeſt, that a magnitude which meaſureth two magnitudes, meaſureth alſo their greateſt common meaſure, as may be gathered from that part of the Demonſtration, by which it is ſhewn that F B *is the greateſt common meaſure of* A B, *and* C D, *for there it is ſhewn that if* G *meaſure* A B *and* C D, *it ſhall alſo meaſure their greateſt common meaſure* F B.

PROP. 4. PROBL. 2.



Three magnitudes commenſurable A, B, *and* C, *being given, to find their greateſt common meaſure.*

Conſtruction FInde [a] in the firſt place D, the greateſt common meaſure of A and B, if D alſo meaſure C, it appears that D is the greateſt common meaſure of the three A. B, and C.

a) 3. 10.

Demonſtration IF a magnitude greater then D meaſure A, B, and C, the ſame [b] ſhall alſo meaſure D, the greateſt common meaſure of

b) Co. 3. 10.

of A and B, the greateſt, the leaſt, which is abſurd: but if D meaſure not C at leaſt, C and D ſhall be commenſurable.

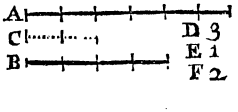
For ſeeing that A, B, and C, are commenſurable, any whatſoever, their
c) Co. 3. 10. common meaſure, c ſhall meaſure D the greateſt common meaſure of A and B: therefore the ſame common meaſure meaſuring alſo C, D and C ſhall be commenſurable: Let d E therefore be the great
d) 3. 10. eſt common meaſure of C & D; I ſay, that E is the greateſt common meaſure of A, B, & C; For aſmuch as E meaſures D and C,
e) 2. c. f. and D meaſures A and B, E e ſhall alſo meaſure A and B; But E alſo meaſures C: therefore E is the greateſt common meaſure of A, B, and C, I ſay, that E is alſo their greateſt oommon meaſure: For otherwiſe, ſuppoſe F a greater common meaſure then E.

f) Co. 3. 10. Foraſmuch then, as F meaſures A and B, it f ſhall alſo meaſure D their
g) Co. 3. 10. greateſt common meaſure: But F alſo meaſures C, it g ſhall then meaſure alſo E the greateſt common meaſure of C and D, the greateſt, the leaſt, which is abſurd: Therefore, Three, &c. Which was to be done.

COROLLARIE.

It is manifeſt alſo that a magnitude which meaſureth three magnitudes, doth alſo meaſure their greateſt common meaſure. For it hath been demonſtrated that ſeeing that F *meaſures* A, B, *and* C, *it ſhall alſo meaſure their greateſt common meaſure* E: *In like manner, if there be more then three magnitudes commenſurable given, their greateſt common meaſure may be found, and if there were four, the queſtion will be firſt of three, if of five, the queſtion will be of four, &c. and this ſame Corollarie ſhall take place.*

PROP. 5. THEOR. 3.



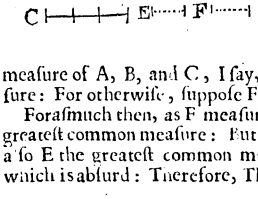
Magnitudes commenſurable A, *and* B, *have the ſame rate, the one to the other, as a number to a number.*

a) 3. 10. *Demonſtration* LEt a C be their greateſt common meaſure, and as often times as C meaſures A, let unity E meaſure as often times a number, as D, and as often times as C meaſures B, let unity alſo meaſure the number F.

Foraſmuch then as the magnitude C, and unity E, meaſures the magnitude A, and the number D equally, the magnitude A ſhall contein C, as many times as the number D conteineth unity: Therefore, as A is to C, ſo the

the number D is to unity; But as C is to B, ſo unity is to F, (ſeeing that C meaſures B; and unity meaſures F equally,) b therefore in equal rate A ſhall be to B, as the number D, to the number F: Therefore, The, &c. Which was to be demonſtrated. b) 22. 5.

PROP. 6. THEOR. 4.



If two magnitudes A, *and* B, *have the ſame rate to one another, as a number* G, *to a number* D; *they ſhall be commenſurable.*

Demonſtration FOr, Let the magnitude A be underſtood to be divided into as many equal parts as C conteins unites, and let one of the ſaid parts be equal to the magnitude E, and let there be another magnitude F meaſured as many times by E, as unity meaſures D.

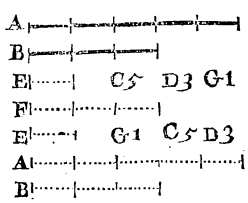
Foraſmuch as E is as many times in A, as unity in the number C, the magnitude A, and the number C, ſhall contein E, and unity equally; therefore A ſhall be to E, as C to unity. But E is to F as unity to D, (E meaſuring F, and unity meaſuring D equally,) therefore a in equal rate, A ſhall be to a) 22. 5.
F, as the number C, to the number D: but A is alſo to B, as the number C to the number D, by Suppoſition; therefore, b as A to B, ſo is A to F: there- b) 11. 5.
fore, c B and F are equal: therefore E meaſuring F ſhall alſo meaſure B, e- c) 9. 5.
qual to F: but E ſhould alſo meaſure A: therefore E meaſures A and B; therefore d A and B are commenſurable: Therefore, If Two, &c. Which was d) 1. d.
to be demonſtrated.

Otherwiſe, Let the magnitude A be ſuppoſed to be divided into as many equal parts as there are unites in the number C, to one of which parts let the magnitude E be equal.

Foraſmuch then, as E is to A, ſo is unity to the number C, (E meaſuring A, and unity the number C equally,) but A is propoſed to be to B, as the number C to the number D; therefore in equal rate, as E is to c B, ſo is e) 22. 5.
unity to D: but unity meaſures the number D: Therefore E ſhall meaſure B, and E alſo meaſure A: therefore, f A and B, having A for common f) 1 def.
meaſure, ſhall be commenſurable, which is propoſed on the ſecond part of the figure.

COROLLARIE.

From the firſt part of this Theorem it appears how you may finde a right line, to which any other right line propoſed, may be as a number to a number: For, (ſee the firſt figure,) let there be found a line, to which the propoſed line A *may be as the number* C *to the number* D; *divide* A *into as many equal parts*



parts as there are unites in C, *and take* F, *conteining as many of the same parts of* A *as there are unites in* D, *and so* A *shall be to* F, *as the number* C, *to the number* D, *as is demonstrated.*

It also appeares by what means may be found a right line, to whose square, the square of another right line given may be, as a number to a number: As for example, let there be found a line, to whose square, let the square of the line A *be as the number* C, *to the number* D, ; *in the first place, let* F *be found, to which let* A *be as* C *to* D, *as is said, then* [a] *let there be found the mean proportional between* A *and* F, [b] A *the line found, and* F, *being continually proportional; the square of* A *shall be to the square of the mean, as* A *to* F: *But* A *is to* F, *as the number* C, *to the number* D: *therefore the square of* A, *shall be to the square of the mean proportional, as the number* C *to the number* D.

a) 13. 6.
b) Co. 20. 6.

PROP. 7. THEOR. 5.

A
B

Incommensurable magnitudes A, *and* B, *have not the same rate to one another, as a number to a number.*

Demonstration. FOr [a] if A and B, should have the same rate to one another, as a number to a number, they should be commensurable, which is absurd, being proposed incommensurable: therefore A and B are not to one another, as a number to a number: Therefore the Magnitudes, &c. Which was to be demonstrated.

a) 6. 10.

PROP. 8. THEOR. 6.

B
A

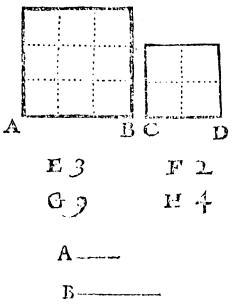
If two magnitudes A *and* B, *have not the same rate to one another, as a number to a number, those magnitudes shall be incommensurable.*

Demonstration. FOr if they were commensurable, they [a] should be to one another, as a number, to a number, which is absurd, they being

a) 15. 0.

being not to one another, as a number to a number, by Supposition: Therefore A and B, are not commensurable: Therefore, If Two magnitudes, &c. Which was to be demonstrated.

PROP. 9. THEOR. 7.



The squares A B, *and* C D, *described on the right lines* A B, *and* C D, *commensurable in length, have the same rate to one another, as a square number to a square number, and the squares which have the same rate to one another, as a square number to a square number, shall have also their sides commensurable in length. But the squares described of the right lines incommensurable in length, have not the same rate to one another, as a square number to a square number: and the squares which have not the same rate to one another, as a square number to a square number, shall not have their sides commensurable in length.*

Demonstration. FOr A B and C D, being commensurable in length, [a] A B shall be to C D, as a number to a number, which let be as the number E to the number F, and let the squares of E and F, be G and H.

a) 5. 10.

Therefore, seeing that A B, is to C D, as E to F: but [b] the square of C D, in a double rate of the side A B to the side C D, [c] and the square number G is to the square number H, also in a double rate of the side E to the side F, the square of A B shall be to the square of C D, as the square number G to the square number H, for as much as those two rates are double to the equal rates, which is proposed.

b) 20. 6.
c) 11. 8.

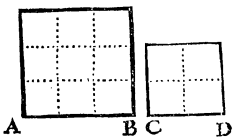
Secondly, Let the square of A B be to the square of C D, as the square number G to the square number H; I say, that A B and C D are commensurable in length: for let E and F be the sides of G and H.

For as much then, as the square of A B, is to the square of C D, as the square number G, to the square number H: but [d] the square of A B is to the square of C D, in a double rate of the side A B, to the side C D, and [e] the square number G to the square number H, also in double rate of the side E to the side F: Therefore A B shall be to C D, as E to F, their double rates being equal, therefore A B and C D, are commensurable in length.

d) 20. 6.
e) 11. 8.

Thirdly,

Thirdly, Let A and B be incommenſurable in length; I ſay, that their ſquares are not to one another, as a ſquare number to a ſquare number: For if the ſquare of A were to the ſquare of B, as a ſquare number to a ſquare number, A and B ſhould be commenſurable in length, as hath been ſhewn, which is contrary to the Suppoſition; therefore the the ſquare of A is not to the ſquare of B, as a ſquare number to a ſquare number, which is propoſed.



Laſtly, Let not the ſquares of A and B be to one another, as a ſquare number; to a ſquare number; I ſay, that A and B are incommenſurable in length: For if it be not ſo, their ſquares are to one another, as a ſquare number to a ſquare number, as hath been ſhewn, which is contrary to Suppoſition: Therefore A and B are not commenſurable in length, which is propoſed: Therefore, The ſquares, &c. Which was to be demonſtrated.

COROLLARIE.

From theſe things thus demonſtrated, it is manifeſt, that right lines commenſurable in length, are alſo commenſurable in power: But that thoſe which are commenſurable in power, are not all ſo in length, and that thoſe which are incommenſurable in length are not all ſo in power: But thoſe that are incommenſurable in power, are all commenſurable in length.

a) 9. 10. b) 6. 10. c) 3. d. *For the ſquares of lines* [a] *commenſurable in length, being to one another, as a ſquare number to a ſquare number; that is to ſay, as a number to a number ſimply;* [b] *they ſhall be commenſurable, and* [c] *therefore their ſides commenſurable in power; therefore right lines commenſurable in length, are alſo all of them commenſurable in power.*

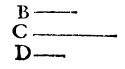
d) 3. d. e) 9. 10. f) 9. 10. *Again, ſeeing that lines whoſe ſquares are not to one another, as a ſquare number to a ſquare number, but ſimply, as a number to a number,* [d] *are commenſurable in power, (* [e] *their ſquares being commenſurable)* [f] *but not in length: It is manifeſt that the lines that are commenſurable in power, are not ſo alwayes in length, but onely the lines that are commenſurable in power, whoſe ſquares are to one another as a ſquare number to a ſquare number are alſo commenſurable in length, as appears by the ſecond part of this Theorem.*

Again, foraſmuch as thoſe lines whoſe ſquares are not to one another, as a ſquare number to a ſquare number, but onely as

a number to a number, are incommenſurable in length, but commenſurable in power, as hath been ſaid; it appears that the lines incommenſurable in length, are not all ſo in power, but onely the lines incommenſurable in length, whoſe ſquares are not to one another, as a number to a number, are incommenſurable in power, their ſquares being incommenſurable.

Laſtly, it is manifeſt that the lines incommenſurable in power, are ſo alſo in length; for if they were commenſurable in length, they ſhould not be ſo in power, as is evident by the firſt part of this Corollarie; which is abſurd, being propoſed incommenſurable in power.

PROP. 10. THEOR. 8.

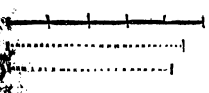


If four magnitudes A, B, C, *and* D, *be proportional, and that the firſt* A, *be commenſurable to the ſecond* B, *the third* C, *ſhall be alſo commenſurable to the fourth* D, *and if the firſt* A, *be incommenſurable to the ſecond* B, *the third* C, *ſhall be alſo incommenſurabe to the fourth* D.

Demonſtration. FOraſmuch as A and B, are commenſurable; [a] they ſhall be to one another, as a number to a number: Let then A be to B, as the number E, to the number F: But as A is to B, ſo C is to D, by Suppoſition: Therefore C ſhall be alſo to D, as the number E, to the number F: therefore [b] C and D, are commenſurable to one another. a) 5. 10. b) 6. 10.

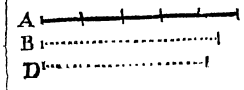
Now let A be incommenſurable to B; I ſay that, C is alſo incommenſurable to D: For ſeeing that A and B are incommenſurable: [c] A ſhall not be to B, as a number to a number, but as A is to B, ſo C is to D by Suppoſition: Therefore C ſhall not be to D, as a number to a number; [d] therefore C and D, are incommenſurable to one another: Therefore, &c. Which was to be demonſtrated. c) 7. 10. d) 8. 10.

PROP. 11. PROBL. 4.



To find two right lines B *and* D, *incommenſurable to a right line propoſed* A; *to wit, the one* D *onely in length, and the other* B *in length and* [illegible]. *That is to ſay, to the right line called* Rational.

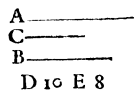
Construction LEt there be found two numbers E and C being not to one another, as a square number to a square number: then a finde the line D, to whose square let the square of A be as the number E, to the number C: I say, that D is incommensurable to A in length onely; For the square of A is to the square of D, as the number E to the number C: But E and C are not square numbers: Therefore the square of A shall not be to the square of D; as a square number to a square number; therefore, b A and D shall be incommensurable in length: and seeing that the squares of A and D are to one another, as a number to a number, they shall be commensurable: therefore c A and D are commensurable in power, and therefore they are onely incommensurable in length.



Now to find a right line incommensurable in length and power to the proposed line A: Let d B be found, a mean proportional between A and D; I say, that B is the line required: For A, B, and D, being continually proportional, e the square of A shall be to the square of B, as A is to D, but A is incommensurable in length to D; as is shewn: therefore f the square of A shall be incommensurable to the square of B; therefore A and B are incommensurable in power, by the Definition; and g by consequence also in length; wherefore we have found two right lines, &c. Which was to be done.

a) Co. 6. 10.
b) 9. 10.
c) 6. 10.
d) 13. 6.
e) Co. 20. 6.
f) 10. 10.
g) Co. 9. 10.

PROP. 12. THEOR. 9.



The magnitudes A *and* B, *commensurable to one and the same magnitude* C, *are also commensurable to one another.*

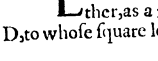
Demonstration FOr seeing that A and C are commensurable, a A shall be to C, as a number to a number, and let it be as the number D to the number E; and C and B being also commensurable, C shall be to B, as a number to a number. Let it be then, as the number F is to the number G, and let b the three numbers H, I, and K, be found continually proportional, and the least in the rate of D to E, and of F to G; and H shall be to I, as D to E, and I to K, as F to G: that is to say, as C to B.

Therefore A being to C, as H to I, and C to B, as I to K: in equal rate, A shall be to B, as H to K: therefore A and B, shall be commensurable; c being to one another, as the number H to the number K: Therefore, &c. Which was to be demonstrated.

a) 5. 10.
b) 4. 8.
c) 6. 10.

PRO

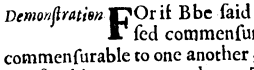
PROP. 13. THEOR. 10.



If there be two magnitudes A *and* B, *and that the one* A *be commensurable, and the other* B *incommensurable to one and the same magnitude* C, *those magnitudes shall be incommensurable to one another.*

Demonstration FOr if B be said to be commensurable to A, C being proposed commensurable to the same A; B and C should be commensurable to one another, which is absurd, being proposed incommensurable to one another: Therefore A is not commensurable to B: Therefore, If, &c, Which was to be demonstrated. 12. 10.

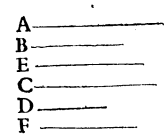
PROP. 14. THEOR. 11.



If there be two magnitudes commensurable A *and* B, *and that one of them* A, *be incommensurable to some magnitude* C, *the other* B *shall be also incommensurable to the same* C.

Demonstration FOr if B be said to be commensurable to C, a A and C, should be commensurable to one another, which is absurd, being proposed incommensurable: Therefore B and C are incommensurable to one another: Therefore, If there be, &c. Which was to be demonstrated. a) 12. 10.

PROP. 15. THEOR. 12.



If four right lines A, B, C, *and* D, *are proportional, and the first* A *be more in power then the second* B, *by the square of a line* E, *which shall be commensurable in length thereto; the third* C *shall be also more in power then the fourth* D, *by the square of a line* F, *which shall be commensurable thereto in length; and if the first* A *be more in power then the second* B, *by the square of a line* E, *incommensurable in length thereto, the third* C *shall be also more in power then the fourth* D, *by the square of a line* F *incommensurable in length thereto.*

Demonſtration FOr ſeeing that A is to B, as C to D: [a] alſo as the ſquare of A ſhall be to the ſquare of B, ſo the ſquare of C to the ſquare of D: But the ſquare of A is equal to the ſquares of B and E, by Suppoſition, and the ſquare of C, to the ſquares of D and F; therefore as the ſquares of B and E together, ſhall be to the ſquare of B, ſo the ſquares of D and F to the ſquare of D, and by dividing, as the ſquare of E to the ſquare of B, ſo the ſquare of F to the ſquare of D.

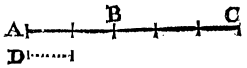
A
B
E
C
D
F

[a] 22. 6.

[b] 22. 6. Therefore [b] as E is to B, ſo is F to D, and alternately, as B is to E, ſo D is to F, in equal rate as A ſhall be to E, ſo C ſhall be to F: Therefore, [c] if A be commenſurable in length to E, alſo C ſhall be commenſurable in length to F, and if incommenſurable, incommenſurable: Therefore, if four lines, &c. Which was to be demonſtrated.

[c] 10. 10.

PROP. 16. THEOR. 13.



If two commenſurable magnitudes are compound, the whole A C *ſhall be alſo commenſurable to each of them: And if the whole* A C *be commenſurable to one of them, the magnitudes propoſed at firſt,* A B, *and* B C, *ſhall be commenſurable to one another.*

Demonſtration LEt [a] D be the common meaſure of A B and B C: Foraſmuch then as D meaſures A B and B C; it [b] ſhall alſo meaſure the whole A C: therefore ſeeing that D meaſures A C and A B, A C and A B ſhall be commenſurable: In like manner, ſeeing that D meaſures A C and B C: A C and B C ſhall be alſo commenſurable: Therefore A C ſhall be commenſurable to each of them A B and B C.

[a] 3. 10.
[b] 1. c. f.

Now let the whole A C be commenſurable to one of them A B or B C, of which it is compounded; to wit to A B: I ſay, that A B and B C are commenſurable to one another.

For let [c] D be the common meaſure of A C and A B: Therefore D meaſuring the whole A C, and the part cut off A B: it [d] ſhall alſo meaſure the reſt B C: Therefore A B and B C, are commenſurrble to one another, D meaſuring both the one and the other: Therefore if two magnitudes, &c, Which was to be demonſtrated.

[c] 3. 10.
[d] 3. c. f.

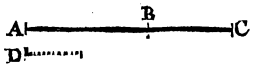
COROLLARIE.

Hence it follows, that if a magnitude compounded of two, be commenſurable to one of them, it ſhall be alſo commenſurable to the other, as if A C *be commenſurable to* A B, *it ſhall be alſo commenſurable to* B C *the remainder, for as hath been ſhewn in the latter part of this Theorem:* D *common meaſure of the*

the whole A C, *and of* A B *the part cut off,* [c] *ſhall alſo meaſure the remainder* B C, *therefore* A C *and* B C *are commenſurable.*

[c] 3. c. f.

PROP. 17. THEOR. 14.



If two magnitudes incommenſurable A B *and* B C *are compounded, the whole* A C *ſhall be alſo incommenſurable to each of them* A B *and* B C; *And if the whole* A C *be incommenſurable to one of them, the magnitudes propoſed at firſt,* A B *and* B C, *ſhall be incommenſurable.*

Demonſtration FOr if it be denied, it ſhould be commenſurable at leaſt to one of them: Suppoſe it then commenſurable to A B, and [a] let D be their common meaſure: Therefore ſeeing that D meaſures the whole A C, and the part cut off A B; [b] it ſhall alſo meaſure the reſt B C: Therefore A B and B C, ſhall be commenſurable, having D for their common meaſure, which is abſurd, for they are propoſed incommenſurable.

[a] 3. 10.
[b] 3. c. f.

Now ſuppoſe the whole A C to be incommenſurable to one of them, to wit, to A B; I ſay, that A B and B C are incommenſurable.

For if you ſay it is not ſo: [c] the whole A C ſhall be commenſurable to each of them A B, and B C, which is abſurd, and contrary to the Suppoſition: Therefore A B, and B C, are incommenſurable, even ſo it might be demonſtrated, that A C and A B, are incommenſurable: Therefore, If, &c. Which was to be demonſtrated.

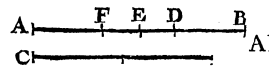
[c] 16. 10.

COROLLARIE.

It follows from theſe Demonſtrations, that if a magnitude compounded of two, be incommenſurable to one of them, it is ſo alſo to the other, as if A C *be incommenſurable to* A B, *it ſhall be ſo alſo to* B C: *For if* A C *were commenſurable to* B C: [d] A C *ſhould be alſo commenſurable to the reſt* A B, *which is abſurd, for that they are propoſed incommenſurable: Therefore* A C *is not commenſurable to* B C, *which is propoſed.*

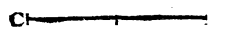
[d] C. 16. 10.

PROP. 18. THEOR. 15.



If there be two unequal right lines AB, *and* C, *and that to the greateſt* AB, *be applyed a Paralellogram equal*

equal to the fourth part of the ſquare of the leaſt C, *wanting of* A B, *by a ſquare figure, & that it divide the ſame greater, in parts commenſurable in length, the greateſt* A B *ſhall be more in power then the leaſt* C, *by the ſquare of a line* F D, *which ſhall be commenſurable in length thereto; And if the greateſt* A B *be more in power then the leaſt* C, *by the ſquare of a line which ſhall be commenſurable in length thereto, and that there be applyed a Paralellogram upon the greateſt of them* A B, *equal to the fourth part of the ſquare of the leaſt* C, *wanting of* A B *by a ſquare figure; it ſhall divide the ſame in parts commenſurable in length.*



Demonſtration FOraſmuch as A B is divided equally by E, and unequally by D, the Rectangle conteined under A D and B D, with the ſquare of E D, ſhall be equal to the ſquare of E B: And therefore four times the Rectangle under A D, and D B, and four times the ſquare of E D ſhall be equal to four times the ſquare of E B: But the ſquare of C is equal to four times the Rectangle under A D and D B, (the Rectangle under A D and D B, being propoſed equal to a quarter of the ſquare of C,) and the ſquare of F D is equal to four times the ſquare of E D, the halfe of F D, and the ſquare of A B equal to four times the ſquare of E B, the half of A B: Therefore the ſquares of C and F D, are equal to the ſquare of A B: Wherefore A B is more in power then C, by the ſquare of F D: I ſay alſo that F D is commenſurable in length to A B: For A D and D B being propoſed commenſurable in length,
a) 16. 10. [a] the whole A B ſhall be alſo commenſurable in length to his part D B, but the compounded of D B and A F, is alſo commenſurable in length to D B, (the compounded of A F and D B equals, being double to B D,) therefore A B, and the compounded of A F and D B, being commenſurable in length
b) 12. 10. each to D B, [b] A B and the compounded of A F and D B, ſhall be commenſurable in length to one another.

Therefore, A B compounded of A F and D B, as of one, and of F D, being commenſurable in length, to the compounded of A F and D B his
c) C. 16. 10. part, the ſame A B [c] ſhall be commenſurable in length to the remainder F D, therefore A B is more in power then C, by the ſquare of F D commenſurable in length to A B.

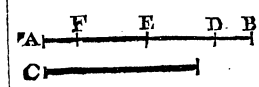
Now let A B be more in power then C by the ſqnare of a line commenſurable in length thereto, and let there be applyed thereto a Paralellogram, as is ſaid before, making the ſegments A D and D B: I ſay, that A D and D B are commenſurable in length to one another: For the ſame conſtruction ſubſiſting, it may be demonſtrated in like manner, that A B is more in power then C, by the ſquare of F D: Wherefore A B being propoſed to be more in power then C, by the ſquare of a Right-line commenſurable in length thereto, A B ſhall be commenſurable in length to F D.

Therefore the whole A B, compounded of F D, and A F and D B as of one, being commenſurable in length to F D, the ſame A B [d] ſhall be alſo commenſurable in length to the remainder, compounded of A F and D B; but the compounded of A F and D B, is alſo commenſurable in length to D B, (being the double thereof,) [e] therefore A B and D B being commenſurable in length to the compounded of A F and D B, they ſhall be commenſurable to one another: Therefore the whole A B compounded of A D and D B, being commenſurable in length to D B: A D and D B, ſhall be commenſurable in length to one another: Therefore, If there be, &c. Which was to be demonſtrated.

d) C. 16. 10.

e) 12. 10.

PROP. 19. THEOR. 16.

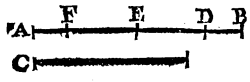


If there be two unequal right lines A B, *and* C, *and that to the greateſt* A B *there be applyed a Rectangle, equal to the quarter part of the ſquare of the leaſt* C, *wanting by a ſquare figure, and that it divide the ſame in parts incommenſurable in length* A D, *and* D B, *the greateſt* A B, *ſhall be more in power then the leaſt* C, *by the ſquare of a line, which ſhall be incommenſurable in length thereto: And if the greateſt* A B, *be more in power then the leaſt* C, *by the ſquare of a line incommenſurable in length thereto: And that there be applyed to the greateſt a Paralellogram equal to the quarter part of the ſquare of the leaſt, wanting a ſquare figure: It ſhall divide the ſame into parts incommenſurable in length.*

Demonſtration FOr having made the Conſtruction as in the former, we ſhall ſhew as there is done, that A B is more in power then C by the ſquare of F D. And I ſay that F D is incommenſurable in length to A B, for A D and D B, being propoſed incommenſurable in length, [a] the
a) 17. 10. whole A B ſhall be incommenſurable in length to his part D B: But D B is commenſurable in length to the compounded of A F and D B, the one being double to the other,) therefore ſeeing that of two lines commenſurable D B, and the compounded of A F and D B, the one, to wit, D B, is commenſurable in length to A B, [b] the remaining, compounded of A F and
b) 14. 10. D B, ſhall be alſo incommenſurable in length to A B, therefore A B compounded of A F and D B, as of one, and of F D, being incommenſurable in length to the compounded of A F and D B, the ſame A B [c] ſhall
c) C. 17. 10. be alſo incommenſurable in length to the remainder F D: Therefore A B is more

more in power then C, by the ſquare of F D, which is incommenſurable in length thereto.

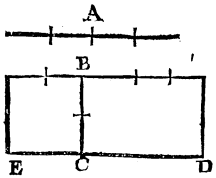
Now let A B be more in power then C, by the ſquare of a line which ſhall be incommenſurable in length thereto, and let there be applyed to A B a Paralellogram equal to the quarter part of the ſquare of C, wanting a ſquare figure, and making in A B, the parts A D and D B; I ſay, that A D and D B are incommenſurable in length.



For obſerving the ſame conſtruction, we ſhall ſhew, as before, that A B is more in power then C, by the ſquare of F D, wherefore ſeeing that A B is propoſed to be more in power then C, by the ſquare of a line incommenſurable in length thereto: A B ſhall be incommenſurable in length to F D; therefore the whole A B, compounded of F D, and of A F and D B, as of one, being incommenſurable in length to F D: [d] the ſame A B ſhall be incommenſurable in length to the remainder, compounded of A F and D B: but the compounded of A F and D B, is commenſurable in length to D B (the one being double to the other,) therefore ſeeing that the two lines commenſurable; to wit, the compounded of A F and D B, and D B: the ſame compounded of A F and D B is incommenſurable in length to A B, [e] the remainder D B ſhall be incommenſurable in length to the ſame A B: Therefore the whole A B, compounded of A D and D B, being commenſurable in length to D B: [f] the ſame A D and D B, ſhall be incommenſurable in length to one another: Therefore, If there be, &c. Which was to be demonſtrated.

d) C. 17. 10.
e) 14. 10.
f) C. 17. 10.

PROP. 20. THEOR. 17.



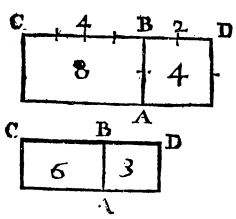
The Rectangle B D, *conteined under two Rational Lines* B C *and* C D, *commenſurable in length, according to ſome of the kinds aforeſaid, is Rational.*

Demonſtration FOr of the one of them, as of B C: Let there be deſcribed the ſquare B E: Foraſmuch as B C is Rational, commenſurable to the Rational propoſed A, either in length and power, or in power onely, [a] his ſquare B E ſhall be commenſurable to the ſquare of A: But the ſquare of A is rational; [b] therefore B E commenſurable thereto, ſhall be Rational: but B C, that is to ſay, E C and C D, being commenſurable in length, (for they are propoſed Rational, commenſurable in length to one another,) and as [c] E C is to C D: ſo E B to B D: [d] E B and B D ſhall be alſo commenſurable in length: wherefore B D commenſurable to the ſpace Rational B E, is alſo Rational: Therefore, &c. Which was to be demonſtrated.

a) 3. d.
b) 9. d.
c) 1. 6.
d) 10. 10.

PROP.

PROP. 21. THEOR. 18.



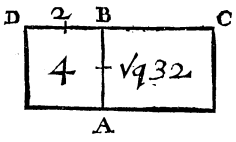
If a Rational Rectangle A C, *be applyed to a Rational line* A B, *it makes the breadth* B C *Rational, commenſurable in length to the line* A B, *to which it is applyed.*

THat is to ſay, if a rational Rectangle have one ſide Rational, it will alſo have the other ſide rational, and thoſe ſides ſhall be commenſurable in length to one another.

Demonſtration FOr on A B let there be deſcribed the ſquare A D, [a] which ſhall be rational: But as the rectangle C A is to the ſquare A D, ſo [b] C B is to B D, or to B A his equal. Now the rectangle A C and the ſquare A D, are rational: and therefore commenſurable: Therefore, [c] C B and B D, or B A, ſhall be commenſurable in length: But A B is rational; therefore B C ſhall be alſo rational: Therefore, If, &c. Which was to be demonſtrated.

a) 8. d.
b) 1. 6.
c) 10. 10.

PROP. 22. THEOR. 19.

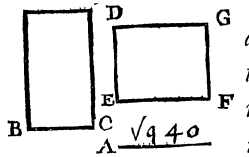


The Rectangle A C, *conteined under two rational right lines* A B *and* B C, *commenſurable in power onely, is irrational, and the right line equal in power to the ſaid irrational, may be called Medial.*

Demonſtration FOr on A B rational, let be deſcribed the ſquare A D, therefore [a] A D ſhall be rational; and foraſmuch as A B, or D B, his equal, is incommenſurable in length to B C; and that as D B is to B C; ſo [b] D A is to A C: [c] A D ſhall be alſo incommenſurable to A C, but the ſquare A D is rational: therefore [d] the Rectangle A C ſhall be irrational: Therefore the right line which is equal in power to the Rectangle A C, that is to ſay, whoſe ſquare is equal to A C is irrational: Let that line be called Medial, or mean proportional, between A B and B C: Foraſmuch as his ſquare [e] is equal to the Rectangle A C conteined under A B and B C: Therefore the Rectangle, &c. Which was to be demonſtrated.

a) 8. d.
b) 1. 6.
c) 10. 10.
d) 10. d.
e) 17. 6.

PROP. 23. THEOR. 20.



The ſquare of a Medial Line A, *applyed to a rational Line* B C, *makes the breadth rational, and incommenſurable in length to the Line to which it is applyed.*

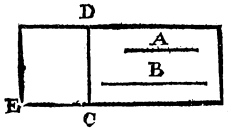
Demonſtration FOraſmuch as A is a Medial, it [a] is equal in power to a Rectangle conteined under two rational lines commenſurable in power onely: the which let be E G, under E F and F G rationals: But it is equal alſo in power to B D: therefore B D and E G are equal: and being [b] equi-angled, they will have their ſides reciprocal about the equal angles, to wit, as B C to E F, ſo F G to C D: Therefore as the ſquare of B C to the ſquare of E F, ſo the ſquare of F G to the ſquare of C G, but [c] the ſquare of B C is commenſurable to the ſquare of E F, B C and E F being rationals: therefore [d] the ſquare of F G, is alſo commenſurable to the ſquare of C D: therefore F G and C D are at leaſt commenſurable in power: But F G is propoſed rational; therefore C D ſhall be alſo rational.

I ſay alſo, that it is incommenſurable in length to B C: For aſmuch as E F and F G, are rational, commenſurable in power onely, and as E F to F G; ſo the ſquare of E F to the Rectangle E G; [e] the ſquare of E F ſhall be incommenſurable to the rectangle E G: and therefore alſo to the Rectangle B D.

But the ſquare of E F, is commenſurable to the ſquare of C D, (E F and C D, being ſhewn rationals,) therefore [f] the ſquare of C D, and the Rectangle B D, are incommenſurable: but the ſquare of C D is to the Rectangle B D, as C D to B C; therefore C D and B C are incommenſurable in length: therefore [g] C D is rational, commenſurable in power onely to the rational B C; that is to ſay, incommenſurable in length: Which was to be demonſtrated.

a) 22. 10.
b) 14. 6.
c) 22. 6.
d) 10. 10.
e) 10. 10.
f) 13. 10.
g) 10. 10.

PROP. 24. THEOR. 21.



The Right Line B *commenſurable to a Medial Line* A, *is alſo Medial.*

Demonſtration FOr let C D a rational, be propoſed, to which let there [a] be applyed the Rectangle D E equal to the ſquare of the Medial A, making the breadth E C: Therefore [b] E C is rational, incommenſurable in length to C D. Again, to C D let there be applyed the Rectangle D F, equal to the ſquare of B, making the breadth C F: Then foraſmuch as A and B are propoſed commenſurable; their ſquares, that is to ſay, the Rectangles D E and D F, equal

a) 45. 1.
b) 13. 10.

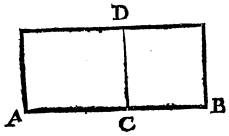
equal to them, ſhall be commenſurable: But [c] as E D to D F, ſo the line E C is to C F: Therefore [d] E C and C F are commenſurable in length: But E C is demonſtrated rational, and incommenſurable in length to C D; [e] therefore C F is alſo rational, and incommenſurable in length to C D: Therefore D C, and C F, are rational, commenſurable in power onely; and [f] therefore the right line B, equal in power to the Rectangle D F, conteined under the ſame C D and C F, is Medial: Which was to be demonſtrated.

c) 1. 6.
d) 10. 10.
e) 14. 10.
f) 22. 10.

COROLLARIE.

From this Demonſtration it is manifeſt that a ſpace Medial, commenſurable to a ſpace Medial, is alſo Medial: For after it hath been demonſtrated that D F *is commenſurable to the medial Rectangle* D E, *from thence hath been ſhewn that* D F *is alſo a medial, conteined under the rationals* C D *and* C F, *commenſurable in power onely.*

PROP. 25. THEOR. 22.

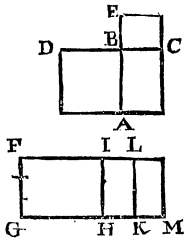


The Rectangle A D, *conteined under two medial right lines* A C *and* C D, *commenſurable in length, is medial.*

Demonſtration FOr on the medial C D; let there be deſcribed the ſquare B D, which ſhall be medial: And ſeeing that as A C is to C B, ſo A D is to D B: But A C and C B, that is to ſay, A C and C D, are commenſurable in length; A D and D B, ſhall be alſo commenſurable: Therefore the ſpace A D, commenſurable to the medial ſpace D B, [a] is medial: Therefore the Rectangle, &c. Which was to be demonſtrated.

1. 6.
a) C. 24 10.

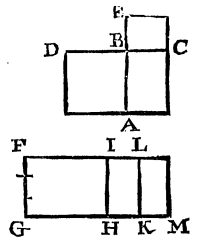
PROP. 26. THEOR. 23.



The Rectangle A C, *conteined under two medial right lines* A B *and* B C, *commenſurable in power onely, is Rational, or medial.*

Demonſtration FOr of A B and B C Medials, let there be deſcribed the ſquares A D and C E, which ſhall be Medials; and let the rational F G, be expoſed, [a] to which let there be applyed the Rectangles F H, H L, L M, equal to A D, A C, and C E: F M ſhall be a Rectangle

a) 45. 1.



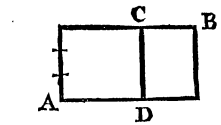
gle alone; and A D and C E, being Medials, their equal Rectangles F H, and L M, shall be also Medials, which applyed to [b] F G & K L rationals, (for K L is equal to F G,) & making the breadths G H & K M, [c] the lines G H & K M, shall be rational, incommensurable in length to F G, or K L; and forasmuch as A B, and B C, are commensurable in power to one another, their squares A D and C E, and therefore their equal Rectangles F H, and L M, shall be commensurable: But as [d] F H is to L M, so G H is to K M; therefore [e] G H and K M, shall be commensurable in length.

b) 34. 1.
c) 23. 10.
d) 1. 6.
e) 10. 10.

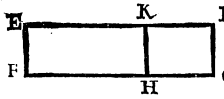
Therefore, seeing that it is shewn that G H and K M, are rationals, and commensurable in length to one another, and commensurable in power onely to the rational proposed; being shewn incommensurable in length thereto; [f] the Rectangle under G H and K M, shall be rational. and [g] seeing that as D B is to B C, so A D is to A C, and as A B to B E, so A C to C E: (D B and B C, being equal to A B and B E, each to his correspondent,) as A D shall be to A C, so A C to C E: therefore A D, A C, and C E, shall be proportional: Wherefore F H, H L, and L M, their equal Rectangles shall be continually proportional: But [h] G H, H K, and K M, are in the same rate, as the Rectangles, F H, H L, and L M, they shall be then proportional, [i] and the Rectangle conteined under G H and K M, is equal to the square of H K: but the Rectangle under G H and K M, is shewn rational, therefore the square of H K, shall be also rational, therefore H K shall be rational: and [k] therefore commensurable to F G, expoſed rational, or to his equal H I, to wit, in length and power, or in power alone. And if H K be commensurable in length to H I; that is to say, to his equal F G, [l] the Rectangle I K, or his equal, shall be rational, and if in power onely, [m] the Rectangle I K shall be a Medial, or his equal A C: therefore the Rectangle A C conteined under A B and A C Medials, commensurable in power onely, is rational or medial: Therefore, &c. Which was to be demonstrated.

f) 20. 10.
g) 1. 6.
h) 1. 6.
i) 17. 6.
k) 6. d.
l) 20. 10.
m) 22. 10.

PROP. 27. THEOR. 24.



A ſpace or figure medial A B, *exceedeth not a figure medial* A C, *by a Rational figure* D B.



Demonſtration FOr let D B be a rational (if poſſible,) and let the rational E F be expoſed, on which let there be applyed the Rectangle E G, equal to the Medial A B, being equal to the Medials A B, A C, and H I, rational, and the Rectangle E H equal to the Medial A C, and ſo the reſt H I, ſhall be equal to the rational D B, and E G, and E H, ſhall be Medials, being equal

45. 1.

equal to the rational figure D B, and E G and E H being applyed on the rational E F; they [a] ſhall have the other ſides F G, and F H rational, incommenſurable in length to E F.

a) 23. 10.

Again, ſeeing that H I rational, is applyed to the rational E F: [b] that is to ſay, to H K, his equal, the line [c] H G ſhall be rational, commenſurable in length to the ſame E F, or H K; but F H is rational, incommenſurable in length to E F: Therefore F H is incommenſurable in length to H G, [c] but as F H is to H G, ſo the ſquare of F H is to the Rectangle conteined under F H and H G; therefore the ſquare of F H is incommenſurable to the Rectangle conteined under F H and H G: But the ſquares of F H and H G, are commenſurable to one another: (For the one and the other, are deſcribed on the rationals F H and H G;) and [f] therefore the two ſquares F H, and H G, together, are commenſurable to the ſquare of F H, and twice the Rectangle under F H, and H G, is commenſurable to the Rectangle F H, and H G, the one being double to the other: Therefore the two ſquares of F H, and H G, together, ſhall be incommenſurable to twice the Rectangle conteined under F H, and H G. Therefore the compounded of the ſquares F G, and H G, and of the Rectangle twice under F H, and H G; that is to ſay, the ſquare of F G, [g] is equal to thoſe ſquares and Rectangle, [h] is incommenſurable to the compounded of the ſquares F H, and H G, which is rational, being commenſurable to the rational ſquare of F H.

b) 34. 1.
c) 21. 10.
e) 14. 10.
f) 16. 10.
g) 4. 2.
h) 17. 10.

Therefore ſeeing that the ſquare of F G is incommenſurable to the compounded of the two ſquares F H, and H G, which is rational, [i] it ſhall be alſo irrational, and [k] F G irrational, which is abſurd, for it hath been ſhewn to be rational, commenſurable in length to E F: Therefore H I, or his equal D B, is not rational: Therefore, a figure, &c. Which was to be demonſtrated.

i) 10 d.
k) 11. d.

PROP. 28. PROBL. 4.

A
B
C
D

To find two medial lines A *and* B, *commenſurable in power onely, conteining a rational Rectangle.*

Conſtruction LEt the two Rationals A, and B, be propoſed, commenſurable in power onely; and [a] let there be found C, a mean proportional between A and B, and as A is to B, ſo let [b] C be to D: I ſay, that C and D are Medials; commenſurable in power onely, which do contein a rational Rectangle.

a) 13. 6.
b) 12. 6.

Demonſtration FOr ſeeing that A and B are propoſed rational, commenſurable in power onely; [c] the Rectangle conteined under A and B, ſhall be irrational, called medial: But [d] C is equal in power to that Rectangle, therefore it ſhall be medial: and ſeeing that as A is to B, ſo C is to D, and A & B, are commenſurable in power onely, C and D [e] ſhall be alſo commenſurable in power onely; and therefore D commenſurable to the medial C, ſhall be alſo medial: Therefore we have found C and D medials, commenſurable in power onely: I ſay, they do contein a rational Rectangle: For ſeeing that A is to B, as C to D; alternately, A ſhall be to C, as B to D: But as A is to C, ſo is C to B: therefore C ſhall be alſo to B, as B to D: Therefore B a mean proportional between C and D, is equal in power to the

c) 22. 10.
d) 17. 6.
e) 10. 10.

24. 10.

17. 6. the Rectangle conteined under C and D: But the square of B a rational line, is rational: Therefore the Rectangle under C and D, shall be also rational: Therefore we have found C and D medials, commensurable in power onely, which conteins a rational Rectangle: Which was to be done.

PROP. 29. PROBL. 5.

A
D
B
C
E

To find two medial lines commensurable in power onely, which doe contein a medial Rectangle.

Construction LEt there be taken the three rational lines A, B, and C, commensurable in power onely; and a between A and B, let there be found a mean proportional D; then let it be made as B is to C, so D to E: I say that D and E are medials, commensurable in power onely, and do contein a medial Rectangle.

a) 13. 6. 12. 6.

Demonstration FOr seeing that A and B are rationals, commensurable in power onely; the Rectangle conteined under A and B, and therefore the square of D equal b thereto, shall be irrational, called medial, and the line D equal in power to it medial: And seeing that B is to C as D to E, and B and C are rational, commensurable in power onely, D and E shall be also commensurable in power onely: Therefore seeing that D is a medial, c D and E commensurable in power onely to D, shall be also medial: I say that D and E do contein a medial Rectangle.

b) 17. 6. 22. 10.

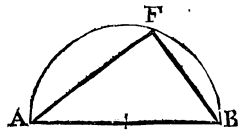
c) 24. 10.

Forasmuch as B is to C, as D to E, alternately, as B shall be to D: so C to E; But as B to D, so D to A: Therefore as D shall be to A, so C shall be to E: Therefore d the Rectangle conteined under D and E, is equal to the Rectangle conteined under A and C: But the Rectangle conteined under A and C rationals commensurable in power onely, is irrational, and medial: therefore e the Rectangle under D and E, is also medial, therefore D and E are medials required: Which was to be done.

d) 16. 6.

e) 22. 10.

PROP. 30. PROBL. 6.



C .... 4 E ..... 5 D

To find two rational Lines commensurable in power onely, in such sort as the greatest may be more in power then the least, by the square of a right line commensurable in length thereto.

Construction LEt the rational A B be exposed, and let there be found two square numbers C D and C E whose excesse E D may not be a square; seeing a as the number C D, is to the number D E: so the square of

a) Co. 6. 10.

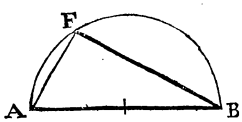
of A B, to the square of some line A F: Let A F be fitted in the Semicircle described on A B, and let F B be joyned; forasmuch as the angle F b is a right angle in the Semicircle, c the square of A B shall be equal to the two squares of A F and F B; that is to say, that A B shall be more in power then A F, by the square of F B: and the square of A B being to the square of A F as C D to D E, a number to a number; d the said squares of A B and A F shall be commensurable, and seeing that the square of A B is rational, being described on the rational A B, the square of A F shall be also rational: and therefore A F rational; therefore A B and A F are rational, and commensurable at least in power; and the squares of A B and A F, being not to one another, as a square number to a square number, the same A B and A F, shall be incommensurable in length: But they are shewn to be commensurable in power: Therefore A B and A F are rational, commensurable in power onely.

b) 31. 3. c) 47. 1. d) 6. 10.

Now seeing that as C D is to D E, so the square of A B, is to the square of A F, by conversion of reason, as the square number C D shall be to the square number C E: so the square of A B to the square of B F: (For as C D exceedeth E D, by C E a square number, so the square A B exceeds the square of A F, by the square of F B:) Therefore the right lines A B and B F shall be commensurable in length: therefore we have found A B and A F rationals, commensurable in power onely, such as A B the greatest is more in power then A F by the square of B F, commensurable in length thereto: Which was to be done.

9. 10.

PROP. 31. PROBL. 7.



C 144 E 16 D C 160 D
C ..... 6 ... E ... 3 D

To find two rational lines commensurable in power onely, in such sort as the greatest be more in power then the least, by the square of a line incommensurable in length thereto.

Construction LEt the rational A B be exposed, and find the two square numbers C E and E D, such as that C D compounded of them be not a square number: Or let C D a square number be divided into two numbers not square numbers, C E and E D, so the whole shall not be to any of them, as a square number to a square number; then a let there be found A F, to whose square let the square of A B be as C D is to D E, and let A F be fitted in the Semicircle described on A B, and let F B be joyned: Therefore b A B shall be more in power then A F, by the square of F B and A B, A F shall be rational, commensurable in power onely, as appears by the fore-going Demonstration.

a) Co. 6. 10.

b) 31. 3. 47. 1.

But forasmuch as by conversion of reason, (as before) as C D is to C E: so the square of A B is to the square of B F: But C D and C E are not to one another, as a square number to a square number: in like manner, the squares of A B and B F, shall not be to one another, as a square number to a square number: Therefore c the lines A B and F B, are incommensurable in length:

c) 9. 10.

length: Therefore we have found two rationual lines AB and A F, commenſurable in power onely, ſuch as A B the greateſt, is more in power then A F the leaſt, by the ſquare of F B incommenſurable in length thereto: Which was to be done

PROP. 32. PROBL. 8.

A
C
B
D

To find two medials commenſurable in power onely, which contein a rational rectangle, in ſuch manner, as the greateſt may be more in power then the leaſt, by the ſquare of a line commenſurable in length thereto.

Conſtruction. a) 30. 10. LEt [a] there be found A and B rationals, commenſurable in power onely, ſuch as A the greateſt may be more in power then B the leaſt, by the ſquare of a line commenſurable in length thereto; b) 13. 6. [b] and let C be a mean proportional between A and B, and as A is to B, ſo let C be to D. 12. 6.

Demonſtration. FOraſmuch as A and B are rationals, commenſurable in power onely: c) 12. 10. the [c] Rectangle conteined under A and B, is irrational, and the line C equal in power to the ſame medial; and A being to B as C to D, d) 10. 10. and A and B commenſurable in power onely: [d] C and D ſhall c) 24. 10. be alſo commenſurable in power onely; therefore [c] D commenſurable to the medial C, is a medial; therefore we have found two medials, C and D commenſurable in power onely: I ſay, that they comprehend a rational rectangle.

For ſeeing that A is to B, as C to D, alſo alternately A ſhall be to C, as B to D: But as A is to C, ſo C is to B: C ſhall be in like manner to B, as B to D: Therefore the rectangle under C and D, ſhall be equal to the ſquare of B: wherefore the ſquare of B rational being rational; the rectangle under C and D, 17. 6. his equal ſhall be alſo rational: therefore C and D do contein a rational rectangle.

But foraſmuch as A is to B, as C to D; and A is more in power then B, by the ſquare of a line commenſurable in length to the ſame; by the Conſtruction, f) 15. 10. [f] C ſhall be alſo more in power then D, by the ſquare of a line commenſurable thereto in length; therefore we have found two medials C and D, commenſurable in power onely, which do contein a rational rectangle, and the greateſt C is more in power then the leaſt D, by the ſquare of a line commenſurable in length thereto, which was to be done.

g) 31. 10. But if A and B [g] are found rational, commenſurable in power onely, in ſuch ſort as A may be more in power then B, by the ſquare of a line incommenſurable in length thereto, and that the reſt be done as before, it will be ſhewn, in like manner, that C and D are medials, commenſurable in power onely, which conteineth a rational rectangle: in ſuch ſort as C is more in power then B, by the ſquare of a line, which is incommenſurable in length thereto.

PROP.

PROP. 33. PROBL. 9.

To find two medial lines commensurable in power onely, the which doe contein a medial Rectangle: in such sort, as that the greatest may be more in power then the least, by the square of a line which shall be commensurable in length thereto.

Construction LEt there be found the three rationals A, B, and C, commensurable in power onely, in such sort, as A may be more in power then C, by the square of a line which shall be commensurable in length to it: Having found two rationals A and C, commensurable in power onely; in such sort as the greatest may be more in power then the least, by the square of a line which shall be commensurable thereto in length, let B be found commensurable in power onely, to the one and the other: then let D be taken a mean proportional between A and B, and as D is to B, so let C be to E.

Demonstration FOrasmuch as A is to C, as the Rectangle under A and B, is to the Rectangle under B and C: but the square of D is equal to the Rectangle under A and B: and the Rectangle under D and E, is equal to the Rectangle under B and C: Seeing that D and B, C and E are proportional; in like manner, as A is to C, so the square of D, shall be to the Rectangle under D and E: But as the square of D is to the Rectangle of D and E, so D is to E: Therefore as A is to C, so D is to E: but A and C are commensurable in power onely: therefore D and E shall be so also: But forasmuch as D is equal in power to the Rectangle of A and B rationals, commensurable in power onely; is rational and medial, E commensurable to D, shall be also medial: We have therefore found two medials D and E, commensurable in power onely. 22. 10. 24. 10.

And forasmuch as the Rectangle under D and E is shewn to be equal to the Rectangle of B and C medial; (seeing that B and C are rational, commensurable in power onely,) the Rectangle under D and E shall be also medial. 22. 10.

Lastly, seeing that we have shewn that as A is to C, so D is to E: But A is more in power then C, by the square of a line commensurable in length to it, by the Construction, D shall also be more in power then E by the square of a line which shall be commensurable in length thereto. 15. 10.

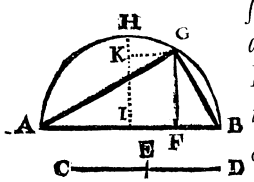
We have therefore found two medials D and E, commensurable in power onely; which do contein a medial Rectangle; and the greatest D is more in power then the least E, by the square of a line which is commensurable in length thereto: Which ought to be done.

But if the Rationals before mentioned had been found, in such sort as A might be more in power then C, by the square of a line incommensurable in length thereto: the medial D would be more in power then E, by the square of a line incommensurable in length thereto; and having finished

the Conſtruction, as before is ſaid, it might be demonſtrated in like manner, that D and E are found medials, commenſurable in power onely, conteining a medial Rectangle, in ſuch ſort as the greateſt may be more in power then the leaſt, by the ſquare of a line incommenſurable in length thereto, &c.

PROP. 34. PROBL. 10.

To find two right lines incommenſurable in power, which do make ſome compound of their rational, ſquares: But the Rectangle conteined under them a medial.



a) 31. 10. *Conſtruction* LEt [a] there be found two Rationals A B and C D, commenſurable in power onely, ſuch as the greateſt A B may be more in power then the leaſt C D; by the ſquare of a line which ſhall be incommenſurable in length thereto; and having divided C D in two equal parts in E: let there be applyed to A B a Rectangle equal to the ſquare of C E, wanting by a ſquare figure; and let it be that which is conteined under A F and F B, and having deſcribed the Semicircle A G B: let there be drawn F G at right angles, and let A G and G B, be joyned together.

Demonſtration FOraſmuch as A and B is more in power then C D by the ſquare of a line incommenſurable in length thereto; and that there is applyed to A B a Rectangle conteined under A F and F B, equal to the quarter part of the ſquare of C D wanting by a ſquare figure;

b) 19. 10. [b] A F ſhall be incommenſurable in length to F B.

But as A F is to F B, ſo the Rectangle under A B and A F, to the Rectangle under A B and F B: but the Rectangle under A B and A F, is equal to the ſquare of A G, and the Rectangle under A B and F B, equal to the ſquare of B G: Seeing that the angle A G B, is right [c] being in the Semicircle: Therefore as A F to F B; ſo the ſquare of A G to the ſquare of G B: and

c) 31. 3.

d) 10. 10. e) 4. d. therefore ſeeing that A F and F B, are incommenſurable in length, [d] the ſquare of A G and G B, ſhall be incommenſurable: Therefore [e] the lines A G and G B, are incommenſurable in power. And foraſmuch as the ſquare of the rational A B is rational; and [f] equal to the ſquares of A G and G B, the compounded of the ſquares of A G and G B, is alſo rational.

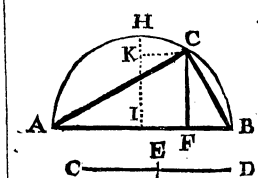
f) 47. 1. g) 17. 6. Again, Foraſmuch as the Rectangle under A F and F B, which is made equal to the ſquare of C E, [g] is equal to the ſquare of F G: (Seeing that the angle A G B is right, and F G perpendicular to A B,) the ſquares of F G and C E, ſhall be equal; and therefore the lines F G and C E equal: wherefore C D the double of C E, ſhall be alſo double to F G: therefore the Rectangle under A B and C D, ſhall be double, the Rectangle under A B and F G: but the Rectangle conteined under the Rationals A B and C D, commenſurable in power onely, is medial: therefore the Rectangle under

under A B and F G commenſurable thereto, being its halfe, is [h] alſo medial: But the Rectangle conteined under A G and G B is equal to the Rectangle conteined under A B and F G: Therefore the Rectangle conteined under A G and G B, is alſo medial, but the compounded of the ſquares of A G and G B, is ſhewn rational: Therefore we have found two right lines A G and G B, incommenſurable in power, which make the compound of their ſquares Rational, and the Rectangle conteined under them medial: Which was to be done.

h) C. 24. 10.

PROP. 35. PROBL. 11.

To find two right lines incommenſurable in power, which do make the Compound of their ſquares medial, and the Rectangle conteined under them Rational.



Conſtruction LEt [a] there be found two medials A B and C D, commenſurable in power onely; conteining a Rectangle rational; in ſuch ſort as A B the greateſt, may be more in power then C D the leſſer, by the ſquare of a line incommenſurable thereto in length; and do for the reſt as in the 34 Propoſition is ſhewn, ſo as in the ſame 34 Prop. that A C and G B, are incommenſurable in power, and the ſquare of the medial A B, (equal [b] to the ſquares of A G and G B,) being medial, the compounded of them, the ſquares of A G and G B, ſhall be alſo medial.

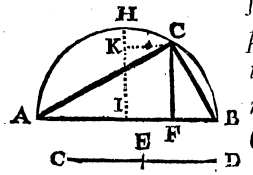
a) 32. 10.

b) 47. 1.

Again, as in the 34 Propoſition, the Rectangle under A B and C D is ſhewn double, the Rectangle under A B and F G, and therefore commenſurable: Wherefore the Rectangle under A B and C D, being Rational, by the Conſtruction, the Rectangle under A B and F G, commenſurable thereto, ſhall be alſo rational; but the Rectangle under A G and G B, is equal to the Rectangle under A B and F G: Therefore the Rectangle under A G and G B, is alſo Rational.

Now the Compound of the ſquares of A G and G B, is ſhewn to be medial; therefore A G and G B are found incommenſurable in power, making the the Compound of their ſquares medial: But the Rectangle under them Rational: Which was to be done.

PROP. 36. PROBL. 12.



To find two right lines incommensurable in power, making the Compound of their squares medial, and the Rectangle contained under them medial, and incommensurable to the Compound of their squares.

Construction LEt [a] there be found A B and C D medials, commensurable in power onely; containing a rectangle medial, in such sort as A B the greatest may be more in power then C D the least, by the square of a line incommensurable thereto in length: and having done the rest, as in the 34 Proposition: I say, that A G and G B are the lines required.

a) 33. 10.

Demonstration FOr as in the 34 Proposition is shewn that A G, and G B, are incommensurable in power; and forasmuch as the square of A B medial is medial, (being [b] equal to the squares of A G and G B,) the Compound of the squares A G and G B, shall be also medial: Again, the Rectangle under A B and C D, is double to [c] the Rectangle under A B and F G, (as [c] is shewn,) and therefore commensurable: therefore by the Construction, the Rectangle under A B and C D, being medial, the Rectangle under A B and F G his equal; (that is to say, the Rectangle under A G and G B, his equal,) is also medial; but A B being proposed incommensurable in length to C D, and C E commensurable in length to the same C D, C D being double to C E: [d] A B and C E shall be incommensurable in length.

b) 47. 1.

c) 34. 10.

d) 13. 10.

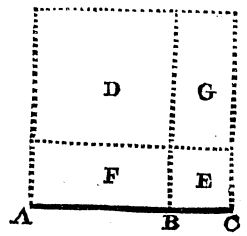
But as A B to C E, so the square of A B to the Rectangle under A B and C E; that is to say, to the Rectangle under A B and F G, that is to say, under A G and G B, (for the Rectangles under A B and F G, and under A G and G B, are equal:) Therefore [e] the square of A B is incommensurable to the Rectangle under A G and G B: Therefore A G, and G B, are found incommensurable in power, making the Compound of their squares medial, and the Rectangle under them medial, and incommensurable to the Compound of their squares: Which was to be done.

e) 10. 10.

Here

Here begins the Senaries of Irrational Lines by Composition.

PROP. 37. THEOR. 25.



If two rational Lines A B, *and* B C, *commensurable in power onely, are compounded, the whole* A C *shall be irrational: and is called a Binomium, or a Binomial line.*

Demonstration FOr A B being incommensurable in length to B C; and as A B to B C, so the Rectangle under A B and B C to the square of B C; to wit [a] F to E, the Rectangle under A B and B C, shall be incommensurable to the square of B C. But twice the Rectangle under A B and B C is commensurable to the same Rectangle under A B and B C: to wit, F and G together, to G or F, and the square of A B, is commensurable to the square of B C; to wit, D to E, (being rationals,) wherefore [b] the Compound of the squares of A B and B C, is commensurable to the square of B C.

a) 10. 10.

b) 16. 10.

Therefore the Compound of twice the Rectangle under A B and B C, and the Compound of the squares of A B, and B C, are incommensurable; wherefore the square of A C, compounded of the squares of A B and B C, and of twice the Rectangle under A B and B C; (for [c] the square of A C is equal to the two squares of A B and B C, and to the two Rectangles under A B and B C, as appears,) is [d] incommensurable to the Compound of the squares of A B and B C: but the Compound of the squares of A B and B C, is rational, being shewn to be commensurable to the square of the Rational B C: therefore the square of A C incommensurable to the square of the Rational B C, is [e] irrational: Therefore A C shall be irrational: Now let that line be called by two names, or as others expresse it *Binomial*: Forasmuch as it is compounded of two names, that is to say, of two rational lines A B and B C, commensurable in power onely: Therefore if two lines, &c. Which was to be demonstrated.

c) 4. 2.

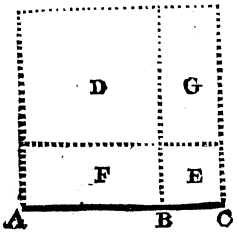
d) 17. 10.

e) 10. d.

A|——B——|C

PROP.

PROP. 38. THEOR. 26.

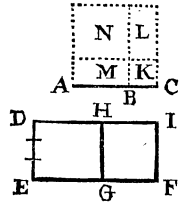


If two medial lines A B, *and* C D, *commensurable in power onely, containing a rational Rectangle, are compounded, the whole* A C *shall be irrational: And is called a first Bimedial line.*

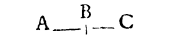
Demonstration. FOr A B being to B C, as the Rectangle conteined under A B and B C, is to the square of B C, and [a] A B and B C incommensurable in length, the Rectangle under A B and B C shall be incommensurable to the square of B C: to wit, as F to E: But twice the Rectangle under A B and B C is commensurable to the Rectangle under A B and B C, and the Compound of the squares of A B and B C is commensurable to the square of B C, (for A B and B C, being commensurable in power, their squares shall be commensurable; and [b] therefore the Compound of them shall be commensurable to the square of B C:) Therefore twice the Rectangle under A B and B C, and the Compound of the squares A B and B C, are commensurable: Therefore the Compound of the squares of A B, and B C and of twice the Rectangle under A B & B C, (which is [c] the square of A C) is [d] incommensurable to the Compound of twice the Rectangle under A B and B C: but the Rectangle under A B and B C is commensurable to twice the Rectangle under A B and B C, being the half thereof: Therefore [e] the square of A C is incommensurable to the Rectangle under A B and B C: but the Rectangle under A B and B C, is proposed rational: therefore the square of A C is irrational; and therefore [f] A C shall be irrational: Then let the line be called first of two medials, or a Prime *Bimedial*: Therefore, &c. Which was to be demonstrated.

a) 10. 10.
b) 16. 10.
c) 4. 2.
d) 17. 10.
e) 13. 1.
f) 10. d. 11. d.

PROP. 39. THEOR. 27.



If two medial lines A B *and* B C, *commensurable in power onely, conteining a medial Rectangle, are compounded, the whole* A C *shall be irrational: And is called a second Bimedial line.*



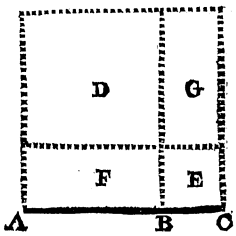
Demonstration. FOr let the Rational D E be exposed, to which let there be [a] applyed the Rectangle D F, equal to the square

a) 45. 1.

ſquare A C: and D G equal to the Compound of the ſquares of A B and B C; to wit N K, and the ſquare of A C, (or D F, his equal Rectangle (being [b] equal to the two ſquares of A B and B C, and twice the Rectangle under A B and B C, the reſt H F ſhall be equal to the Rectangle under A B and B C, twice, to wit, to M L: But the Rectangle under A B and B C, being propoſed medial, the double of that which is commenſurable thereto, to wit, [c] H F ſhall be alſo medial. b) 4. 2. c) C. 24. 10.

Again, ſeeing that the ſquares of A B and B C medials, commenſurable in power, are commenſurable; [d] their Compound, to wit, D G, ſhall be commenſurable to each of them, and being medials, [e] D G ſhall be a medial; therefore ſeeing that the medials D G and H F, are applyed to the Rational D E; and H G [f] his equal, [g] the breadth E G and G F ſhall be Rational, incommenſurable in length to D E: Again, A B and B C, being incommenſurable in length, and as A B to B C, ſo the ſquare of A B to the Rectangle under A B and B C: to wit, as N to L, the ſquare of A B ſhall be incommenſurable to the Rectangle under A B and B C: but the Compound of the ſquares of A B and B C is ſhewn to be commenſurable to the ſquare of A B, and twice the Rectangle under A B and B C, being the double thereof: Therefore the Compound of the ſquares of A B and B C, and twice the Rectangle under A B and B C, or their equal Rectangle D G and H F, are incommenſurable: Wherefore [h] D G being to H F as E G to G F; [i] E G and G F ſhall be incommenſurable in length, and being ſhewn Rationals, they ſhall be rational, commenſurable in power onely; therefore [k] the whole E F ſhall be irrational: Wherefore the Rectangle D F, under D E Rational, and E F Irrational, ſhall be Irrational; and the ſquare of A C equal to the ſame D F, is alſo Irrational: Wherefore A C is Irrational, which let be called ſecond of two Medials, or a ſecond Bimedial line: Therefore, If two, &c. Which was to be demonſtrated. d) 16. 10. e) C. 24. 10. f) 34. 1. g) 23. 10. h) 1. 6. i) 10. 10. k) 17. 10.

PROP. 40. THEOR. 28.

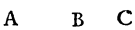


If two right lines A B *and* B C, *incommenſurable in power, making the Compound of their Squares rational, and the Rectangle conteined under them Medial, are compounded, the whole ſhall be irrational: And is called the Greater, or Major.*

Demonſtration For aſmuch as the Rectangle conteined under A B and B C, is propoſed Medial, twice the Rectangle conteined under A B and B C, commenſurable thereto, [a] ſhall be alſo Medial, and therefore Irrational: Now the Compound of the ſquares of A B and B C, is propoſed rational: Therefore [b] twice the Rectangle under A B and B C, ſhall be incommenſurable thereto: Therefore the ſquare of A C compounded of the ſquares of A B and B C, and twice the Rectangle under A B and B C, ſhall a) C. 10. 10. b) 10. d. 17. 10.

ſhall be incommenſurable to the Compound of the ſquares of A B and B C: and this Compound of the ſquares of A B and B C being propoſed rational; [c] the ſquare of A C incommenſurable thereto, ſhall be Irrational, [d] and A C Irrational: Let this line be called Major: Therefore, If, &c. Which was to be demonſtrated.

PROP. 41. THEOR. 29.

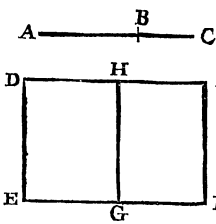


If two right lines A B *and* B C, *incommenſurable in power, are compounded, the which do make the Compound of their Squares medial; and the Rectangle conteined under them rational; the whole ſhall be Irrational: And is called a line equal in power to a rational, and a medial Superficies.*

Demonſtration FOr the Compound of the ſquares of A B and B C, being medial: but twice the Rectangle under A B and B C rational, his half, to wit, the Rectangle once, under A B and B C, being propoſed rational, [a] the Compound of the ſquares of A B and B C, is incommenſurable to twice the Rectangle under A B and B C: therefore [b] the Compound of the ſquares of A B and B C, and twice the rectangle under A B and B C; that is to ſay, the ſquare of A C is incommenſurable to twice the rectangle under A B and B C, which being rational, as before ſaid, the ſquare of A C incommenſurable thereto, is Irrational, and therefore A C Irrational: And is equal in power to a rational and a medial: Therefore, &c. Which was to be demonſtrated.

a) 10. d.
b) 17. 10.
10. d.
11. d.

PROP. 42. THEOR. 30.



If two right lines A B *and* B C, *incommenſurable in power, are compounded, which make the Compound of their ſquares medial, and the Rectangle conteined under them medial, and incommenſurable to the Compound of their ſquares; the whole ſhall be irrational: And is called a line equal in power to two Medials.*

Demonſtration FOr let the rational D E be expoſed, [a] to the which let there be applyed the rectangle D F, equal to the ſquare of A C, and D G equal to the Compound of the ſquares of A B and B C, and

a) 45. 1.

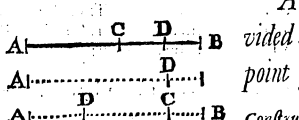
and the ſquare of A C [b] being equal to the Compound of the ſquares of A B and B C, and to twice the Rectangle under A B and B C, the reſt H F ſhall be equal to twice the reſt Rectangle under A B and B C. b) 4. 2.

But ſeeing that as well the Compound of the ſquares of A B and B C; that is to ſay, the Rectangle D G, as the Rectangle under A B and B C: [c] therefore the Compound of twice the Rectangle under A B and B C: that is to ſay, H F (being commenſurable thereto,) is propoſed medial, D G and H F, applyed to the Rational D E, ſhall make the breadths E G and G F Rational, incommenſurable in length to D E. c) C. 24. 10. 23. 10.

Again, the Compound of the ſquares of A B and B C, that is to ſay, the Rectangle DG being propoſed incommenſurable to the Rectangle under AB and BC, & twice the Rectangle under AB & BC; to wit HF, being commenſurable to the ſame Rectangle under A B and B C, (being double thereto,) [d] D G and H F ſhall be incommenſurable: Wherefore [e] E G being to G F, as DG to H F; [f] E G and G F ſhall be incommenſurable in length, and being ſhewn rational, the ſame E G and G F ſhall be rational, commenſurable in power onely: Therefore [g] the whole E F compounded of them is irrational: Therefore the Rectangle D F, conteined under D E rational, and E F irrational, is irrational, and therefore the ſquare of A C equal thereto, is alſo irrational, and the line A C equal in power to the ſame is irrational: And is called a line equal in power to two Medials. Therefore, If, &c. Which was to be demonſtrated. d) 13. 10. e) 1. 6. f) 10. 10. g) 37. 10.

PROP. 43. THEOR. 31.

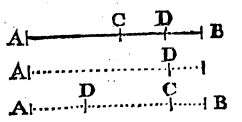
A Binomial line A B, *may be divided in its names onely in one point* C.



Conſtruction LEt A B a *Binomial* be divided in the point C, in its names, in ſuch ſort as A C and C B may be rationals, commenſurable in power onely, according to the 37 Propoſition of the Tenth: I ſay the line A B cannot be divided into other names, at another point: that is to ſay, in other lines then A C and C B, which may be rational commenſurable in power onely.

Demonſtration FOr (if poſſible) let it be againe divided in his names at D: It is manifeſt that A B is not divided at C in two equal parts, for A C and C B ſhould then be commenſurable in length, which is contrary to the Suppoſition, nor in like manner at D: Therefore A C is divided as well in C as in D unequally: Therefore A C and C B, are parts leſſe unequal then A D and D B, or more unequal: Wherefore the ſquares of A C and C B, ſhall be leſſe then the ſquares of A D and D B, or greater, (for the parts A D and D B, ſhall not be equal to the parts A C and C B, each to his correſpondent; to wit, the greateſt to the greateſt, and the leſſer to the leſſer, from what part ſoever the point D ſhall fall, (otherwiſe A B ſhould be divided at the ſecond Diviſion, at the ſame point as in the firſt.)

But forasmuch as, if from equal things there be taken unequal; the exceſſe [a] of the remainders, is equal to the exceſſe of the parts cut off; but the ſquares of A C and C B, with twice the Rectangle under A C and C B, are equal to the ſquares of A D and D B, with twice the rectangle under A D and D B: [b] Seeing that as well the one as the other, are equal to the ſquare of A B: It follows that the exceſſe of the Compound of the ſquares of A C and C B, and of the Compound of the ſquares of A D and D B, ſhall be the ſame with the Compound rectangle under A C and C B, and the rectangle under A D and D B.

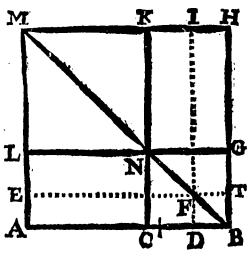


But the exceſſe of the Compounds of the ſquares of A C and C B, and A D and D B, is rational, (for A C and C B being rational, and therefore their ſquares rational, and their compound [c] being commenſurable to each of them, ſhal be rational; by the ſame reaſon the compound of the ſquares of A D and D B ſhall be rational: therefore ſeeing that one rational exceeds another rational by a rational: It is manifeſt that the exceſſe of the compounds of the ſquares of A C and C B, and of A D and D B, is rational: Therefore the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, is alſo rational: But [d] the Rectangle under A C and C B being medial, twice the Rectangle under A C and C B commenſurable thereto [e], is alſo medial: In like manner, twice the rectangle under AD & DB, is alſo medial: Wherefore [f] a medial that exceeds not a medial by a rational, the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, ſhall not be a ſpace rational. But we have ſhewn that it is rational, which is abſurd: Therefore A B cannot be divided in its names in any other point then C: Therefore it is diided in its names onely in one point. Which was to be demonſtrated.

a) Cor. 2. 1.
b) 4. 2.
c) 16. 10. 10. d.
d) 22. 10.
e) C. 24. 10.
f) 27. 10.

¶ This Demonſtration is alſo eaſie to be underſtood by the Figure hereafter expreſſed.

PROP. 44. THEOR. 32.



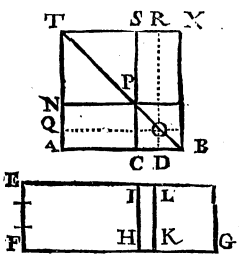
A firſt Bimedial line A B, *is divided in his names onely in one point* C.

Demonſtration FOr (if poſſible,) let the ſame be divided in D alſo, in ſuch ſort as A D and D B may be medials, commenſurable in power onely, and conteining a rational Rectangle, from what part ſoever the point D ſhall fall, it will be ſhewn as in the precedent, that the exceſſe of twice the Rectangle under A C and C B, and twice the rectangle under A D and D B, is the ſame as the

the exceſſe of the Compounds of the ſquares of A C and C B, and of A D and D B: But the exceſſe of the ſaid Rectangle is rational, (for the Rectangle under A C and C B, being rational, twice the Rectangle under A C and C B being commenſurable thereto, is alſo rational, and in like manner, twice the Rectangle under A D and D B ſhall be rational, and a figure rational, exceeding a figure rational, by a figure rational; the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, is rational:) Therefore the exceſſe of the Compounds of the ſquares of A C and C B, and of the ſquares of A D and D B, is rational: A C C B being medials commenſurable in power, their ſquares L K and C G ſhall be medials, and commenſurable: Wherefore [a] their Compound ſhall be commenſurable to each of them, and therefore medial; by the ſame reaſon, the Compound of the ſquares of A D and D B, which are E I and D I, is medial; and [b] a ſpace medial not exceeding a medial by a rational; the exceſſe of the Compounds of the ſquares of A C and C B, and of A D and D B, ſhall not be rational, but we have ſhewn it rational; which is abſurd: Therefore A B the firſt of two Medials, or a Prime Bimedial, cannot be divided in his names at any other point then C: And therefore is divided in his names onely in one point: Which was to be demonſtrated.

a) 16. 10. C. 24. 10.
b) 27. 10.

PROP. 45. THEOR. 33.



A ſecond Bimedial line A B, *is divided in his names, onely in one point* C.

Demonſtration FOr if poſſible, let it be divided in D, in ſuch ſort as A D and D B, may be alſo medials commenſurable in power onely, and conteining a medial rectangle: We ſhall ſhew as in the 43 Prop. that the ſquares of A C and C B, which are N S and C V, are greater or leſſer then the ſquares of A D and D B, to wit, Q R, and O B.

Let the rational E F be expoſed, to which let there be [a] applyed the Rectangle E G equal to the ſquare of A B, and E H, equal to the ſquares of A C and C B: Therefore the reſt I G ſhall be equal to twice the Rectangle under A C and C B: Seeing [b] that the ſquare of A B is equal to the ſaid ſquares of A C and C B, and twice the Rectangle under A C and C B, as appeares by the figure.

In like manner, if to E F there be applyed E K, equal to the ſquare of A D and D B, the reſt L G, ſhall be equal to twice the Rectangle under A D and D B; and ſeeing that the ſquares of A C and C B, are unequal to the ſquares of A D and D B, the Rectangles E H and E K, which are equal unto them, ſhall be alſo unequal: and therefore F H and F K, ſhall be unequal.

Again, the ſquares of A C and C B, being greater then twice the Rectangle conteined under A C and C B, E H ſhall be greater then I G, and therefore E H greater then the halfe of E G, and the line F H greater then

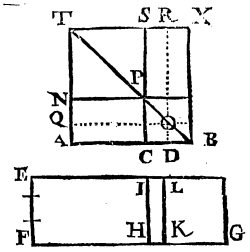
a) 45. 1.
b) 4. 2.

the halfe of F G, by the ſame reaſon, F K ſhall be ſhewn greater then the halfe of F G: Therefore the parts F H and H G, are unequal to the parts of F K and K G, each to his correſpondent: But foraſmuch as A C and C B, are medials, commenſurable in power, their ſquares are medials, and commenſurable: Therefore [c] the compound of them E H ſhall be commenſurable to every one of them; and [d] therefore medial: by the ſame reaſon, E K ſhall be medial; therefore [e] E H and E K applyed to the rational E F, make the breadths F H and F K rational, incommenſurable in length to E F.

c) 16. 10.
d) C. 24. 10
e) 23. 10.

In like manner, ſeeing that the Rectangle under A C and C B is propoſed medial, twice the Rectangle under A C and C B commenſurable thereto; that is to ſay, [f] I G is alſo medial: Therefore being applyed to the rational H I, H G ſhall be rational, incommenſurable [g] in length to H I. In like manner, we ſhall ſhew L G to be a medial, and K G rational, incommenſurable in length to K L.

f) C. 24. 10.
g) 23. 10.



10. 10.

Again, ſeeing that A C and C B are incommenſurable in length, and that as A C is to C B: ſo the ſquare of A C to the Rectangle under A C and C B, the ſquare of A C ſhall be incommenſurable to the Rectangles under A C and C B: But the Compound of the ſquares of A C and C B is commenſurable to the ſquare of A C: (for A C and C B being commenſurable in power; that is to ſay, their ſquares being commenſurable; the [h] Compound of them ſhall be commenſurable to each of them; to wit, to the ſquare of A C: But twice the Rectangle under A C and C B, is commenſurable to the Rectangle under A C and C B: Therefore the Compound of the ſquares of A C and C B; that is to ſay, the Rectangle E H is incommenſurable to twice the Rectangle under A C and C B, that is to ſay, to I G.

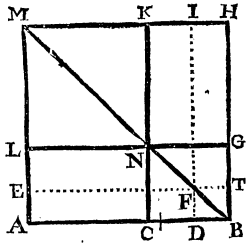
h) 16. 10.

Therefore ſeeing that E H [i] is to I G, as F H to H G: [k] F H and H G ſhall be incommenſurable in length: But they are ſhewn rationals, and therefore rationals commenſurable in power onely: [l] Therefore the whole F G is irrational, which is called by two names, and divided in his names in the point H: In like manner, we ſhall alſo demonſtrate that F G of two names, is divided into other names at another point K, [m] which is impoſſible: Therefore A B a Second Bimedial, is not divided in his names in any other point then C: And therefore is divided in his names at a point onely. Which was to be demonſtrated.

i) 1. 6.
k) 10. 10.
l) 37. 10.
m) 43. 10.

PROP.

PROP. 46. THEOR. 36.



A major Line A B *is divided in his names in one onely point* C.

Demonſtration FOr (if poſſible) let it be divided into other names q D, [a] as it hath been demonſtrated, the exceſſe of the compound of the ſquares of A D and D B, is the ſame as that of twice Rectangle under A D and D B, but the exceſſe of the compounds of the ſaid ſquares is rational: (for ſeeing both the one and the other compound is propoſed rational, their exceſſe ſhall be alſo rational;) therefore the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B is rational: But foraſmuch as the Rectangle under A C and C B, is propoſed medial, twice the Rectangle of [b] A C and C B is alſo medial, being commenſurable: In like manner, the Rectangle under A D and D B twice, is medial: [c] Therefore ſeeing that a medial exceedeth not a medial by a rational, the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B ſhall not be rational: But we have ſhewn that it is alſo rational, which is abſurd: Therefore A B cannot be divided in his names in any other point then C: Therefore it is divided in his names in one onely point. Which was to be demonſtrated.

a) 43. 10.
b) C. 24. 10.
c) 27. 10.

PROP. 47. THEOR. 35.

The line equal in power to a Rational and a Medial is divided its names onely in a point C.

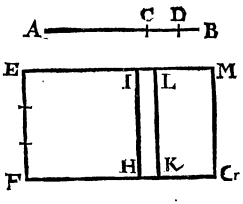
Demonſtration FOr, if poſſible, Let it be divided into other names at D, as we have ſhewn in the 43 Propoſition, that the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, is the ſame as the exceſſe of the compound of the ſquares of A C and C B, and the compound of the ſquares of A D and D B: but the exceſſe of the ſaid Rectangles is rational, [a] as hath been ſhewn: Therefore the exceſſe of the compound of the ſquares of A C and C B, and of A D and D B, is alſo rational: But the compounds of the ſaid ſquares are propoſed Medial, [b] therefore their exceſſe ſhall not be rational: But it is ſhewn rational, which is abſurd; Therefore A B is not divided in his names in any other point then C: And therefore onely in one point: Which was to be demonſtrated.

a) 44. 10.
b) 27. 10.

PROP.

PROP. 48. THEOR. 36.

The line A B *equal in power to two medials, is divided in his names in one point onely* C.



Demonstration FOr, if possible, Let it be divided into other names by D; using the same Construction as in the 45 Proposition, we shall shew here, (as there) that the parts F H and H G, are unequal to the parts F K and K G, each to his correspondent: Forasmuch then as the compound of the squares of A C and C B is proposed Medial, E H his equal is also medial: and seeing that the Rectangle under A C and C B, is proposed medial also, I G equal
a) C.24.10. to the double thereof, [a] is medial, being commensurable: Therefore E H and I G medials, applyed to the rational E F, make F H and H G, rationals, incommensurable in length to E F; (for H I is equal to E F,) so we shall prove that F K, and K G, are rational, incommensurable in length to E F, but seeing that the compound of the squares of A C and
b) 1.6. C B, that is to say, E H is incommensurable to twice the rectangle under A C and C B, by the Suppositon; that is to say, I G: and as [b] E H to I G,
c) 10.10. so F H to H G, F H and H G are [c] incommensurable in length: But they are shewn rational; therefore F H and H G are rational, commensurable in power onely: Therefore the whole F G is rational, called by two names; and divided in his names in H, but we shall shew that F G is also divided in
d) 43.10. other names at another point K, which is demonstrated to be impossible: [d] Therefore A B equal in power to two Medials, is not divided in his names at any other point then C: And therefore in one onely point: Which was to be demonstrated.

SECOND DEFINITIONS.

A Rational line being exposed, and a line of two names, (which is called Binomial,) divided in his names, of which the greatest name is more in power than the least, by the square of a right line which is commensurable in length thereto.

1 *If the greatest name be commensurable in length to the Rational exposed: the whole shall be called first of two names, or* A Prime Binomial.

2 *But*

2 *But if the least name be commensurable in length to the Rational exposed: Let it be called the second of two names, or* A Second Binomial.

3 *That if neither the one or the other of those names be commensurable in length to the Rational exposed: Let it be called the third of two names, or* A Third Binomial.

Again if the greatest name be more in power then the least, by the square of a right line which may be incommensurable in length thereto.

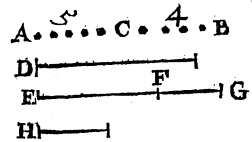
4 *If the greatest name be commensurable in length to the Rational exposed: Let it be called the fourth of two names, or* A Fourth Binomial.

5 *But if the lesser name be commensurable in length to the Rational exposed: Let it be called the fifth of two names, or* A Fifth Binomial.

6 *That if neither of those names be commensurable in length to the Rational exposed: Let it be called the Sixth of two names, or* A Sixth Binomial.

PROP. 49. PROBL. 13.

To find out a first Binomial line.



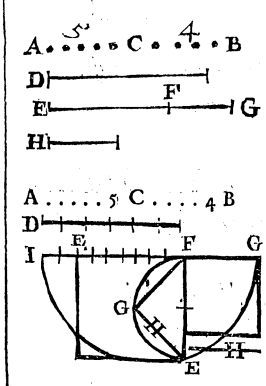
Construction HAving found two square numbers A B and C B, whose excesse A C may be no square number, in such sort as A B and C B may be to one another as a square number to a square number; but A B and A C may not be to one another as a square number to a square number; and let there be exposed a Rational line D, to the which let there be taken E F, commensurable in length; E F commensurable to D rational, shall be also [a] rational: then let there be drawn F G, to whose square a) Co.6.10.

square let the square of E F be as the number A B to the number A C, (which will be easily done, by dividing E F into as many equal parts as there are unites in A B;) and taking F I of five such parts, and F G a mean proportional between them, as appeares by the Figure; I say, that E G the total, is a first Binomial.

Demonstration FOr seeing that the squares of E F and F G being to one another as the numbers A B and A C,[b] are commensurable at least in power: But E F is shewn rational: Therefore F G is rational also: and A B being not to A C, as a square number to a square number, the squares of E F and F G shall not be to one another as a square number to a square number: Therefore E F and F G are incommensurable in length; and therefore Rational, commensurable in power onely; and the whole E G is irrational, called a Binomial: I say that it is also first, for the square of E F being to the square of F G as A B to A C, a number to a number; and A B being greater then A C, the square of E F shall be greater then the square of F G: Let it then be greater by the square of H: and as appeares by the Figure. For-asmuch as the number A B is to the number A C, as the square of E F to the square of F G; by conversion of reason, as A B shall be to C B; (to wit, to the excesse, therefore A B the antecedent, exceeds A C the consequent:) so the square of E F is to the square of H; to wit, to the excesse; therefore the antecedent square of E F exceeds the consequent square of F G: But A B and C B are square numbers; therefore the squares of E F and H, are to one another as a square number to a square number therefore E F and H are commensurable in length: therefore seeing that the greatest number E F is more in power then the least F G, by the square of H, commensurable in length thereto: and the same E F is also commensurable in length to D a rational exposed: E G shall be first of two names, or the first Binomial, according to the Definition: Therefore we have found, &c. Which was to be done.

b) 6. 10.

9. 10.



PROP. 50. PROBL. 14.

To find out a second Binomial line.

Construction HAving found two square numbers A B and C B, as in the fore-going Proposition, & the rational D being exposed; let F G be taken commensurable in length thereto; F G shall be then also rational: and as the number A C is to the number A B: So [a] let the square of F G be to the square of F; which will be easie, as in the precedent Proposition, in taking a line as F I, of nine such parts as F G is five thereof, and F E a mean proportional between them, as appeares by the Figure; I say that E G is a second Binomial.

a) C. 6. 10.

Demon-

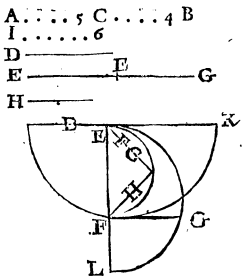
Demonstration FOr the squares of F G and E F being to one another as the numbers A C and A B; [b] they shall be commensurable; and the lines F G and E F, commensurable at least in power, and F G is shewn rational: therefore E F is also rational, But seeing that neither A C nor A B are to one another as a square number to a square number, the squares of F G and E F also shall not be as a square number to a square number; and [c] therefore E F and F G incommensurable in length, and so are rationals commensurable in power onely; and the whole E G irrational, which is called a Binomial: I say also that it is the second Binomial.

For seeing that the square of F G is to the square of E F, as the number A C to the number A B, and by conversion of reason, the square of E F to the square of F G as A B to A C: and as A B is greater then A C, the square of E F shall be greater then the square of F G: Let it then be greater by the square of H, we shall shew as in the fore-going that H is commensurable in length to E F: Now seeing that the greatest name E F is more in power then the least F G, by the square of H commensurable in length thereto, and the least name F G is commensurable in length to the Rational exposed D. E G by the Definition, shall be the second Binomial: Therefore we have found, &c. Which was to be done.

b) 6. 10.

c) 9. 10. 37. 10.

PROP. 51. PROBL. 15.



To find a third line of two names, or a third Binomial.

Construction HAving taken two numbers A B and C B, as in the 49 Proposition:, Let there be taken another number I, which may not be to A B or to A C, as a square number to a square number: which may be done in taking I, the number not square, the next greater then A C: that being done, it shall not be to A B, as a square number to a square number.

Again, there being no neer greater then A C, it shall differ from A C by one or two: Therefore between I and A C there will not fall a mean proportional; therefore they shall not be like Plaines, nor shall be to one another as a square number to a square number: Now let the Rational D be proposed, and as I is to A B, so the square of D may [a] be to the square of E F, which will also be easie in taking E K, of nine such parts as D is six of them, and E F a mean proportional between the two D and E K: therfore the squares of D and E F shall be commensurable, and D and E F commensurable at least in power, and D being a Rational, E F shall be so also; and forasmuch as I is not to A B; that is to say, the square of D is not to the square of E F as a square number to a square number [b] D, and E F shall be incommensurable in length.

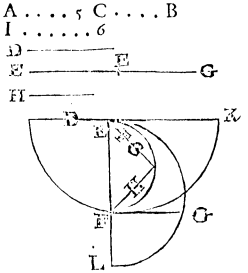
Again, let F G, to whose square let the square of E F be as A B to A C; [c] which will be done by dividing E F in nine equal parts, and making F L of five

a) Co 6. 10.

b) Co. 9. 10.

c) 6. 10.

five such parts; then taking F G a mean proportional between E F and F L, as appeares by the figure, those squares shall be commensurable, and E F and F G, at least commensurable in power: Therefore E F being shewn rational, F G shall be also rational; and forasmuch as A B is not to A C; that is to say, the square of E F is not to the square of F G as a square number to a square number, E F and F G [d] shall be incommensurable in length: Therefore E F and F G are rational, commensurable in power onely: Therefore [e] the whole E G is irrational, called a Binomial: I say, that it is the third.



d) 9. 10.

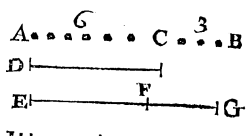
e) 37. 10.

Demonstration FOr seeing that as I to A B so the square of D to the square of E F, and as A B to A C; so the square of E F to the square of F G, in equal reason, as I is to A C, so the square of D to the square of F G: But I and A C are not to one another as a square number to a square number; therefore the squares of D and F G also, shall not be as a square number to a square number; therefore [f] D and F G are incommensurable in length: But forasmuch as A B is to A C as the square of E F is to the square of F G: But A B is greater then A C, therefore the square of E F shall be greater then the square of F G: Let it then be greater by the square of H, we shall shew, as in the 49 Proposition; that H is commensurable in length to E F: Forasmuch then as the greatest name is more in power then the least F G, by the square of H, commensurable in length thereto; and that neither E F nor F G is commensurable in length to the Rational proposed D, as is demonstrated: E G by the Definition, shall be the third of two names: Therefore we have found, &c. Which was to be done.

f) 9. 10.

PROP. 52. PROBL. 16.

To find a line fourth of two names, or a fourth Binomial.



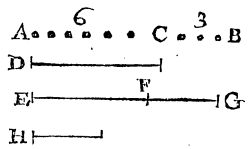
Construction HAving found two numbers A C and C B, such as the Compound A B be not to either of them as a square number to a square number: Let the rational D be proposed, and E F taken commensurable in length thereto; therefore the same E F shall be also Rational; and having made the rest as in the 49 Propositon; we shall shew as there that the whole E G is a Binomial: I say, it is the fourth.

Demonstration FOr the square of E F, (as in the 49,) shall be greater then the square of F G: Let it be so then, by the square of H, and by conversion of reason, as in the 49, as A B is to C B: so the square of E F is to the square of H: But A B being not to C B as a square number to a square number, the square of E F shall not be to the square of H, as a square number to a square number; therefore [a] E F and H are incommensurable in length: and seeing that the greatest name E F is more in power then the least F G, by the square of H incommensurable in length thereto; and the greatest name E F is commensurable in length to the Rational D, E G by the Definition, shall be the fourth Binomial: Therefore, &c. Which was to be done.

a) 9. 10.

PROP. 53. PROBL. 17.

To find a fifth line of two names, or a fifth Binomial.

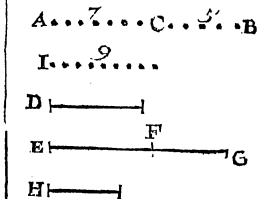


Construction HAving found two numbers A C and C B, as in the precedent; Let the Construction be made as in the 50th. that is to say; let F G be taken commensurable in length to the exposed rational D, it shall be shewn, as there, that E G is a Binomial: I say also that it is the fifth; for it shall be shewn as in the 50th. that the square of E F shall be greater then the square of F G: Let it then be greater by the square of H.

Demonstration BUt seeing by conversion of reason (as is demonstrated in the 49, as A B is to C B, so the square of E F, is to the square of H,) it shall be shewn as in the precedent, that E F and H, are incommensurable in length: therefore seeing that the greatest name E F is more in power then the lesser F G, by the square of H, incommensurable in length thereto; and the least name F G is commensurable in length to the Rational exposed D, by the Definition E G shall be a fifth Binomial: Therefore we have found, &c. Which was to be done.

PROP. 54. PROBL. 18.

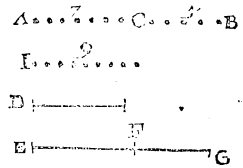
To find a line sixth of two names, or a sixth Binomial.



Construction HAving found two numbers A C and C B Plaines, not alike, neither of them being a square, and the Compound of them A B, also not a square, nor being to one another as a square number to a square number; (which may be done by dividing some number not square, into two numbers,) Primes to one another; [a] for so the whole shall be Prime to each of them; and therefore shall not be to either of them, as a square number to a square num-

a) 30. 7.

number,) and let there be taken some other number I, that may not be to A B nor to A C, as a square number to a square number, and a rational D be exposed, and as I is to A B; so let the square of D be made [b] to the square of E F, and let the rest be done as in the 51[th]. We shall shew (as there,) that D and E F, are incommensurable in length, and that the whole E G is a Binomial: I say also that it is the sixth.

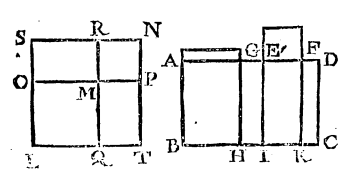


Demonstration FOr it shall be demonstrated as in the 51 Proposition, that D and F G are incommensurable in length, and that the square of E F is greater then the square of F G: Let it then be greater by the square of H, we shall demonstrate also, as in the 49[th]. that by conversion of reason, as A B is to C B, so the square of E F is to the square of H: therefore seeing that A B is not to C B, and therefore the square of E F; to the square of H, as a square number to a square number: [c] E F and H shall be incommensurable in length: Therefore seeing that the greatest name E F, is more in power then the least F G, by the square of H, incommensurable in length thereto; and neither of them E F nor F G is commensurable in length to the exposed Rational D, according to the Definition, E G shall be the Sixth Binomial: Therefore we have found, &c. Which was to be done.

b) Co. 6. 10

c) 9. 10.

PROP. 55. THEOR. 37.



If a Superficies A C *be conteined undera Rational* A B, *and a first Binomial line* A D; *the right line being equal in power to the said Superficies, is irrational; and is called a Binomial line.*

Demonstration FOr let A E be the greatest names of A D; therefore A E and E D, shall be rational, commensurable in power onely, according to the Definition; and A E shall be more in power then E D, by the square of a line commensurable in length thereto: and A E shall be commensurable to the Rational exposed A B: Let E D be divided in two equal parts in F: Forasmuch then as A E is more in power then E D, by the square of a line commensurable in length thereto: if there be applyed at A G a Rectangle equal to a quarter of the square of E D; that is to say, to the square of E F, conteined under A G and G E: and wanting by a square figure, it [a] shall divide the same in parts commensurable in length: Therefore A G and G E are commensurable in length to one another: Now let there be drawn G H, E I, and F K, parallel to A B and D C; and [b] let the square L M be equal to the Parallelogram A H, and the square M N equal to the Parallelogram G I, and let those squares be joyned together in the point M, in such sort as M O and M P may make one right line O P: Therefore Q M R shall also make a right line Q R, as [c] appeares: and having finished the Rectangle L N, O M and M P, being equal to Q M and M R; (because of the squares L M and M N:) and therefore the whole O P, to the whole Q R, and O P [d] being also equal; as well to S N as to L T, and Q R to L S and T N, the Rectangle L N shall be equilateral, and therefore a square.

a) 18. 10.

b) 14. 2.

c) 15, 13, & 14. 1.

d) 34. 1.

But the Rectangle under A G and G E; being made equal to the square of E F; as [c] A G to E F: so E F is to G E; and [f] therefore as A H to E K, so E K to G I: Therefore E K is a mean proportional between A H and G I; that is to say, between their equal squares L M and M N: But T M is a mean proportional between L M and M N: Therefore T M is equal to E K: Therefore M S being equal to T M, and [g] F C equal to E K, M S shall be also equal to F C, wherefore the whole square L N shall be equal to the whole Rectangle A C: Therefore the whole O P is equal in power to the Superficies A C conteined under the Rational A B and A D the first Binomial: I say, that O P is irrational, called a Binomial: For seeing that A G and G E are shewn to be commensurable in length, the whole A E shall be commensurable in length to each of them: But A E (being the greatest name of A D the first Binomial,) is commensurable in length to the Rational A B; Therefore A G and G E are commensurable in length to the same A B, [h] as appeares, therefore A B being Rational, A G and G E shall be Rational: Therefore [i] the Rectangles A H and G I conteined under the Rational commensurable in length, are Rationals, as also their equal squares L M and M N: and therefore O M and M P shall be Rational.

e) 17. 6.

f) 1. 6.

g) 43. 1. 36. 1.

16. 10.

h) 12. 10.

i) 20. 10.

And A E being incommensurable in length to E D, and A G is shewn commensurable in length to A E, and E F commensurable in length to E D, being half thereof, A G and E F shall be commensurable in length to one another: Wherefore A H and E K being in the same rate as A G and E F; that [k] is to say, their equals L M and M T are incommensurable: Therefore O M and M P are incommensurable in length, ([l] having the same reason as L M to M T:) but O M and M P are shewn rationals; commensurable in power onely: Wherefore the whole O P equal in power to the Superficies A C is irrational called a Binomial: Therefore, If a Superficies, &c. Which was to be demonstrated.

k) 10. 10.

l) 37. 10.

PROP. 56. THEOR. 38.

If a Superficies A C *be conteined under a rational line* A B, *and a second Binomial line* A D, *the right line equal in power to the said Superficies is irrational, and is called a first Bimedial line.*

Demonstration FOr let A E be the greater name of A D; therefore A E and E D, are rational, commensurable in power onely, according to the Definition; and A E is more in power then E D, by the square of

of a line commensurable in length thereto; and E D shall be commensurable in length to the Rational expoſed A B, Let E D be divided in two equal parts in F; and having done the reſt as in the precedent, it ſhall be demonſtrated (as in that Propoſition,) that O P is equal in power to the Superficies conteined under A B Rational, and A D a ſecond Binomial: I ſay, that O P is irrational, and is called a firſt Bimedial.

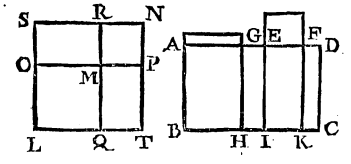
For A E being incommenſurable in length to E D, the leſſer name of A D a ſecond Binomial, commenſurable in length to A B rational, [a] A E and A B are incommenſurable in length, and A G and G E being ſhewn [b] commenſurable in length, [c] the whole A E ſhall be commenſurable in length to each of them: Wherefore A E the greateſt name of A D being rational, A G and G E ſhall be alſo rational: Therefore ſeeing that as well A G as G E, is commenſurable in length to A E; but A E incommenſurable in length to A B rational; [d] both A G and G E are incommenſurable in length to the ſame A B: Therefore as well A B and A G as A B and G E are rational, commenſurable in power onely: Therefore [e] the Rectangles A H and G I, or their equal ſquares L M and M N are medials: and therefore O M and M P medials: But A G and G E being commenſurable in length, [f] A H and G I being in the ſame rate, and therefore their equal ſquares L M and M N ſhall be commenſurable: Therefore O M and M P are commenſurable at leaſt in power.

And ſeeing that A E and E D are incommenſurable in length, and A G is ſhewn commenſurable in length to A E: and E F is commenſurable in length to E D; being the halfe thereof, A G and E F ſhall be incommenſurable in length: Therefore A H and E K having the ſame rate, [g] their equal Rectangles L M and M T are incommenſurable; and [h] therefore O M and M P are incommenſurable in length, having the ſame rate; therefore O M and M P being ſhewn medials, and commenſurable, they ſhall be medial, commenſurable in power onely.

Laſtly, E D the leaſt name being commenſurable in length to A B; that is to ſay[c] to E I: but E F is alſo commenſurable in length to E D, [i] E I and and E F ſhall be commenſurable in length; wherefore E I being rational, E F ſhall be alſo rational: Therefore [k] E K is a rational Rectangle: But M T conteined under O M and M P, is equal to E K, therefore M T is rational. Therefore O M and M P are medials, commenſurable in power onely, conteining a Rectangle rational; and ſo [l] O P is irrational, and is called the firſt Bimedial: Therefore if a Superficies, &c. Which was to be demonſtrated.

a) 13. 10.
b) 55. 10.
c) 6. 10.
d) 14. 10.
c) 22. 10.
f) 10. 10.
g) 10. 10.
h) 1. 6.
i) 12. 10.
k) 20. 10.
l) 38. 10.

PROP. 57. THEOR. 39.



If a Superficies A C *be conteined under a Rational line* A B, *and a third Binomial line* A D, *the right line equal in power to the ſaid Superficies* A C, *is irrational, and is called a ſecond Bimedial.*

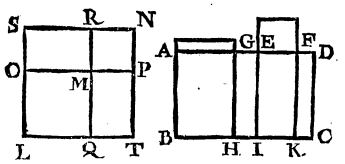
Demon-

Demonſtration FOr let A E be the greateſt name of A D; therefore A E and E D ſhall be rational, commenſurable in power onely; according to the Definition; and A E ſhall be more in power then E D, by the ſquare of a line commenſurable in length thereto, and neither A E nor E D ſhall be commenſurable in length to the rational expoſed A B.

Let E D be divided in two equal parts in F, and do the reſt as is ſhewn in the 55 Propoſition, it ſhall be demonſtrated (as there,) that O P is equal in power to the Superficies A C, and as in the precedent Propoſition 56, that O M and M P are medials, commenſurable in power onely: Seeing that A E is propoſed incommenſurable in length to A B rational; as the ſame A E was there incommenſurable in length to A B, but E D and E F being commenſurable in length, and E D incommenſurable in length to the rational A B; that is to ſay E I: [a] A E F ſhall be alſo incommenſurable in length to E I, but E F and E I are rationals, E F being the halfe of E D rational, and E I equal to the rational A B: Therefore E F and E I are rational, commenſurable in power onely; therefore [b] E K is a medial: Therefore M T his equal is alſo medial, conteined under O M and M P medials: Therefore O M and M P being medials, commenſurable in power onely, conteining a Superficies medial, [c] O P ſhall be irrational, and is called a ſecond Bimedial: Therefore, &c, Which was to be demonſtrated.

a) 14. 10.
b) 22. 10.
c) 22. 2.

PROP. 58. THEOR. 40.



If a Superficies A C *be conteined under a Rational line* A B, *and a fourth Binomial line* A D, *the right line equal in power to the Superficies* A C, *is irrational, and is called a Major line.*

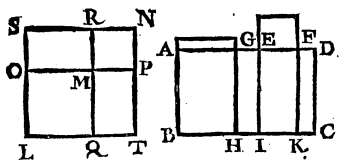
Demonſtration FOr let A E be the greateſt name of A D: Therefore A E and E D ſhall be rational, commenſurable in power onely, according o the Definition, and A E ſhall be more in power then E D; by the ſquare of a line incommenſurable in length thereto, and A E ſhall be commenſurable in length to A B: For having divided E D in two equal parts in F; Let the reſt be done as by the 55 Propoſition: Therefore [a] A G and G E ſhall be incommenſurable in length: Now we ſhall ſhew (as there,) that O P is equal in power to the Superficies A C: I ſay, that O P is irrational, called Major: For A G and G E being incommenſurable in length: [b] A H and G I having the ſame rate, are incommenſerable in length, and therefore their equal ſquares L M and M N incommenſurable: Wherefore O M and M P are incommenſurable in power: But A E the greateſt name being commenſurable in length to A B rational, ſhall be alſo rational: [c] and the Rectangle A I under them rational: which being equal to the Compound of

a) 19. 10.
b) 10. 10.
c) 20. 10.

of the ſquares L M and M N, the ſame Compound of L M and M N is Rational.

And foraſmuch as E D the leaſt name, is incommenſurable in length to the Rational A B; E F its halfe, ſhall be alſo incommenſurable in length to A B: and E F is rational, being commenſurable to E D rational: Therefore E F and A B are rational commenſurable in power onely; d) 22. 10. d Wherefore E K conteined under them, and therefore M T his equal, conteined under O M and M P, is a medial; wherefore O P and M P being incommenſurable in power, and making the Compound of their ſquares L M and M N rational, and the Rectangle M T under the ſame medial, the whole c) 40. 10. c O P ſhall be irrational, and is called Major: Therefore, &c. Which was to be demonſtrated.

PROP. 59. THEOR. 41.



If a Superficies A C *be conteined under a Rational line* A B, *and a fifth Binomial line, the right line equal in power to the ſaid Superficies* A C *is irrational, and is called a line equal in power to a rational and a medial Superficies.*

Demonſtration. FOr let A E be the greateſt name of A D: Therefore by the Definition A E and E D, are rational, commenſurable in power onely, and A E is more in power then E D, by the ſquare of a line incommenſurable thereto in length, and E D is commenſurable in length to A B the rational expoſed: Let E D be divided in two equal parts at F, a) 19. 10. and let it be done as in the 55 Propoſition: Therefore a A G and G E ſhall be incommenſurable in length, and we ſhall ſhew (as there) that O P is equal in power to the Superficies A C: I ſay, that O P is irrational, and is called a line equal in power to a Rational and a Medial: For as in the precedent Propoſition, O M and M P ſhall be incommenſurable in power, and A E the greateſt name being rational, and incommenſurable in length to the rational A B: A E and A B ſhall be rational, commenſurable in power onely; b) 22. 10. b wherefore the Rectangle A I under them is medial: But A I is equal to the Compound of the ſquares L M and M N; therefore the ſame Compound ſhall be medial.

Again, E D being propoſed rational, commenſurable in length to the rational A B, being the leaſt name of A D a fifth Binomial, E F the halfe of E D ſhall be alſo commenſurable in length to A B, and rational: therefore c) 20. 10. c E K conteined under E I and E F rationals, commenſurable in length, and therefore his equal Rectangle M T under O M and M P is rational; therefore O M and M P being incommenſurable in length, and making the Compound of their ſquares L M and M N medial: but the Rectangle M T under them rational; d) 40. 10. the d whole O P ſhall be irrational, and is called a line equal in power to a Rational, and a medial: Therefore, If a Superficies, &c. Which was to be demonſtrated. PROP.

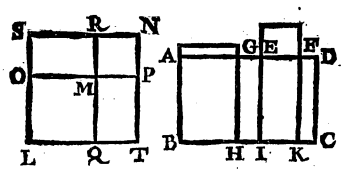
PROP. 60. THEOR. 42.

If a Superficies A C *be conteined under a rational line* A B, *and a ſixth Binomial line* A D, *the right line equal in power to the ſame Superficies* A C, *is irrational, and is called a line equal in power to two medials.*

Demonſtration. FOr let A E be the greateſt name: Therefore A E and E D are rationals, commenſurable in power onely, according to the Definition, and A E ſhall be more in power then E D, by the ſquare of a line which ſhall be incommenſurable thereto in length, and neither the one or the other of them AE and ED ſhall be commenſurable in length to the Rational A B: Having done as in the precedent Propoſition, A G a) and G E ſhall be incommenſurable in length, and it ſhall be ſhewn as in the 58 Propoſition, that O P is equal in power to the Superficies A C, and that O M and M P are incommenſurable in power. a) 19. 10.

Again, as in the precedent Propoſition, the Compound of the ſquares L M and M N ſhall be medial; but as in the 58 Propoſition, M T under O M and M P ſhall be alſo medial: And foraſmuch as of A E and E F, the one A E is incommenſurable in length to E D, and the other E F is commenſurable to E D; b) A E and E F ſhall be incommenſurable in length; b) 13. 10.
therefore c) A I and E K are incommenſurable; having d) the ſame rate as A E c) 10. 10.
and E F; wherefore the Compound of the ſquares L M and M N, equal d) 1. 6.
to A I, and M T equal to E K, are incommenſurable: Therefore O M and M P being commenſurable in power, and making the Compound of their ſquares L M and M N medial, and M T the Rectangle under them medial, and incommenſurable to the Compound of their ſquares, the e) whole O P c) 42. 10.
ſhall be Irrational, and is called a line equal in power to two Medials: Therefore, If, &c. Which was to be demonſtrated.

PROP. 61. THEOR. 43.



The Square of a Binomial line A B, *applyed unto a Rational line* D E, *makes the breadth* D G, *a firſt Binomial.*

Demonſtration. FOr to D E Rational let there be applyed D H, equal to the ſquare of A C: and to H I let there be applyed I K, equal to the ſquare of C B: Therefore the reſt L F ſhall be equal to twice the Rectangle under A C and C B, a) the ſquares of A C and C B, and twice a) 4. 2.
the Rectangle under A C and C B; being equal to the ſquare of A B, even as the Rectangle D F is equal to the ſame ſquare of A B, by the Conſtruction

Let L G be divided in two equal parts at M, and let M N be drawn parallel to L K and G F, both the one and the other Rectangles L N and M F, shall be equal to the Rectangle under A C and C B: And [b] A C and C B being Rationals, commensurable in power onely, the whole A B being a Binomial, the squares of A C & C B shall be rational, & therefore commensurable, [c] and their compounds, being commensurable to each of them, the said compound shall be also rational, to which D K being equal, by the construction D K shall be a rational: & seeing that the same D K is applyed to the rational, it shall make the breadth D L rational, & commensurable in length to D E.

b) 37. 10.
c) 16. 10.

Again, seeing that A C and C B are rationals commensurable in power onely; the [d] Rectangle under them shall be medial, and therefore twice the Rectangle under A C and C B, commensurable thereto; that is to say, L E [e] shall be medial; therefore L F applyed to L K Rational, [f] makes the breadth L G rational, incommensurabe in length to L K, that is to say, to D E: but D L is shewn commensurable in length to D E; therefore [g] D L and L G are incommensurable in length: Therefore they are rational, commensurable in power onely; and [h] therefore D G is a Binomial, or of two names: I say that it is the first.

d) 22. 10.
e) C. 24. 10.
f) 23. 10.
g) 13. 10.
h) 37. 10.

For the Rectangle under A C and C B, being a mean proportional between the squares of A C and C B, L N shall be also a mean proportional between D H and I K; therefore D I, L M, and I L having the same rate as D H, L N, I K, and L M, shall be also mean proportionals between D I and I L: [i] Therefore the Rectangle under D I and I L shall be equal to the square of L M, and the squares of A C and C B being commensurable, (seeing that A C and C B are proposed commensurable in power,) D H and I K equal to them, shall be commensurable: Therefore [k] D I and I L having the same rate, shall be commensurable in length.

i) 17. 7.
k) 10. 10.

But seeing that D K is greater then L F, the squares of A C and C B being greater then twice the Rectangle under A C and C B; D L shall be greater then L G: ([l] D L and L G having the same rate as D K and L F;) therefore seeing that D L is greater then L G, and at D L is applyed the Rectangle under D I and I L, equal to the square of L M; that is to say, to a quarter of the square of L G, the lesser wanting a square figure, (for the Rectangle under D I and I L, is shewn equal to the square of L M,) and D L is divided at I, in D I and I L, commensurable in length, as hath been demonstrated: Therefore [m] D L the greatest, shall be more in power then L G the lesser, by the square of a line commensurable in length thereto; wherefore D G being shewn to be a Binomial, and that D L the greatest name is more in power then the lesser L G, by the square of a line which is commensurable in length thereto, and that the same D L the greatest, is commensurable in length to the Rational proposed D E, by the Definition D G shall be first Binomial: Therefore, &c. Which was to be demonstrated.

l) 1. 6.
m) 18. 10.

PROP. 62. THEOR. 44.

The square of a first Bimedial line A B, *applyed to a Rational line* D E, *makes the breadth* D C *a second Binomial line.*

Demonstration LEt there be done as in the precedent Proposition, in such sort as that D H and I K may again be equal to the squares of

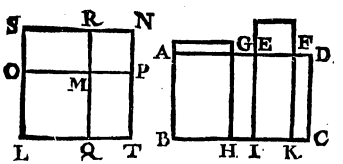
of A C and C B, and L N and M F, equal each of them to the Rectangle under A C and C B, forasmuch as A C and C B compounding A B a first Bimedial, are [a] medials, commensurable in power onely; conteining a rationa Rectangle: the squares of A C and C B, and therefore their equal Rectangles D H and I K are commensurable and medials; and [b] therefore the whole D K being commensurable to each of them, D H and I K [c] shall be also medial, the which being applyed to the Rational D E, [d] its breadth D L shall be Rational, incommensurable in length to D E.

a) 38. 10.
b) 16. 10.
c) C. 24. 10.
d) 23. 10.

Again, the Rectangle under A C and C B being Rational, the double thereof, to wit, L F shall be also Rational, which being applyed to the rational L K [e] its latitude L G shall be rational, commensurable in length to L K; that is to say, to D E; therefore seeing that L G is commensurable in length to D E, and D L incommensurable in length to D E, [f] D L and L G shall be incommensurable in length, and being shewn rational, they shall be rational, commensurable in power onely; and [g] therefore the whole D G is a Binomial, and as in the precedent we shall shew that D L is the greatest name, and is more in power then the least L G; by the square of a line commensurable thereto in length, and the least name L G being shewn commensurable in length to the rational exposed D E, according to the Definition, D G shall be second of two names, or a second Binomial: Therefore, &c. Which was to be demonstrated.

c) 21. 10.
f) 13. 10.
g) 37. 10.

PROP. 63. THEOR. 45.



The Square of a second Bimedial line A B, *applyed to a Rational line* D E, *makes the breadth* D G, *a third Binomial line.*

Demonstration FOr having made the Construction, as in the 61 Proposition; Forasmuch as A C and C B, making the whole A B a second Bimedial, [a] are medials, commensurable in power onely; conteining a medial; the squares of A C and C B; and therefore their equal Rectangles D H and I K, are commensurable, and medials: Wherefore the whole D K being commensurable to each of them, [b] shall be also medial, and being applyed to the Rational D E; [c] shall make the breadth L D rational, commensurable in length to D E.

a) 39. 10.
b) C. 24. 10.
c) 23. 10.

Again, the Rectangle under A C and C B being medial; the double thereof, to wit L F, shall be also medial: which being applyed to the Rational L K, [d] the other side or breadth L G shall be Rational, commensurable in length to L K; that is to say, to D E; and seeing that A C is incommensurable in length to C B; and as A C to C B, so the square of A C to the Rectangle under A C and C B: [e] the square of A C is also incommensurable to the Rectangle under A C and C B. [f] But the Compound of the squares of A C and C B, is commensurable to the square of A C, (those squares

d) 23. 10.
e) 10. 10.
f) 16. 10.

ſquares being commenſurable, deſcribed of the lines propoſed commenſurable in power,) and twice the Rectangle under A C and C B is commenſurable to the Rectangle under A C and C B, being double thereto: Therefore the Compound of the ſquares of A C and C B, that is to ſay, the Rectangle D K is commenſurable to twice the Rectangle under A C and C B; that is to ſay, to the Rectangle L F; therefore D L and L G having the ſame rate as D K and L F, g are incommenſurabre in length: But they are ſhewn rationals; Therefore D L and L G are rationals, commenſurable in power onely: Therefore h the whole D G is a Binomial, and as in the 61 Propoſition, we ſhall ſhew, that D L is the greateſt name, and is more in power then L G the leſſer, by the ſquare of a line which ſhall be incommenſurable thereto in length, and neither the one nor the other, D L nor L G, being ſhewn not to be commenſurable in length to the Rational propoſed, by the Definition, D G ſhall be a third Binomial: Therefore, &c. Which was to be demonſtrated.

g) 10. 10.

h) 37. 10.

PROP. 64. THEOR. 46.

The Square of a Major line A B, *applyed to a Rational line* D E, *makes the breadth* D G *a fourth Binomial line.*

Demonſtration FOr let the Conſtruction be made as in the 61 Propoſition: Foraſmuch as A C and C B, making the Major A B, a are incommenſurable in power, and do make the Compound of their ſquares rational, and the Rectangle under them medial: D K equal to the Compound of thoſe ſquares, ſhall be alſo rational: But L F the double of the Rectangle under A C and C B medial: Foraſmuch then as D K is applyed to the Rational D E: b D L ſhall be rational, commenſurable in length to D E; in like manner, L F medial being applyed to the Rational L K, c L G ſhall be rational, incommenſurable in length to L K; that is to ſay, to D E: Therefore ſeeing that D L is commenſurable in length to D E, and L G incommenſurable: D L and L G ſhall be incommenſurable in length: but they are ſhewn Rational: Therefore D L and L G are rational, commenſurable in power onely; therefore d D G is a Binomial.

a) 45. 10.

b) 21. 10.

c) 22. 10. 13. 10.

d) 37. 10.

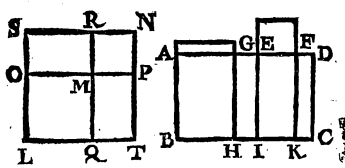
Now, as is ſhewn in the 62 Propoſition, the Rectangle under D I and I L is equal to the Square of L M: but foraſmuch as the ſquares of A C and C B are incommenſurable, (A C and C B being incommenſurable in power,) their equal Rectangles D H and I K, ſhall be alſo incommenſurable; and e therefore D I and I L being in the ſame reaſon, are incommenſurable in length: But as is ſhewn in the 61 Propoſition, D L is greater then L G, and to D L there hath been applyed a Rectangle conteined under D I and I L, equal to the ſquare of L M: that is to ſay, to the quarter part of the ſquare of L G, wanting a ſquare figure, which divideth D L at I, in parts incommenſurable in length; f D L ſhall be more in power then L G, by the ſquare of a line which is incommenſurable thereto in length: Wherefore D L (the greateſt name of D G a Binomial,) being ſhewn commenſurable in length to D E the rational propoſed: D G by the Definition, ſhall be a fourth Binomial: Therefore, &c. Which was to be demonſtrated.

c) 10. 10.

f) 19. 10.

PROP.

PROP. 65. THEOR. 47.



The Square of a line A B, *equal in power to a Rational* A C, *and a medial* C B, *applyed to a Rational line* D E, *makes the breadth* D G, *a fifth Binomial line.*

Demonſtration FOraſmuch as A C and C B, which make A B equal in power to a rational and a medial, are incommenſurable in power, making the Compound of their ſquares medial, and the Rectangle under them rational, D K equal to the Compound of the ſquares of A C and C B, ſhall be médial: and L N equal to the Rectangle under A C and C B: and therefore the double thereof to wit L F, ſhall be rational: Therefore D K a medial, applyed to D E a rational, makes a D L the breadth rational, incommenſurable in length to D E rational: but L F rational applyed to the Rational L K, b makes the breadth L G rational, commenſurable in length to L K, that is to ſay, to D E.

41. 10.

a) 23. 10.

b) 21. 10.

Foraſmuch c then as L G is commenſurable in length to D E rational, and D L incommenſurable in length, D L and L G ſhall be incommenſurable in length; wherefore D L and L G are rational, commenſurable in power onely; and therefore d D G is a Binomial.

c) 13. 10.

d) 37. 10.

Now we ſhall ſhew as in the 61 Propoſition, that the Rectangle under D I and I L is equal to the ſquare of L M: and as in the precedent, D L is the greateſt name, and is more in power then L G, by the ſquare of a line which ſhall be incommenſurable in length thereto; wherefore the leaſt name L G being ſhewn commenſurable in length to the rational expoſed D E, by the Definition, D G ſhall be a fifth Binomial: Therefore, &c. Which was to be demonſtrated.

PROP. 66. THEOR. 48.

The ſquare of a line A B, *equal in power to two medials, applyed to a Rational line* D E, *makes the breadth* D G, *a ſixth Binomial.*

Demonſtration FOr having made the Conſtruction as in the 61 Propoſition: Foraſmuch as A C and C B, making A B equal in power to two medials, are incommenſurable in power, and make the Compound of their ſquares medial, and the Rectangle under them medial, and incommenſurable to the Compound of their ſquares: as well D K equal to the ſame Compound, as L F equal to twice the Rectangle under A C and C B, is medial: and D K and L F medials, being applyed to the Rational D E,

42. 10.

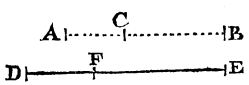
DE, [a] their breadths DL and LG ſhall be rational, incommenſurable in length to DE rational.

And [b] foraſmuch as the Compound of the ſquares of AC and CB is incommenſurable to the Rectangle under AC and CB, and twice the Rectangle under AC and CB, is commenſurable to the ſame Rectangle under AC and CB, being double thereto; [c] the Compound of the ſaid ſquares, that is to ſay, DK, is commenſurable to twice the Rectangle under AC and CB; that is to ſay, to LF: Wherefore DL and LG, having the ſame rate as DK and LF, [d] are incommenſurable in length; therefore DL to LG are rational, commenſurable in power onely [e]; Therefore DG is a Binomial.

And we ſhall demonſtrate as in the precedent, that DL is the greateſt name, and more in power then LG, by the ſquare of a line which is incommenſurable in length thereto; and being ſhewn that neither DL nor LG is commenſurable in length to DE the rational expoſed, by the Definition, DG ſhall be a ſixth Binomial: Therefore, &c. Which was to be demonſtrated.

a) 23. 10.
b) 42. 10.
c) 13. 10.
d) 10. 10.
e) 37. 10.

PROP. 67. THEOR. 49.



The line DE *commenſurable in length to the line of two names* AB, *or Binomial, is alſo a line of two names, and of the ſame order.*

Demonſtration. FOr as the whole AB is to the whole DE; ſo [a] the part cut off AC, to the part cut off DF; therefore [b] the reſt CB ſhall be to the reſt FE, as the whole AB to the whole DE: And foraſmuch as DE is propoſed commenſurable in length to AB; [c] DF ſhall be alſo commenſurable in length to AC, and FE to CB: But [d] AC and CB names of AB Binomial, are rational; therefore DF and FE are alſo rational.

Again, ſeeing that as AC is to DF, ſo CB to FE; and alternately, as AC to CB, ſo DF to FE: but AC and CB are commenſurable in power onely: therefore [e] DF and FE are alſo commenſurable in power onely: Therefore DF and FE are rational, commenſurable in power onely; and [f] therefore DE is a Binomial: I ſay, that DE is alſo of the ſame order as AB, for AC is more in power then CB; by the ſquare of a line which is commenſurable or incommenſurable thereto; and if AC be more in power than CB, by the ſquare of a line that may be commenſurable in length thereto; [g] DF ſhall be more in power than EF, by the ſquare of a line which ſhall be commenſurable in length thereto.

And if AC be commenſurable in length to the Rational expoſed, in ſuch ſort as AB may be a firſt Binomial, [h] DF ſhall be alſo commenſurable in length to the ſame rational, both the one and the other, being rational; And DF being alſo commenſurable in length to AC: Wherefore DE according to the Definition, is a firſt Binomial; that is to ſay, of the ſame order.

But if CB be commenſurable in length to the Rational expoſed: in ſuch ſort

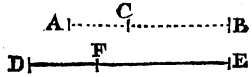
a) 12. 6.
b) 19. 5.
c) 10. 10.
d) 37. 10.
e) 10. 10.
f) 37. 10.
g) 15. 10.
h) 12. 10.

ſort as AB is a ſecond Binomial: In like manner, FE ſhall be commenſurable in length to the Rational expoſed: wherefore DE by the Definition, is a ſecond Binomial.

Laſtly, if neither AC nor AB be commenſurable in length to the rational expoſed: in ſuch ſort, as AB may be a third Binomial; neither the one nor the other DF nor FE, is commenſurable in length to the Rational expoſed: Therefore DE by the Definition, is a third Binomial.

But if AC be more in power then CB, by the ſquare of a line which ſhall be incommenſurable thereto in length, DF ſhall be more in power than FE, by the ſquare of a line incommenſurable in length thereto: wherefore we ſhall ſhew (as before,) that DE is a fourth, fifth, or ſixth Binomial, according as AB is a fourth, fifth, or ſixth Binomial: Therefore, &c. Which was to be demonſtrated.

PROP. 68. THEOR 50.



The right line DE *commenſurable to a Bimedial line* AB, *the ſame is alſo Bimedial, and of the ſame order.*

Demonſtration. FOr as [a] the whole AB is to the whole DE, ſo the part cut off AC, to the part cut off DF: Therefore [b] the reſt CB ſhall be to the reſt FE, as the whole AB to the whole DE; wherefore DE being propoſed commenſurable in length to AB, DF [c] ſhall be commenſurable in length to AC, and FE to CB: But AC and CB are medials; therefore [d] DF and FE commenſurable unto them, are alſo medials.

Again, AC being to DF as CB to FE; alternately, as AC is to CB, DF ſhall be to FE: but [e] AC and CB are commenſurable in power onely: Therefore [f] DF and FE ſhall be alſo commenſurable in power onely: and being ſhewn medials, the ſame DF and FE ſhall be medials, commenſurable in power onely: wherefore [g] DE ſhall be a Bimedial.

I ſay, that it is of the ſame order as AB: for ſeeing that as AC to CB, ſo is DF to FE: but as AC to CB, ſo the ſquare of AC to the Rectangle under AC and CB, and as DF to FE, ſo the ſquare of DF to the Rectangle under DF and FE; as the ſquare of AC ſhall be to the Rectangle under AC and CB: ſo the ſquare of DF ſhall be to the Rectangle under DE and EF: and alternately, as the ſquare of AC to the ſquare of DF: ſo the Rectangle under AC and CB, to the Rectangle under DF and FE: but the ſquare of AC is commenſurable to the ſquare of DF: AC and DF being ſhewn commenſurable in length: [h] Therefore the Rectangle under AC and CB ſhall be commenſurable to the Rectangle under DF and FE: Wherefore if the Rectangle under AC and CB be rational; in ſuch ſort as AB may be a firſt Bimedial: the Rectangle under DF and FE, commenſurable thereto, ſhall be alſo rational; and [i] therefore DE ſhall be a firſt Bimedial.

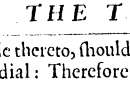
But if the Rectangle under AC and CB were medial; in ſuch ſort as AB were a ſecond Bimedial, the Rectangle under DF and FE, commenſurable

a) 12. 6.
b) 19. 5.
c) 10. 10.
d) 24. 10.
e) 38. 10.
f) 10. 10.
g) 39. 10.
h) 10. 10.
i) 38. 10.

surable thereto, ſhould [k] be alſo medial: Therefore D E ſhould be a ſecond Bimedial: Therefore, &c. Which was to be demonſtrated.

k) 39.10.

PROP. 69. THEOR. 51.



The right line D E *commenſurable to a Major line* A B, *is alſo a Major line.*

Demonſtration. LEt there be done as before, in ſuch ſort as A C and C B may bear the ſame rate to D F and F E, as the whole A B to the whole D E: Foraſmuch then as A B and D E are commenſurable either in length and power, or in power alone, as well [a] A C and D F, as C B and F E ſhall be commenſurable after the ſame manner.

a) 10.10.

Again, foraſmuch as A C is to D F, as C B is to F E, and alternately, as A C to C B, ſo D F to F E: as [b] the ſquare of A C ſhall be to the ſquare of C B, ſo the ſquare of D F to the ſquare of E F: And in compounding, as the Compound of the ſquares of A C and C B, to the ſquare of C B, ſo the Compound of the ſquares of D F and F E, to the ſquare of E F; and by converſion of reaſon, as the ſquare of C B to the Compound of the ſquares of A C and C B, ſo the ſquare of F E to the compound of the ſquares of D F and F E; and alternately, as the ſquare of C B to the ſquare of F E, ſo the compound of the ſquares of A C and C B to the compound of the ſquares of D F and F E: but the ſquare of C B is commenſurable to the ſquare of F E: C B and F E, being ſhewn commenſurable, be it in length and power, or in power onely: Therefore [c] the compound of the ſquares of A C and C B, is commenſurable to the compound of the ſquares of D F and F E; but the compound of the ſquares of [d] A C and C B is Rational: ſeeing that A C and C B compound the Major A B: therefore the compound of the ſquares of D F and F E is alſo Rational.

b) 22.6.

c) 10.10.

d) 40. 10. 9 Def.

Again, ſeeing that as A C is to C B ſo D F to F E; but as A C to C B, ſo the ſquare of A C to the rectangle under A C and C B; and as D F to F E, ſo the ſquare of D F to the rectangle under D F and F E, as the ſquare of A C ſhall be to the Rectangle under A C and C B, ſo the ſquare of D F ſhall be to the rectangle under D F & F E; and alternately, as the ſquare of A C to the ſquare of D F, ſo the rectangle under A C & C B, to the Rectangle under D F and F E.

But the ſquare of A C is commenſurable to the ſquare of D F, A C and D F being ſhewn commenſurable either in length and power, or in power alone; therefore [e] the Rectangle under A C and C B, is alſo commenſurable to the Rectangle under D F and F E: but [f] the Rectangle under A C and C B is medial; therefore the Rectangle under D F and F E, commenſurable thereto, is alſo medial: But A C being to C B as D F to F E; and [g] A C and C B, are incommenſurable in power, (compounding the Major A B,) D F and F E ſhall be alſo incommenſurable in power: Therefore D F and F E being incommenſurable in power, and making the compound of their ſquares Rational, and the Rectangle under them Medial, as hath been ſhewn: [h] the whole D E ſhall be a Major line: Therefore, &c. Which was to be demonſtrated.

c) 40. 10.
f) C. 24. 10.
g) 40. 10. 10. 10.
h) 40. 10.

PROP.

PROP. 70. THEOR. 52.

The line D E *commenſurable to the line* A B, *equal in power to a Rational and a Medial, is alſo a line equal in power to a Rational and a Medial.*

Demonſtration. FOr having made the ſame Conſtruction as before, we ſhall ſhew as in the precedent, that the compound of the ſquares of A C and C B is commenſurable to the compound of the ſquares of D F and F E: but the compound of the ſquares of A C and C B is a Medial; ſeeing that A C and C B do compound A B equal in power to a Rational and a Medial; therefore the compound of the ſquares of D F and F E, commenſurable thereto, is Medial.

In like manner, as in the precedent Propoſition, the Rectangle under A C and C B, which [a] is rational, ſhall be commenſurable to the Rectangle under D F and F E; and therefore the Rectangle under D F and F E ſhall be Rational.

a) 41.10. 9.d.

Laſtly, as in the precedent Propoſition, D F and F E ſhall be incommenſurable in power; therefore D F and F E being incommenſurable in power, and making the compound of their ſquares Medial, and the Rectangle under them Rational, as is ſhewn, [b] the whole D E ſhall be a line equal in power to a Rational and a Medial: Therefore, &c. Which was to be demonſtrated.

b) 41.10.

PROP. 71. THEOR. 53.

The line D E *commenſurable to the line* A B, *equal in power to two medials, is alſo a line equal in power to two medials.*

Demonſtration. FOr, the Conſtruction made, as before, it ſhall be ſhewn as in the 69 Propoſition, that the compound of the ſquares of A C and C B is commenſurable to the compound of the ſquares of D F and F E, and the Rectangle under A C and C B commenſurable to the Rectangle under D F and F E: wherefore [a] as well the compound of the ſquares of A C and C B, or the Rectangle under A C and C B, being medial: alſo [b] as well the compound of the ſquares of D F and F E, as the Rectangle under D F and F E, ſhall be medial: But D F and F E are incommenſurable in power, as before is ſaid.

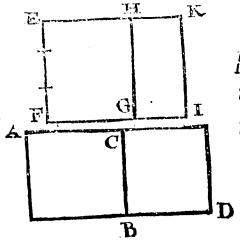
a) 42.10.
b) C. 24. 10.

Laſtly, [c] the compound of the ſquares of A C and C B, and the Rectangle under A C and C B, being incommenſurable, by Suppoſition, ſeeing that A B is equal in power to two medials: But the compound of the ſquares of D F and F E, is ſhewn commenſurable to the compound of the ſquares of A C and C B, and the Rectangle under D F and F E ſhewn

c) 42.10.

commensurable to the Rectangle under A C and C B, the compound of the squares of D F and F E, and the Rectangle under D F and F E shall be incommensurable: Therefore D F and F E being incommensurable in power, and making the compound of their squares medial, and the Rectangle under them medial, and incommensurable to the compound of their squares,
d) 42.10. [d] D E shall be a line equal in power to two medials: Therefore, &c. Which was to be demonstrated.

PROP. 72. THEOR. 54.



If a rational Superficies AB, *be compounded with a medial* CD, *there will be made four Irrationals, to wit, that which is called a Binomial, or a first Bimedial, or a line major, or a line equal in power to a rational and a Medial.*

Demonstration FOr the Space A B, shall be greater then C D, or lesse, for it cannot be equal: For A B being rational, if C D were equal thereto, it should be also rational; contrary to the Supposition.

Let first of all A B be greater then C D, and to the rational proposed E F, let there be applyed the Rectangle E G equal to A B, and to H G let there be applyed H I equal to C D, insuch sort as the whole E I be equal to the whole A D, and A B being a rational, and C D medial; E G shall be also rational, and H I medial, which applyed to the rational E F: E H
a) 21.10. [a] shall be rational, commensurable in length to E F: [b] but H K rational,
b) 23.10. incommensurable in length to E F.

Again, E G and H I being incommensurable, as appeares by the Definition, E G being rational, and H I irrational, to wit, medial; [c] E H and H K, having the same rate, [d] shall be incommensurable in length, and therefore are rational, commensurable in power only: [e] Therefore E K is a
c) 1.6. binomial, and A B being proposed greater then C D; that is to say, E G
d) 10.10. greater then H I: E H shall be also greater then H K, E H and H K having
c) 37.10. the same rate as E G and H I: Now E H the greatest name, shall be more in power then H K the lesser, by a square of a line commensurable in length or incommensurable thereto.

If E H be more in power then H K, by the square of a line commensurable in length thereto, E K shall be a first Binomial, by the Definition: E H being the greater name, shewn commensurable in length to the rational E F: Wherefore the line equal in power to the Space E I, conteined under the rational E F, and E K is a first Binomial; and therefore the Space A B compounded of the rational A B, and of the Medial C D
f) 55.10. [f] is irrational, called a Binomial.

But if E H be more in power then H K, by the square of a line which may be incommensurable in length thereto: E K shall be a fourth Binomial, by the Definition, E H being the greatest name, shewn commensurable in length

length to the rational E F: Therefore the right line equal in power to the square E I, conteined under E F rational, and E K a fourth binomial, and therefore also the square A D g is irrational, called Major.
g) 58.10.

Now let A B be the greatest irrational, and medial, and C D the least rational: having made the Construction as before, E H shall be incommensurable in length to E F, and H K commensurable, and therefore E H and H K rational, commensurable in power only, and the whole E K a Binomial; therefore E H the greatest name, shall be more in power then H K, by the square of a line which shall be commensurable thereto in length, or incommensurable: if it be commensurable, and E H is shewn incommensurable in length to the rational proposed, E K by the Definition, shall be a second Binomial: Therefore the line equal in power to the Superficies E I, a second Binomial, is irrational, [h] called a first Bimedial, if incom-
h) 56.10. mensurable, E K shall be a fifth Binomial; therefore the right line equal in power to the Superficies E I, (which is A B,) conteined under E F rational, and E K a fifth Binomial, [i] is irrational, called a line equal in power to a rational, and a medial: Therefore, &c. Which was to be demonstrated.
i) 59.10.

PROP. 73. THEOR. 55.

If two Superficies medials AB *and* C D, *incommensurable to one another, are compounded, two other irrational lines are made; to wit, either the second Bimedial, or the line equal in power to two medials.*

Demonstration FOr A B shall be greater then C D, or lesse, and not equal; otherwise A B and C D should be commensurable, which is contrary to the Supposition.

Let it in the first place be supposed greater, and let it be done as in the precedent Proposition, and A B and C D being proposed medials, and incommensurable, their equals E G and H I, shall be also medials, and incommensurable: Therefore [a] E H and H K having the same rate, shall [b] be
a) 1.6. b) 10.10. incommensurable in length: (and E G and H I medials,) being applyed to the rational E F, both [c] E H and H K are rational, incommensurable in
c) 23.10. length to the rational E F: Therefore E H and H K are rational, commensurable in power onely, therefore the whole E K is a Binomial.

But seeing that A B is proposed greater then C D, as in the precedent Proposition, E H shall be the greatest name of E K: and therefore shall be more in power then H K, by the square of a line which is commensurable in length thereto, or incommensurable: If E H be more in power then H K by the square of a line which may be commensurable in length thereto, as well E H as H K being shewn incommensurable in length to E F rationals, by the Definition, E K shall be a third Binomial: Therefore the right line equal in power to the Superficies E I, conteined under E F a rational, and E K a third Binomial, and therefore the line equal in power to the Superficies A D is irrational, called a second Bimedial.

If E H be more in power then H K, by the square of a line incommensurable in length thereto, as well E H as H K, being incommensurable in length to E F rational, E K shall be a sixth Binomial, by the Definition;

Therefore the line equal in power to E I, or A D is irrational, called a line equal in power to two Medials: and if A B were lesse then C D, it may be demonstrated in like manner: Therefore, &c. Which was to be demonstrated.

COROLLARIE.

From all these things may be gathered easily that the line of two names, or Binomial, and the other irrational lines following it are different among themselves; and also from the medial line.

For the square of a medial line applyed to a rational line, a *makes the breadth rational, incommensurable in length to the said rational to which it is applyed.* a) 23. 10.

But the square of a line of two names applyed to a rational line, b *makes the breadth the first of two names, or first Binomial.* b) 61. 10.

And the square of a first Bimedial applyed to a Rational line c *makes the breadth a second Binomial.* c) 62. 10.

And the square of a second Bimedial, applyed to a rational line, d *makes the breadth a third Binomial.* d) 63. 10.

But the square of a Major line applyed to a Rational, e *makes the breadth a fourth Binomial.* e) 64. 10.

And the square of a line equal in power to a Rational and a Medial, applyed to a Rational line, f *makes the breadth a fifth Binomial.* f) 65. 10.

Lastly, the square of a line equal in power to two Medials, applyed to a Rational line, g *makes the breadth a sixth Binomial.* g) 66. 10.

Now seeing that all these breadths do differ from the breadth of the Medial, and among themselves (to wit, from the breadth of the Medial, the one being rational, to wit, the medial, and the others irrational:) But forasmuch as they are not Binomials, of the same order they differ from one another; It is manifest that all the irrational lines whereof we have spoken hitherto, are different among themselves.

Here begins the Senaries of Irrational lines by intersection.

PROP. 74. THEOR. 56.

A C B *If from a Rational line* A B *be cut off a Rational line* A C, *commensurable in power only to the whole* A B, *the remainder* B C *is irrational: and may be called an* Apotome, *or a* Residual.

Demonstration For as A B is to A C, so the square of A B to the Rectangle under A B & A C, and A B and A C, being incommensurable in length; the a square of A B shall be incommensurable to the Rectangle under A B and A C: but the compound of the squares of A B and A C is commensurable to the square of A B, (for A B and A C being proposed commensurable in power, the squares of A B & A C shall be commensurable: b and therefore their Compound commensurable to the square of A B:) but twice the Rectangle under A B and A C is commensurable to the Rectangle under A B and A C: therefore the Compound of the squares of A B and A C, and twice the Rectangle under A B and A C are incommensurable: Now c the Compound of the squares of A B and A C is equal to twice the Rectangle under A B and A C, and to the square of B C: therefore d the Compound of the squares of A B and A C is incommensurable to the remaining square of B C: wherefore the Compound of the squares of A B and A C being rational: (as being commensurable to the square of the rational A B;) the square of B C shall be irrational, and therefore B C irrational: And let it be called an *Apotome*, which others call a *Residual*: Therefore, &c. Which was to be demonstrated.

a) 10. 10.
b) 16. 10.
c) 7. 2.
d) C. 17. 10.

PROP. 75. THEOR. 57.

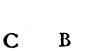
A C B *If from a medial line* A B, *be cut off a medial line* A C, *commensurable only in power to the whole* A B, *which doth contein a Rational Rectangle with the whole* A B, *the rest* C B *is irrational: And let it be called a first Residual Medial.*

Demonstration For A B and A C being commensurable in power, the squares of A B and A C shall be commensurable; and a therefore their compound shall be commensurable to the square of A C: but the square of A C medial is irrational, and medial: therefore b the compound of the squares of A B and A C, commensurable thereto, is irrational, and medial, and the Rectangle under A B and A C being proposed Rational, twice the Rectangle under A B and A C shall be Rational; wherefore the compound of the squares of A B and A C shall be incommensurable to twice the Rectangle of A B and A C: Therefore c seeing that the compound of the squares of A B and A C, is equal to twice the Rectangle under A B and A C, and to the square of B C; twice the Rectangle under A B and A C, with the square of B C, shall be also incommensurable to twice the Rectangle under A B and A C: Wherefore d twice the Rectangle under A B and A C, and the square of B C are incommensurable, and twice the Rectangle under A B and A C, being rational, the square of B C shall be irrational; and therefore B C irrational: And let it be called an *Apotome*, or first *Residual*: Therefore, &c. Which was to be demonstrated.

a) 16. 10.
b) C. 24. 10.
c) 7. 2.
d) 17. 10.

PROP.

PROP. 76. THEOR. 58.



If from a medial line A B, *be cut off a medial line* A C, *commensurable in power onely to the whole* A B, *which conteins with the whole* A B *a medial Rectangle, the remainder* B C *is irrational: And is called a Second* Apotome, *or* Residual *of a medial.*

Demonstration FOrasmuch as the squares of A B and A C commensurable in power, are commensurable, their [a] compound shall be commensurable to each of them: but each of those squares is medial; (A B and A C being proposed medial:) therefore [b] their compound is medial, being commensurable to each of them.

Again, seeing that the Rectangle under A B and A C is proposed medial: [c] the double thereof, to wit, twice the Rectangle under A B and A C, shall be also medial: Therefore [d] seeing that the compound of the squares of A B and A C is equal to twice the Rectangle under A B and A C, and to the square of B C, the compound of the squares of A B and A C which is medial, shall exceed twice the Rectangle under A B and A C, (which is also medial,) by the square of B C: But [e] a medial doth not exceed a medial by a Rational: Therefore the square of B C shall not be rational: therefore irrational, and B C irrational: And is called a second *Apotome* of a medial line: Therefore, &c. Which was to be demonstrated.

a) 16. 10.
b) C. 24. 10.
c) C. 24. 10.
d) 7. 2.
e) 27. 10.

PROP. 77. THEOR. 59.



If from a right line A B, *be cut off a right line* A C, *incommensurable in power to the whole* A B, *making with the whole* A B, *the Compound of their squares rational, and the Rectangle under them medial, the remainder* B C *is irrational: And is called a Minor.*

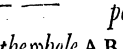
Demonstration FOr the Compound of the squares of A B and A C, being rational: But the Rectangle under A B and A C: and [a] therefore the double thereof which is commensurable thereto; that is to say, twice the Rectangle under A B and A C medial, that is to say, irrational: [b] the compound of the squares of A B and A C, shall be incommensurable to twice the Rectangle under A B and A C; But the compound of the squares of A B and A C is equal to twice the Rectangle under A B and A C, and to the square of B C: therefore the compound of the squares of A B and A C [c] shall be incommensurable to the remaining square of B C: but the compound of the squares of A B and A C is proposed rational: there

a) C. 24. 10.
b) 10. 10.
c) C. 17. 10.

therefore [d] the square of B C incommensurable thereto, is irrational, and the line B C irrational: And is called Minor: Therefore, &c. Which was to be demonstrated.

d) 10. d.

PROP. 78. THEOR. 60.

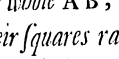


If from a right line A B, *be cut off a right line* C D, *incommensurable in power to the Whole* A B, *making with the Whole* A B *the Compound of their squares medial: But the Rectangle conteined under them rational, the remainder* C B *is irrational: And is called a line making with a rational Space, a whole medial.*

Demonstration FOr the compound of the squares of A B and A C being medial; that is to say irrational, and the Rectangle under A B and A C: and therefore the double thereof, to wit, twice the Rectangle under A B and A C rational: the [a] compound of the squares of A B and A C shall be incommensurable to twice the Rectangle under A B and A C: But [b] the compound of the squares of A B and A C is equal to twice the Rectangle under A B and A C, and to the square of B C: therefore twice the Rectangle under A B and A C, with the square of B C, is incommensurable to twice the Rectangle under A B and A C; therefore [c] twice the rectangle under A B and A C, and the square of B C are incommensurable: Therefore twice the rectangle under A B and A C being rational, the square of B C shall be irrational, and the line B C irrational: And is called a line making with a rational Space a whole medial: Forasmuch as its square added to twice the Rectangle under A B and A C, which is rational, makes the whole medial: Therefore, If, &c. Which was to be demonstrated.

a) 10. d.
b) 7. 2.
c) 17. 10.

PROP. 79. THEOR. 61.



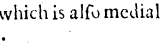
If from a right line A B, *be cut off a right line* A C, *incommensurable in power to the whole* A B, *making with the whole* A B, *the Compound of their squares medial, and the Rectangle contained under them medial, and incommensurable to the compound of their squares: the remainder* C B *is irrational; and let it be called a line making a whole medial with a rectangle medial.*

Demonstration FOrasmuch as the compound of the squares of A B and A C is equal to twice the Rectangle under [a] A B and A C, and to the square of B C, the compound of the squares of A B and A C, proposed medial:

a) 7. 2.

b) C.24.10. medial, shall exceed twice the rectangle under A B and A C, [b] which is medial, (being commensurable to the medial: conteined under A B and A C:) of the square of B C.: But a medial doth not exceed a medial by a rational, wherefore the square of B C is not rational, therefore irrational, and B C irrational: And is called a line making a whole medial with a medial: Forasmuch as the square thereof with the Rectangle under A B and A C, which is medial, makes the whole compound of the squares of A B and A C, which is also medial: Therefore, If, &c. Which was to be demonstrated.

PROP. 80. THEOR. 62.



There belongs only to a Residual *or* Apotome A B, *one only right line* B C *rational, commensurable in power only to the whole* A D.

Demonstration FOr (if possible,) let another rational B D agree to it, commensurable in power only to A D, and B C being rational, A C commensurable in power only thereto, shall be rational: Therefore A C and B C are rational, commensurable in power only, In like manner A D and B D shall be rationals, commensurable in power onely.

But there being the same excesse between the compound of the squares of A C and B C, and twice the Rectangle under A C and B C, as between the compounds of the squares of A D and B D, and twice the Rectangle under A D and C B; (for the excesse between the compound of the squares of A C and B C, and twice the Rectangle under A C and B C is the square of A B: the [a] compound of the squares of A C and B C, being equal to twice the Rectangle under A C and B C, and the square of A B.)

a) 7. 2.

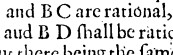
In like manner the square of A B is the excesse between the compound of the squares of A D and B D, and twice the Rectangle under A B and B D, (the compound of the squares of A D and B D, being equal to twice the Rectangle of A D and B D, and the square of A B;) also alternately, there will be the same excesse between the compound of the squares of A C and B C, and the compound of the squares of A D and B D, as between twice the Rectangle of A C and B C, and twice the Rectangle of A D and B D: but the excesse between those compounds is rational, the said compounds being rational: (for A C and C B being rational, commensurable in power onely, their squares shall be rational and commensurable: Therefore their compound shall be commensurable to each of them; and therefore Rational: In like manner, the compound of the squares of A D and B D shall be shewn rational;) therefore the excesse between twice the Rectangle under A C and B C, and twice the Rectangle under A D and B D is a space rational: And seeing that [b] the Rectangle under A C and B C rationals, commensurable in power onely, is medial, [c] the double thereof, to wit, twice the Rectangle under A C and B C, shall be medial: by the same reason, the

b) 22. 10.
c) C.24. 10.

the Rectangle under A D and B D is medial; and [d] a medial exceeds not a medial by a rational; therefore the excesse between twice the Rectangle under A C and B C, and twice the Rectangle under A D and B D, is not a space rational: but it hath been shewn rational, which is absurd: Therefore any other rational agrees not with A B, commensurable in power onely to the whole, besides B C: Therefore, &c. Which was to be demonstrated.

d) 27. 10.

PROP. 81. THEOR. 63.



To a medial first Residual A B, *there agrees onely a medial line* B C, *commensurable in power only to the whole* A D, *conteining with the whole* A D *a rational Rectangle.*

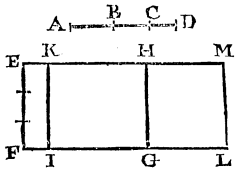
Demonstration FOr, (if possible,) let another medial B D agree therewith, commensurable in power onely to the whole: and let the Rectangle under A D and B D, be rational, and B C being Medial, commensurable in power only to A C, [a] A C shall be medial; therefore A C and B C are medials commensurable in power only.

a) 24.10.

And seeing that there is the same excesse between the compound of the squares of A D and B D, as between twice the Rectangle under A C and B C, and twice the Rectangle under A D and B D, as is shewn in the precedent, but the excesse between those Rectangles is rational, the one and the other being rational: (for the Rectangle under A C and B C being proposed Rational, twice the Rectangle under A C and B C, commensurable thereto is rational, and by the same reason, twice the Rectangle under A D and B D is rational:) Therefore the excesse between the compounds of the squares of A C and B C; and that of the squares of A D and B D is rational, and A C and B C being medials, commensurable in power only, their squares shall be medials, and commensurable: Therefore [b] their compound is commensurable to each of them, and [c] therefore medial; by the same reason, the compound of the squares of A D and B D shall be medial: But a medial not exceeding a medial by a rational, the excesse between the compounds of the squares of A C and B C, and that of the squares of A D and B D is not rational: But it hath been shewn rational, which is absurd: Therefore another medial agrees not to A B, besides B C, which may be commensurable in power onely to the whole, and contein a rational Rectangle with the whole: Therefore, &c. Which was to be demonstrated.

b) 16.10.
c) C.24.10.

PROP. 82. THEOR. 64.



To the medial ſecond Reſidual A B, *there can agree but one only right line* B C *medial, commenſurable in power only to the whole* A C, *and conteining a medial Rectangle under* A C *and* B C, *with the whole.*

Demonſtration FOr, (if poſſible,) let B D medial agree therewith, commenſurable in power only to the whole A D, and let the Rectangle under A D and B D be medial; and let E F the rational be expoſed, to which [a] let there be applyed the Rectangle E G, equal to that which is compounded of the ſquares of A C and B C, and to E F let there be applyed the Rectangle E I, equal to the ſquare of A B; then the Rectangle E L equal to the Rectangle which is compounded of the ſquares of A D and B D: Foraſmuch [b] then as the compound of the ſquares of A C and B C, is equal to twice the Rectangle under A C and B C, and to the ſquare of A B, K G ſhall be equal to twice the Rectangle under A C and B C: ſo K L ſhall be equal to twice the Rectangle under A D and B D, and A C and B C being medials, commenſurable in power only, (for B C being propoſed medial, A C [c] commenſurable in power thereto, ſhall be medial alſo,) their ſquares ſhall be medials, and commenſurable: Therefore [d] their compound is commenſurable to each of them, and therefore medial: Therefore E G equal to the ſame compound is alſo medial, and E G being applyed to E F rational, E H ſhall be rational, incommenſurable in length to E F.

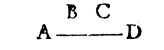
a) 45. 1.
b) 7. 2.
c) 16. 10.
d) C. 24. 10.

Again, the Rectangle under A C and B C being medial, the double thereof, to wit, K G ſhall be medial, and being applyed to K I rational, K H ſhall be rational, incommenſurable in length to K I; that is to ſay, to E F: and ſeeing that A C and B C are incommenſurable in length, and as A C to B C, ſo the ſquare of A C to the Rectangle under A C and B C, the ſquare of A C ſhall be incommenſurable to the Rectangle under A C and B C: but the compound of the ſquares of A C and B C, that is to ſay, the Rectangle E G is commenſurable to the ſquare of A C; (for A C and B C being propoſed commenſurable in power, their ſquares ſhall be commenſurable to each of them; to wit, to the ſquare of A C:) and the double of the Rectangle under A C and B C, to wit K G, is commenſurable to the ſaid Rectangle under A C and B C: Therefore E G is incommenſurable to K G, therefore E H and K H bearing the ſame rate as E G and K G, are incommenſurable in length, and being ſhewn rational, they ſhall be ratiorational, commenſurable in power only: Wherefore from E H rational, having cut off the rational K H, commenſurable in power only to E H, [e] E K ſhall be reſidual, and K H agreeing therewith: So it may be ſhewn that E K is a Reſidual to K M, agreeing therewith: Therefore to the Reſidual there agrees only one rational line, commenſurable in power only to the whole, which is abſurd, as hath been demonſtrated: Therefore, to a medial Reſidual, &c. Which was to be demonſtrated.

c) 74. 10.

PROP.

PROP. 83. THEOR. 65.



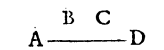
To a minor line A B, *there agrees only one right line* B C *incommenſurable in power to the Whole* A C, *and making with the Whole* A C *the Compound of their ſquares rational, and the Rectangle under them medial.*

Demonſtration FOr otherwiſe let another B D agree with A B incommenſurable in power to the whole A D, making the compound of the ſquares of A D and B D rational, and the Rectangle under A D and B D medial.

Foraſmuch then as hath been demonſtrated in the 80 Propoſition, the exceſſe of the compound of the ſquares of A C & B C, and the compound of the ſquares of A D & B D is the ſame as that between twice the rectangle under A C & B C, and of twice the rectangle under A D & B D: but the exceſſe between thoſe compounds is rational, both the one and the other, being propoſed rational: Therefore the exceſſe between twice the rectangle under A C and B C, and twice the rectangle under A D & B D is a rational Space: But it is alſo irrational, (for the rectangle under A C & B C being propoſed medial, twice the rectangle under A C and B C double thereto, [a] is alſo medial: In like manner, twice the rectangle under A D and D B ſhall be medial; therefore [b] the one ſhall not exceed the other by a rational, which is abſurd: Therefore another line agrees not to A B, beſides B C incommenſurable in power to the whole: Therefore, &c. Which was to be demonſtrated.

a) C. 24. 10.
b) 27. 10.

PROP. 84. THEOR. 66.

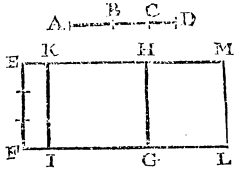


To a line A B, *making with a Space rational the whole medial, there agrees one only right line* B C, *incommenſurable in power to the whole* A C, *and making with the Whole the Compound of their ſquares medial: But the Rectangle conteined under them Rational.*

Demonſtration FOr (if poſſible) let another line B D agree therewith incommenſurable in power to the whole A D, and making the compound of the ſquares A D and B D medial, and the Rectangle under A D and B D rational: Foraſmuch as it is ſhewn in the 80th. Propoſition, that there is the ſame exceſſe between the compound of the ſquares of A C and B C, and the compound of the ſquares of A D and B D, as between twice the Rectangle under A C and B C, and twice the Rectangle under

AD and BD: But the exceſſe between thoſe Rectangles is a rational Space, as hath been ſhewn in the 81 Propoſition; therefore the exceſſe between thoſe compounds of the ſquares is alſo a rational Space: but it is not
a) 27.10. alſo rational, both the one and the other, being medial, [a] one medial exceeding not a medial by a rational, which is abſurd: Therefore any other line then BC agrees not with AB, incommenſurable in power to the whole: Therefore the line, &c. Which was to be demonſtrated.

PROP. 85. THEOR. 67.



To a line AB, *making with a Space medial a whole medial, there agrees one only right line* BC, *incommenſurable in power to the whole* AC, *and making with the whole* AC, *the Compound of their ſquares medial, and the* Rectangle *conteined under them medial, and incommenſurable to the compound of their ſquares.*

Demonſtration FOr (if poſſible, let BD agree thereto, alſo incommenſurable in power to the whole AD, making what is before ſaid; and let there be done as in the 82 Propoſition: Foraſmuch as the compound of the ſquares of AC and BC is medial, EG his equal is alſo me-
a) 23.10. dial; therefore [a] EH is rational, incommenſurable in length to EF: Again foraſmuch as the Rectangle under AC and BC is medial, the double
b) 23.10. thereof KG is alſo medial; therefore [b] KH is rational, incommenſurable in length to EF; and foraſmuch as KG is commenſurable to the Rectangle under AC and BC, being double to the ſaid Rectangle: But the Rectangle under AC and BC is propoſed incommenſurable to the compound of
c) 14.10. the ſquares of AC and BC: [c] KG ſhall be alſo incommenſurable to the compound to the ſaid Compound, that is to ſay to EG; and therefore EH
d) 10.10. and HK being in the ſame rate as EG and KG, are [d] incommenſurable in length; but they are ſhewn rational; therefore are rational, commenſurable in power only.

Taking then from the rational EH, the rational KH, commenſurable in
c) 74. 10. power only to EH, [e] EK ſhall be a Reſidual, and thereto agrees KH: in like manner we ſhall ſhew that EK is a Reſidual, to the which KM agrees. Therefore one only line doth not agree to a Reſidual, &c. Which is ab-
f) 80. 10. ſurd, being [f] ſhewn the contrary: Therefore, &c. Which was to be demonſtrated.

THIRD

THIRD DEFINITIONS.

A rational line being propoſed and a Reſidual, when the whole compounded of the Reſidual and of the agreeing or added line, be more in power then the agreeing line, by the ſquare of a line which is commenſurable thereto in length.

1 *If the whole be commenſurable in length to the rational propoſed, let the reſidual be called a* First Reſidual.

2 *But if the agreeing or added line, be commenſurable in length to the rational propoſed, let it be called a* Second Reſidual.

3 *And if neither the whole nor the agreeing, be commenſurable in length to the rational propoſed, let it be called a* Third Reſidual.

Again, if the whole be more in power then the agreeing, by the ſquare of a right line incommenſurable in length thereto.

4 *If the whole be commenſurable in length to the rational propoſed, let it be called a* Fourth Reſidual.

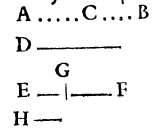
5 *But if the agreeing be commenſurable in length to the rational propoſed, let it be called a* Fifth Reſidual.

6 *And if neither the whole, nor the agreeing, be commenſurable in length to the rational propoſed, let it be called a* Sixth Reſidual.

PROP.

PROP. 86. PROBL. 19.

To finde an Apotome, *or a* First Residual.



Construction HAving found two square numbers, A B and C B, whose excesse A C may be no square; let the rational D be proposed, and let E F be commensurable in length thereto: E F shall
a) C.6.10. be also rational, and [a] as the number A B to A C: so let the square of E F be made to the square of G F: I say that E G is a first Residual.

Demonstration FOrasmuch as the squares of E F and G F, being to one another as the number A B to the number A C are commensurable, E F and G F shall be commensurable at least in power: Therefore seeing that E F is shewn rational, G F shall be rational also.

And seeing that A B and A C are not to one another as square numbers, the squares of E F and G F shall not be to one another as a square number to
b) 9. 10. a square number; therefore [b] E F and G F are incommensurable in length,
c) 74.10. they shall be then rational, commensurable in power only: therefore [c] the remainder E G is a Residual: I say, it is a first Residual: For let E F the greatest, be more in power then G F by the square of H; and forasmuch as the number A B is to the number A C, so the square of E F to the square of G F; by conversion, as A B to C B, so the square of E F to the square of H: but A B and C B are square numbers; therefore the square of E F shal be to the square of H, as a square number to a square number: therefore E F and H are commensurable in length.

Forasmuch then as the whole E F is more in power then G F, agreeing with the square of H, commensurable in length thereto, and the whole E F is commensurable in length to the rational exposed D, by the Definition E G shall be a first Residual: Therefore, &c. Which was to be done.

PROP. 87. PROEL. 20.

To finde a Second Residual.



Construction HAving found two numbers A B and C B, as in the precedent, and proposed the rational D, let G F be taken commensurable in length thereto, G F shall be also rational, and as
a) Co.6.10. [a] the number A C to the number A B, so let the square of G F be made to the square of E F: I say, that E G is a Second Apotome.

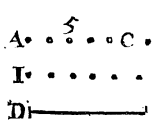
Demonstration FOr the squares of G F and E F being to one another as the numbers A C and A B are commensurable, and G F and E F commen-

commensurable at least in power, and seeing that G F is shewn rational, E F shall be also rational: But forasmuch as the numbers A C and A B, and by the same meanes the squares of G F and E F, are not to one another as a square number to a square number, [b] G F and E F shall be incommensurable in length, therefore G F and E F are rational, commensurable in power only, and therefore [c] E G remainder, is a Residual: I say also, it is a second Residual. For let E F be more in power then G F, by the square of H.
b) 9. 10.
c) 74.10.

Forasmuch then as A C is to A B, so the square of G F is to the square of E F, and by conversion of reason, as A B to A C, so the square of E F to the square of G F. Now we shall shew as in the precedent, that the line H is commensurable in length to E F: wherefore the whole E F being more in power then the agreeing G F, by the square of H, commensurable in length therero; and that the same agreeing G F is commensurable in length to the rational exposed D, by the Definition, E G shall be an *Apotome*, or *Second Residual*: Therefore, &c. Which was to be done.

PROP. 88. PROBL. 21.

To finde an Apotome, *or* Third Residual.

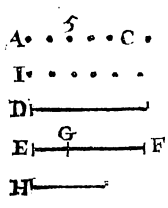


Construction HAving found two numbers A B and C B, as in the 86 Proposition; Let there be taken another number I, as is shewn in the 51 Proposition, which may not be to either of them as a square number to a square number, and having proposed D rational, let it be made as I to A B: so [a] is the
a) C. 6. 10. square of D to the square of E F, and those squares of D and E F shall be to one another as the number I and A B, and therefore commensurable: and therefore D and E F commensurable in power, and D being rational, E F shall be also rational; and forasmuch as the numbers I, and A B, and therefore the squares of D and E F are not to one another as a square number to a square number, [b] D and E F shall be incommensurable in length.
b) 9. 10.

Again, [c] as A B is to A C, so the square of E F to the square of G F: I
c) C. 9. 10. say, that E G is a third Residual: For, [d] seeing that the squares of E F and
d) 6.10. G F being to one another as the numbers A B and A C, are commensurable, E F and G F shall be commensurable at least in power, and E F being shewn rational, G F shall be also rational; and forasmuch as A B and A C, and so likewise the squares of E F and G F, are not to one another as square numbers, E F and G F shall be incommensurable in length: Therefore E F and G F are rational, commensurable in power only; and therefore G F being cut off from E F, commensurable thereto in power only, E G shall be *Residual*, I say, that it is a *Third Residual*.

Demonstration FOrasmuch as I is to A B, as the square of D to the square of E F, and as A B to A C, so the square of E F to the square of G F, in equal rate, as I to A C, so the square of D to the square of G F: Now

Now I and A C are not to one another as a ſquare number to a ſquare number: therefore c the ſquares of D and GF ſhall not be to one another as a ſquare number to a ſquare number: therefore D and G F are incommenſurable in length: therefore neither the one nor the other, E F nor G F, is commenſarable in length to the rational propoſed D.

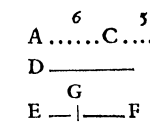


Now let E F be more in power then G F, by the ſquare of H, we ſhall ſhew as in the 86 Propoſition, that H is commenſurable in length to E F: Wherefore ſeeing that E F the whole is more in power then the agreeing G F, by the ſquare of H, commenſurable in length thereto, and that neither the one nor the other, E F nor G F, is commenſurable in length to the rational propoſed D, by the Definition, E G ſhall be a Third Reſiduall: Therefore, &c. Which was to be done.

c) 9. 10.

PROP. 89. PROBL. 22.

To finde a Fourth Reſidual.



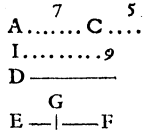
Conſtruction HAving found two numbers A C and C B, ſo as A B compounded of them may not have to either of them ſuch proportion as a ſquare number to a ſquare number: Let the rational D be expoſed, to the which let E F be commenſurable in length; E F ſhall be alſo rational, and having finiſhed the reſt as in the 86 Propoſition, we ſhall ſhew (as there) that E G is a Reſidual; I ſay, that it is alſo a fourth Reſidual.

Demonſtration FOr let E F be more in power then G F, by the ſquare of H: and ſeeing that as A B to A C, ſo the ſquare of E F to the ſquare of G F; by converſion of reaſon, as A B ſhall be to C B, ſo the ſquare of E F to the ſquare of H. Therefore ſeeing that A B and C B are not to one another as ſquare numbers, E F and H ſhall be incommenſurable in length. Foraſmuch then as the whole E F, is more in power then the agreeing G F, by the ſquare of H, incommenſurable in length thereto, and the ſame whole E F is commenſurable in length to the rational D; E G by the Definition, ſhall be a fourth Reſidual: Therefore, &c. Which was to be done.

PROP.

PROP. 90. PROBL. 23.

To finde a Fifth Reſidual.



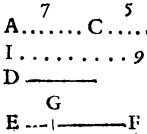
Conſtruction HAving found two numbers A C and C B, as in the precedent Propoſition: Let the ſame Conſtruction be made as in the 87 Propoſition; that is to ſay, let G F be taken commenſurable in length to the rational D, &c. Therefore as in the 87 Propoſition, we ſhall ſhew that E G is a Reſidual; I ſay alſo, that it is a Fifth Reſidual.

Demonſtration FOr let E F be more in power then G F, by the ſquare of H, and foraſmuch as we ſhall ſhew as in the 86 Propoſition, that by converſion of reaſon, as A B is to C B, ſo the ſquare of E F is to the ſquare of H.

Again, as in the precedent Propoſition, E F and H, are incommenſurable in length, and the agreeing G F is commenſurable in length to the rational propoſed; by the Definition, E G ſhall be a Fifth Reſidual: Therefore, &c. Which was to be done.

PROP. 91. PROBL. 24.

To finde a Sixth Reſidual.

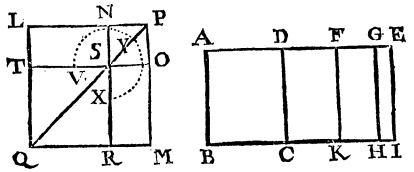


Conſtruction HAving found three numbers A C, C B, and I, as in the 54 Propoſition in ſuch ſort as A B be not to A C, nor to C B, and I be not to A B, nor to A C, as a ſquare number to a ſquare number: Let the rational D be expoſed, and let the reſt be done as in the 88 Propoſition, we ſhall ſhew as there, that D and E F are incommenſurable in length, and that E G is a Sixth Reſidual.

Demonſtration FOr as in the 88 Propoſition, D and G F, ſhall be alſo incommenſurable in length; and therefore neither the one nor the other, E F, nor G F, is commenſurable in length to the rational propoſed D. Now let E F be more in power then G F, by the ſquare of H, which we ſhall ſhew to be incommenſurable in length to E F, as in the 89 Propoſition: Therefore ſeeing that the whole E F is more in power then the agreeing G F, by the ſquare of H incommenſurable in length thereto; and neither E F nor G F, is commenſurable in length to the rational expoſed D, by the Definition, E G ſhall be a Sixth Reſidual: Therefore, &c. Which was to be done.

PROP. 92. THEOR. 68.

If a Space A C, *be conteined under a rational line* A B, *and a firſt Reſidual* A D, *the right line equal in power to the Space* A C, *is a Reſidual.*



Demonſtration. FOr let D E be the agreeing line to A D; therefore A E and D E by the Definition of the firſt Reſidual, ſhall be rational, commenſurable in power only: and the whole A E commenſurable in length to the rational A B, and the whole A E ſhall be more in power then the agreeing D E, by the ſquare of a line incommenſurable in length thereto.

Let D E be divided in two equal parts in F, and let there be applyed to A E a Rectangle, equal to the ſquare of E F; that is to ſay, equal to the quarter of the ſquare of D E, wanting by a ſquare figure, to wit, under A G and G E; and foraſmuch as A E is more in power then D E, by the ſquare of a line commenſurable thereto in length, A G and G E [a] ſhall be commenſurable in length; wherefore [b] the one and the other ſhall be commenſurable in length to the whole A E: but the whole A E is commenſurable in length to the rational A B: therefore [c] as well A G as G E, is commenſurable in length to A B, and therefore [d] rational: therefore having drawn G H and E I parallels to A B, meeting B C prolonged, in H and I, [e] the one and the other Rectangle A H and G I conteined under two rational lines commenſurable in length, ſhall be rational.

Again, foraſmuch as the one and the other, D F and F E, is commenſurable in length to D E, and D E incommenſurable in length to the rational A B, (for if it were commenſurable in length, A E being alſo commenſurable in length to A B: [f] A E and D E, ſhould be alſo commenſurable in length, which is abſurd, for they are propoſed commenſurable in power only,) [g] the one and and the other of them, D E and F E, ſhall be alſo incommenſurable in length to the rational A B.

And foraſmuch as the one and the other D F and F E, is commenſurable to the rational D E, they are rational; therefore as well A B and D F, as A B and F E, ſhall be rational, commenſurable in power only: Therefore having drawn F K parallel to A B, [h] the one and the other Rectangles D K and F I, conteined under the rational, commenſurable in power only, is Medial.

Now [i] let there be made the ſquare L M equal to A H, and N O equal to G I, having one angle common with L M; to wit, L P M; therefore [k] the ſquares L M and N O, ſhall be about one and the ſame Diameter; which let be P Q, and let the figure be finiſhed as you ſee. For-

a) 18. 10.
b) 16. 10.
c) 12. 10.
d) 6. d.
e) 20. 10.
f) 12. 10.
g) 14. 10.
h) 22 10.
i) 14. 10.
k) 26. 6.

Foraſmuch as the Rectangle under A G and G E is equal to the ſquare of E F, by Conſtruction, the three lines A G, F E, and G E, ſhall be proportional; and therefore A H, F I, and G I, having the ſame rate, F I ſhall be a mean proportional between A H and G I; that is to ſay, between the ſquares L M and N O, equal to thoſe Rectangles: But L O is alſo a mean proportional between the ſame L M and N O: therefore the Rectangles F I and L O are equal: But [l] D K is equal to F I, and M N to L O; therefore the whole D I is equal to all the Gnomon R O, O N, N P, with the ſquare N O: But the whole A I is equal to the ſquares L M and N O, by Conſtruction, therefore the remainder A C ſhall be equal to the remaining ſquare T R; therefore T S is equal in power to the Space A C: I ſay, that T S is a Reſidual.

l) 36. 10.

For ſeeing that the ſpaces A H and G I are ſhewn rational, and to them are equal L M and N O, the ſame ſquares L M and N O ſhall be rational, and therefore T O and S O rational.

Again, ſeeing that F I is ſhewn medial, L O equal thereto ſhall be alſo medial: Therefore L O and N O, are incommenſurable, the one being rational, and the other irrational; therefore [m] the lines T O and S O, having the ſame rate as L O and N O, are [n] incommenſurable in length; therefore the remainder T S is a Reſidual: Therefore, &c. Which was to be demonſtrated.

m) 1. 6.
n) 10. 10.

PROP. 93. THEOR. 69.

If a Space A C, *be conteined under a rational line* A B, *and a ſecond Reſidual* A D, *the right line equal in power to the ſame Space* A C, *is a Reſidual firſt Medial.*

Demonſtration. FOr let D E be the agreeing line to A D; therefore A E and D E, by the Definition of the ſecond Reſidual, ſhall be rational, commenſurable in power only, and the agreeing D E commenſurable in length to the rational A B; and laſtly, A E ſhall be more in power then D B, by the ſquare of a line commenſurable in length thereto: Let D E be divided in two equal parts in F, and for the reſt let it be done as in the precedent Propoſition, [a] A G and G E ſhall be commenſurable in length; and therefore [b] each ſhall be commenſurable to the whole A E; but A E is incommenſurable in length to the rational A B; (for if it were commenſurable in length to A B, D E being propoſed commenſurable in length to the ſame A B: [c] A E and D E ſhould be commenſurable in length to one another, which is abſurd, being propoſed commenſurable in power only:) therefore [d] as well A G as G E is incommenſurable in length to A B: But A G and G E commenſurable to the rational A E are rational: Therefore as well A B and A G, as A B and G E, are rational, commenſurable in power onely: Therefore [e] as well the Rectangle A H as G I, conteined under the rational commenſurable in power only, is Medial.

a) 18. 10.
b) 16. 10.
c) 12. 10.
d) 14. 10.
e) 22. 10.

Again, as well D F as F E, being commenſurable in length to D E, propoſed commenſurable in length to the rational A B, D F and F E, ſhall be commenſurable in length to A B rational, as appears by the 12 Propoſition, and therefore rational: therefore [f] as well the Rectangle D K as F I,

f) 20. 10.

F I, conteined under the rational commensurable in length is rational. And it shall be shewn as in the 92 Proposition, that T S is equal in power to the Space A C. I say that T S is also a Residual first Medial.

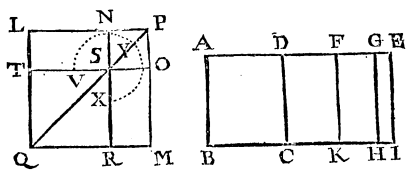
g) 10. 10. For A G and G E, being commensurable in length, [g] A H and G I having the same rate as they shall be commensurable, and therefore also their equal squares L M and N O commensurable: therefore the sides T O and S O, shall be at least commensurable in power. But T O and S O are Medials, being the squares L M and N O equal to A H and G I (shewn medials,) are Medials. And forasmuch as F I, and therefore its equal L O,

h) 10. 10. is rational, and therefore incommensurable to the Medial N O; [h] T O and S O having the same rate as they shall be incommensurable in length, and being shewn Medials, and commensurable, the same T O and S O

i) 75. 10. shall be Medials, commensurable in power only: and conteining the Rectangle L O, shewn rational, [i] T S shall be a first Residual: Therefore, &c. Which was to be demonstrated.

PROP. 94. THEOR. 70.

If a Space A C, *be conteined under a rational line* A B, *and a third Residual* A D, *the right line equal in power to the same Space is a Residual second Medial.*



Demonstration. For let D E be the agreeing line to A D; therefore A E and D E according to the Definition of the third Residual shall be rational, commensurable in power only; and neither A E nor D E shall be comensurable in length to the rational A B, and A E shall be more in power then D E, by the square of a line commensurable in length thereto: Let D E be divided in two equal parts in F, and let the rest of the Construction be finished, as in the 92 Proposition.

a) 16. 10. b) 14. 10. Again, as in the 92 Proposition, A G and G E, shall be commensurable in length to one another [a]: and therefore [b] as well A G as G E, commensurable in length to the whole A E: but A E is proposed incommensurable to the rational A B: therefore as well A G as G E, shall be incommensurable in length to A B: Therefore as well A G as G E, commensurable to A B rational, being rational: as well A B and A G, as A E and G E, are rational, commensurable in power only: therefore

c) 22. 10. [c] the one and the other Rectangle A H and G I, shall be Medial.

d) 14. 10. Again, D E being proposed incommensurable in length to the rational A B, D F and E F commensurable in length to D E, [d] shall be also incommensurable in length to A B: therefore D F and F E, being commensurable to

to D E rational, as well A B and D F, as A B and F E shall be rational, commensurable in power onely; and therefore [c] as well the Rectangle D K as F I, shall be Medial: and as in the 92 Proposition, it shall be shewn that T S is equal in power to the Space A C: I say, that T S is a Residual second Medial. c) 22. 10.

For A H and G I being shewn Medials, their equal squares L M & N O shall be also Medial; and therefore T O and S O Medials; and [f] seeing that A H and G I (having the same rate as A G and G E shewn commensurable,) are commensurable, the squares L M and N O shall be also commensurable; therefore T O and S O are commensurable at least in power. f) 10. 10.

But forasmuch as A E and D E are commensurable in power only; that is to say, incommensurable in length; and G E is shewn commensurable in length to A E, and F E commensurable in length to D E, G E and F E shall be incommensurable in length; and therefore [g] G I and F I, having the same rate as G E and F E; that is to say, their equals N O and L O are incommensurable. Wherefore T O and S O having the same rate as L O and N O are incommensurable in length, and being shewn Medials, and commensurable, they shall be Medials, commensurable in power only; and conteining L O a Medial: (for seeing that L O is equal to F I, as appears by the 92 Proposition, which is shewn medial, L O shall be also a medial;) T S shall be a Residual second Medial: Therefore, &c. Which was to be demonstrated. g) 10. 10.

PROP. 95. THEOR. 71.

If a Space A C *be conteined under a rational line* A B, *and a fourth Residual* A D, *the right line equal in power to the said Space* A C, *is a line Minor.*

Demonstration. For let D E be the agreeing line to A D; therefore A E and D E shall be rational, commensurable in power only, by the Definition of the fourth Residual: and A E commensurable in length to A B rational: and the same A E shall be more in power then D E, by the square of a line incommensurable in length thereto: Let D E be divided in two equal parts in F, and the rest done as is before shewn: Therefore A G and G E [a] shall be incommensurable in length; seeing that A E is more in power then D E, by the square of a line incommensurable in length thereto: And to A E is applyed a Rectangle under A G and G F, equal to a quarter of the square of D E, wanting a square figure, and A E being rational, and commensurable in length to the rational A B, [b] the Rectangle A I shall be rational. a) 19. 10. b) 20. 10.

Again, D E being rational, and incommensurable in length to A B rational: D I [c] and therefore its half F I, shall be medial. And moreover A G and G E being incommensurable in length, A H and G L [d] having the same rate, shall be incommensurable; and as in the 92 Proposition, so here it shall be shewn that T S is equal in power to the Space A C: I say, that T S is a Minor. c) 22. 10. d) 10. 10.

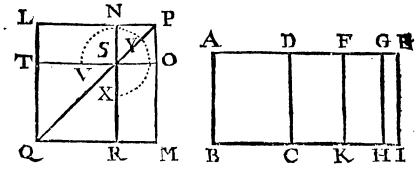
For the compound of the squares L M and N O, described of T O and S O, is equal to the Rectangle A I, by Construction: but the same is shewn

ſhewn rational; therefore the compound of the ſquares of T O and S O ſhall be rational: In like manner F I being ſhewn Medial, L O conteined under T O and S O, equal thereto, ſhall be Medial.

Laſtly, A H and G I, being ſhewn incommenſurable, their equal ſquares L M and N O, ſhall be incommenſurable, and therefore T O and S O incommenſurable in power: Therefore T O and S O being incommenſurable in power, and the compound of their ſquares being rational, and the Rectangle under them medial, the remainder T S ſhall be a Minor: Therefore, &c. Which was to be demonſtrated.

PROP. 96. THEOR. 72.

If a Space A C, *be conteined under a rational line* A B, *and a fifth Reſidual* A D, *the right line equal in power to the ſaid Space* A C, *is a line making with a Space rational, a whole medial.*



Demonſtration LEt D E agree to A D, by the Definition of the fifth Reſidual, A E and D E ſhall be rational, commenſurable in power only, and D E commenſurable in length to the rational A B, ſo A E ſhall be more in power then D E, by the ſquare of a line incommenſurable in length thereto.

Let D E be divided in two equal parts in F, and do the reſt as is before ſhewn. Therefore as in the precedent Propoſition, A G and G E ſhall be incommenſurable in length; and foraſmuch as A E is rational, incommenſurable in length to A B rational, as is ſaid in the 93 Propoſition, A I
a) 22. 10. [a] ſhall be Medial.

Alſo, ſeeing that D E is rational, commenſurable in length to A B rational,
b) 20. 10. [b] the Rectangle D I, and therefore its half F I, is rational.

Again, as in the precedent Propoſition, A H and G I are incommenſurable, and as is ſhewn in the 92 Propoſition, T S equal in power to the Space A C. I ſay that T S is the line with which a Space rational makes a whole Medial.

Foraſmuch as it hath been ſhewn that A I is a Medial, the compound of the ſquares L M and N O, of the lines T O and S O, equal thereto is medial, and F I being a rational, as we have ſhewn, the Rectangle L O conteined under T O and S O, equal thereto, is alſo rational, and T O and S O are incommenſurable in power, as is ſhewn in the precedent: therefore ſeeing that the compound of their ſquares is medial, and the Re-
c) 78. 10. ctangle under them rational, the remainder [c] T S is the line with which a rational makes a whole Medial: Therefore, &c. Which was to be demonſtrated.

PROP.

PROP. 97. THEOR. 73.

If a Space A C *be conteined under a rational line* A B, *and a ſixth Reſidual* A D, *the right line* T S, *equal in power to the ſaid Space* A C, *is that which with a Rectangle Medial makes a whole Medial.*

Demonſtration FOr let D E agree to the ſame A D: therefore A E and D E by the Definition of the Sixth Reſidual, are rational, commenſurable in power only, and neither one nor the other is commenſurable in length to the rational A B; and finally, A E ſhall be more in power then D E, by the ſquare of a line incommenſurable in length thereto.

Let D F be divided in two equal parts in F, and let the reſt be done as before is ſhewn: Therefore as in the 95 Propoſition, A G and G E ſhall be incommenſurable in length.

And ſeeing that as well A E as D E is rational, incommenſurable in length to the rational A B: as well [a] A I as D I, and therefore F I the halfe of D I, ſhall be medial, and A H and G I as in the 95 Propoſition, ſhall be incommenſurable. a) 22. 10.

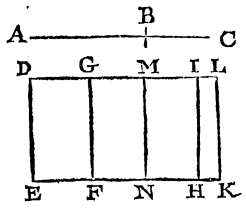
And foraſmuch as A E and D E are commenſurable in power only, [b] A I and D I which are in the ſame rate, are incommenſurable; and D I and F I being commenſurable, F I [c] ſhall be incommenſurable to A I: Now we ſhall ſhew as in the 92 Propoſition, that T S is equal in power to A C: and I ſay that T S is that, which with a Rectangle Medial makes a whole Medial. b) 10. 10. c) 14. 10.

Foraſmuch as A I is ſhewn medial, the compound of the ſquares L M and N O, of the right lines T O and S O, equal thereto, is Medial, alſo F I being ſhewn Medial, S O his equal conteined under T O and S O, is a Medial.

Again, F I being incommenſurable to A I, as is ſhewn L O conteined under T O and S O, ſhall be incommenſurable to the compound of the ſquares of T O and S O: ſeeing that L O is equal to F I, and the compound of the ſquares of T O and S O, equal to A I. Laſtly, T O and S O are incommenſurable in power, as is ſhewn in the 95 Propoſition:. Therefore ſeeing that the compound of their ſquares is medial, and the Rectangle under them Medial, and incommenſurable to the compound of their ſquares, the [d] remainder T S is that which with a Medial makes a whole Medial: Therefore, &c. Which was to be demonſtrated. d) 79. 10.

PROP.

PROP. 98. THEOR. 74.



The ſquare of a Reſidual A B, *applyed to a rational line* D E, *makes the breadth* D G *a firſt Reſidual.*

Demonſtration. FOr to D E let there be applyed the Rectangle D H, equal to the ſquare of A C, and to H I let there be applyed I K, equal to the ſquare of B C, in ſuch ſort as the whole D K may be equal to the compound of the ſquares of A C and C B.

a) 7. 2. And [a] foraſmuch as the compound of the ſquares of A C and B C, is equal to twice the Rectangle under A C and B C, and to the ſquare of A B, if the ſquare of A B and the Rectangle D F be taken away, the Rectangle G K ſhall remaine equal to twice the Rectangle under A C and B C: therefore G L being divided into two equal parts in M, and M N being drawn parallel to D E; M K ſhall be equal to the Rectangle under A C and B C.

And foraſmuch as A C and B C are rationals; their ſquares ſhall be rational, and therefore commenſurable: Therefore [b] ſeeing that the compound of the ſquares of A C and B C is commenſurable to each of them, the ſaid compound, that is to ſay, his equal Rectangle D K ſhall be rational; which being applyed to the rational D E, [c] D L ſhall be rational, commenſurable in length to D E.

b) 16. 10.

c) 21. 10.

Again, ſeeing that A C and B C are rational, commenſurable in power only, the Rectangle under them, and his double G K is Medial, which being applyed to the rational G F, G L [d] ſhall be rational, incommenſurable in length to C F; that is to ſay, to D E: And foraſmuch as D K rational, and G K Medial, that is to ſay irrational, are incommenſurable, D L and G L [e] having the ſame rate as D K and G K, ſhall be incommenſurable in length: Therefore being ſhewn rational, they ſhall be rational, commenſurable in power only, and therefore [f] the remainder D G ſhall be a Reſidual. I ſay it is a firſt Reſidual.

d) 23. 10.

e) 10. 10.

f) 74. 10.

Foraſmuch as the Rectangle under A C and B C, that is to ſay M K, is a mean proportional between the ſquares of A C and B C, that is to ſay, between D K and I K: D H, M K, and I L, are continually proportional, and the lines D I, M L, and I L, continually proportional, being in the ſame rate. Wherefore [g] the Rectangle under D I and I L, is equal to the ſquare of M L; that is to ſay, a quarter of the ſquare of G L.

g) 17. 6.

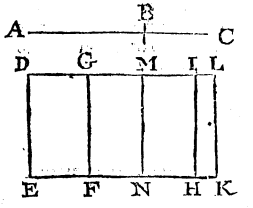
And foraſmuch as the ſquares of A C and B C, or their equals D H and I K are commenſurable: [h] D I and I L, being in the ſame rate, ſhall be commenſurable in length; therefore D L and G L being unequal, and to the greateſt there be applyed a Rectangle uuder D I and I L, equal to the quarter of the ſquare of G L the leaſt, wanting a ſquare figure, and D I being ſhewn commenſurable in length to I L, D L ſhall be more in power then G L, by the ſquare of a line commenſurable thereto in length: Wherefore D G being ſhewn Reſidual, and that the whole D L [i] is more in power then G L, his agreeing line by the ſquare of a line commenſurable in length thereto;

h) 16. 10.

i) 18. 10.

thereto; and that the ſame whole D L is commenſurable in length to the rational D E, by the Definition, D G ſhall be a firſt Reſidual: Therefore the ſquare, &c. Which was to be demonſtrated.

PROP. 99. THEOR. 75.



The ſquare of a Reſidual firſt medial A B, *applyed to a rational line* D E, *makes the breadth* D G, *a ſecond Reſidual.*

Demonſtration. FOr let it be done as in the precedent Propoſition, in ſuch ſort as D H and I K may be equal to the ſquares of A C and B C; and G K equal to twice the Rectangle under A C and B C, and therefore M K equal to once the ſame Rectangle; foraſmuch then as A C and B C are Medials, commenſurable in power only, and their ſquares, that is to ſay, their equals D H and I K medials, and commenſurable: therefore [a] the whole D K ſhall be commenſurable to each of them, [b] therefore medial; And D K being applyed to the rational D E, [c] D L ſhall be rational incommenſurable in length to D E.

a) 16. 10.

b) C. 24. 10.

c) 22. 10.

Again, foraſmuch as the Rectangle under A C and B C is propoſed rational, the double thereof G K ſhall be alſo rational, which being applyed to the rational D E, [d] G L ſhall be rational, commenſurable in length to D E.

d) 21. 10.

But foraſmuch as D K Medial, that is to ſay irrational, and G K rational, are incommenſurable in length, and being ſhewn rational, [e] D L and L G ſhall be rational, commenſurable in power only; and therefore [f] the remainder D G a Reſidual: I ſay that it is a ſecond Reſidual: For (as in the precedent Propoſition) we ſhall ſhew that the whole D L is more in power then the agreeing line G L, by the ſquare of a line commenſurable thereto in length: therefore G L being ſhewn commenſurable in length to the rational D E, the ſame D G by the Definition, ſhall be a ſecond Reſidual: Therefore, &c. Which was to be demonſtrated.

c) 10. 10.

f) 74. 10.

PROP. 100. THEOR. 76.

The ſquare of a Reſidual ſecond medial A B, *applyed to a rational line* D E, *makes the breadth* D G *a third Reſidual.*

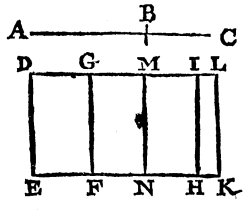
Demonſtration. LEt there be done as is before ſhewn, and it ſhall be ſhewn as in the precedent Propoſition, that D K is a Medial, and therefore D L rational incommenſurable in length to D E: and foraſmuch as the Rectangle under A C and B C is medial, and therefore the double thereof, to wit G K, G L [a] ſhall be alſo rational, incommenſurable in length to D E.

a) 23. 10.

But forasmuch as A C and B C are incommensurable in length, and as A C to B C, so the square of A C to the Rectangle under A C and B C; b) 10.10. b the square of A C shall be incommensurable to the Rectangle under A C and B C: But c) 16.10. c the compound of the squares of A C and B C is commensurable to the square of A C, those squares being commensurable, described of lines commensurable in power, and twice the Rectangle under A C and B C is commensurable to the said Rectangle under A C and B C: Therefore the compound of the squares of A C and B C, that is to say D K, is incommensurable to twice the Rectangle under A C and B C, that is to say G K: and therefore d) 10.10. d D L and G L being in the same rate as D K and G K, are incommensurable in length, and are shewn rational; therefore D L and G L are rational, commensurable in power only; and therefore e) 74.10. e D G the remainder is a Residual: I say it is a third Residual.

For we shall shew as in the 98 Proposition, that the whole D L is more in power then G L, its agreeing line, by the square of a line commensurable thereto in length: Therefore seeing that neither D L nor G L are commensurable in length to the rational D E, as hath been shewn, D G shall be a third Residual, by the Definition: Therefore the square, &c. Which was to be demonstrated.

PROP. 101. THEOR. 77.



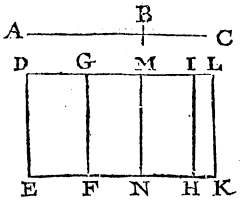
The square of a line minor A B, *applyed to a rational line* D E, *makes the breadth* D G *a fourth Residual.*

Demonstration FOrasmuch as the Compound of the squares of A C and B C, that is to say D K, their equal, is rational; a) 21.10. D L a shall be rational, commensurable in length to D E; and seeing that the Rectangle under A C and B C, and therefore the double thereof G K is medial: b) 23.10. G L b shall be rational, incommensurable in length to D E.

Again, forasmuch as D K rational, and G K irrational; to wit Medial, are incommensurable; c) 10.10. D L and G L c being in the same rate, are incommensurable in length; and being shewn rational, d) 74.10. D L and G L are rational, commensurable in power only: therefore d D G the remainder is a Residual: I say that it is a fourth Residual. For A C and B C being commensurable in power, their squares or their equal Rectangles D H and I K, shall be incommensurable; e) 10.10. and therefore e D I and I L incommensurable in length: And seeing that the Rectangle under D I and I L is equal to the square of M L, that is to say, to a quarter of the square of G L, as is demonstrated in the 98 Proposition, f) 19.10. D L f shall be more in power then G L, by the square of a line incommensurable in length thereto, seeing that to D L the greatest, there is applyed the Rectangle under D I and I L, equal to a quarter of the square of G L the least, wanting a square figure, and dividing D L in

in parts incommensurable in length D I and I L: Therefore seeing that D L the whole is more in power then the agreeing line G L, by the square of a line incommensurable in length thereto, and the whole D L is shewn commensurable in length to the rational D E: by the Definition D G shall be a fourth Residual: Therefore, &c. Which was to be demonstrated.

PROP. 102. THEOR. 78.



The square of a line A B, *which with a Space rational* G K, *makes the whole Medial, applyed to a rational line* D E, *makes the breadth* D G, *a fifth Residual.*

Demonstration LEt it be done as is before shewn; forasmuch as the compound of the squares of A C and B C, or his equal D K is Medial: a) 23.10. D L a shall be rational, incommensurable in length to D E; and forasmuch as the Rectangle under A C and B C, or the double thereof G K is rational, b) 21.10. G L b shall be rational, commensurable in length to D E.

And forasmuch as D K Medial, or irrational, and G K rational, are incommensurable; c) 10.10. D L and G L having the same rate, shall be incommensurable in length: But they are shewn rational: Therefore D L and G L are rational, commensurable in power only: therefore d) 74.10. d the remainder D G is a Residual: I say it is a fifth Residual.

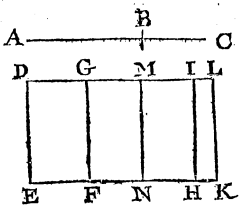
For as in the precedent, it shall be shewn that D L the whole is more in power then G L, the agreeing line, by the square of a line incommensurable in length thereto: Therefore seeing that the agreeing line G L, is shewn commensurable in length to the rational D E, by the Definition, D G shall be a fifth Residual: Therefore, &c. Which was to be demonstrated.

PROP. 103. THEOR. 79.

The square of a line A B, *making with a Space medial, the whole Medial, applyed to a rational line* D E, *makes the breadth* D G, *a sixth Residual.*

Demonstration LEt the same be done as in the former; forasmuch as, as well the compound of the squares of A C and B C, or D K his equal, as the Rectangle under A C and B C: and therefore the double thereof G K is medial, a) 23.10. as well a D L as G L, shall be rational, incommensurable in length to D E, and the Rectangle under A C and B C,

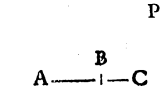
and the double thereof G K commensurable; G K [b] shall be incommensurable to the said compound, that is to say, to D K. And therefore [c] D L and G L having the same rate as D K and G K, shall be incommensurable in length. And they are shewn rational: Therefore D L and G L are rational, commensurable in power only; therefore [d] D G is a Residual: I say it is a sixth Residual.



For as in the 101 Proposition, we shall shew that the whole D L is more in power then the agreeing line G L, by the square of a line incommensurable in length thereto: Therefore seeing that neither D L nor G L is commensurable in length to the rational D E; by the Definition, D G shall be a sixth Residual. Therefore, &c. Which was to be demonstrated.

b) 14. 10.
c) 10. 10.
d) 74. 10.

PROP. 104. THEOR. 80.



The right line D E *commensurable in length to a Residual* A B, *is also a Residual, and of the same order as* A B.

Demonstration. FOr let it be as A B to D E, [a] so B C to E F; therefore [b] the whole A C shall be to the whole D F, as A B to D E; or B C to E F: forasmuch then as A B and D E are proposed commensurable in length; [c] A C and D F, and B C and E F, shall be also commensurable in length; and forasmuch as A C and B C are rational, their commensurables D F and E F shall be rational: Again, forasmuch as A C is to D F, as B C is to E F; and alternately, as A C to B C, so D F to E F: and A C and B C are commensurable in power only: Therefore D F and E F shall be also commensurable in power only, and being shewn rational, the remainder D E is a Residual: I say that it is of the same order as A B. For first let A C be more in power then B C, by the square of a line commensurable in length thereto; that being [d] D F shall be more in power then E F, by the square of a line commensurable in length thereto: Therefore if A C be commensurable in length to the rational exposed, to the end that A B may be a first Residual, D F shall be also commensurable in length to the rational exposed; seeing that as well the rational exposed, as D F, is commensurable in length to the same A C; Therefore, by the Definition, D E shall be a first Residual, to wit, of the same order as A B. But if B C be commensurable in length to the rational, in the same manner E F shall be commensurable in length to the rational; therefore the one and the other A B and D E, by the Definition shall be a second Residual.

Lastly, if neither the one or the other A C nor B C, be commensurable in length to the rational exposed; [e] also neither the one nor the other D F nor E F shall be commensurable in length to the same rational: Therefore as

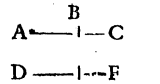
a) 12. 6.
b) 12. 5.
c) 10. 10.
d) 15. 10.
e) 14. 10.

as well the one as the other A B or D E, shall be a third Residual by the Definition.

Now let A C be more in power then B C; by the square of a line incommensurable in length thereto, that being [f] D F shall be more in power then E F, by the square of a line which shall be incommensurable in length thereto. Wherefore we shall shew as before, that D E is a fourth, fifth, or sixth Residual. Therefore, &c. Which was to be demonstrated.

f) 15. 10.

PROP. 105. THEOR. 81.



The right line D E, *commensurable to a Medial Residual* A B, *is also a Residual Medial, and of the same order as* A B.

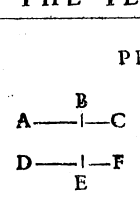
Demonstration. FOr as A B is to D E, [a] so let B C be to E F: Therefore the whole A C shall be to the whole D F, as A B to D E, and B C to E F: forasmuch then as A B and D E are proposed commensurable, [b] B C and E F, and A C and D F shall be also commensurable, and A C and B C being Medials D F and E F [c] commensurable to them, shall be also Medials.

Again, A C being to D F as B C to E F, and alternately, as A C to B C, so D F to E F; but A C and B C are commensurable in power only; Therefore [d] D F and E F shall be commensurable in power only: Wherefore D F and E F being shewn Medials, the remainder [e] D E shall be a Residual Medial: I say that it is of the same order as AB, for seeing that as A C to BC, so DF to E F: and as A C to B C, so the square of A C to the Rectangle under A C and B C, and as D F to E F, so the square of D F to the Rectangle under D F and E F: Therefore as the square of A C to the Rectangle under A C and B C, so the square of D F to the Rectangle under D F and E F: and alternately, as the square of A C to the square of D F: so the Rectangle under A C and B C, to the Rectangle under D F and E F: therefore the square of A C being commensurable to the square of D F, A C and D F being shewn commensurable, [f] the Rectangle under A C and B C shall be commensurable to the Rectangle under D F and E F: therefore if the Rectangle under A C and B C be rational, in such sort as A B may be a first Residual Medial, the Rectangle under D F and E F, commensurable thereto, shall be also rational; and therefore [g] D E shall be a Residual first Medial: but if the Rectangle under A C and B C be Medial, in such sort as A B may be a second Residual Medial: the Rectangle under D F and E F, commensurable thereto, [h] shall be also Medial: Therefore [i] D E shall be a second Residual Medial: Therefore, &c. Which was to be demonstrated.

a) 12. 6.
b) 10. 10.
c) 24. 10.
d) 10. 10.
e) 75, 76. 10.
f) 10. 10.
g) 75. 10.
h) 24. 10.
i) 76. 10.

PROP.

PROP. 106. THEOR. 82.



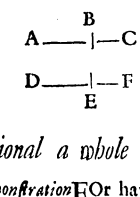
The right line D E, *commensurable to a line minor* A B, *is also a line minor.*

Demonstration. FOr let be done as is before shewn, in such sort as A B and B C, may have the same rate to D E and E F, as A C to D F, as in the former Proposition, D F and E F shall be commensurable to A C and B C, be it in length and power, or in power alone; and seeing that as A C is to D F, so B C is to E F; and alternately, as A C to B C, so D E to E F, [a] as the square of A C to the square of B C, so the square of D F to the square of E F; and in compounding, as the compound of the squares of A C and B C, to the square of B C, so the compound of the squares of D F and E F, to the square of E F: and by permutation, as the compound of the squares of A C and B C to the compound of the squares of D F and E F, so the square of B C to the square of E F: But the square of B C is commensurable to the square of E F, B C and E F being shewn commensurable: Therefore [b] the compound of the squares of A C and B C is commensurable to the compound of the squares of D F and E F: But the compound of the squares of A C and B C is proposed rational; Therefore [c] the compound of the squares of D F and E F is rational.

Again, as is shewn in the precedent Proposition, the Rectangle under A C and B C is commensurable to the Rectangle under D E and E F: but the Rectangle under A C and B C is proposed Medial; therefore [d] the Rectangle under D F and E F, shall be also Medial; and seeing that as A C is to B C, so D F is to E F: and A C and B C are incommensurable in power, D F and E F [e] shall be also incommensurable in power: therefore D F and E F being incommensurable in power, and making the compound of their squares rational, and the Rectangle under them Medial: [f] D E shall be a Minor: Therefore, &c. Which was to be demonstrated.

a) 22. 6.
b) 10. 10.
c) 9. d.
d) C. 24. 10.
e) 10. 10.
f) 77. 10.

PROP. 107. THEOR. 83.



The right line D E, *commensurable to a line* A B, *the which with a Space rational, makes the whole medial, the same is also a line making with a Space rational a whole medial.*

Demonstration. FOr having made the Construction as before, it shall be shewn as in the precedent Proposition, that the compound of the squares of A C and B C is commensurable to the compound of the squares of D F and E F, therefore the one being proposed medial, [a] the other shall be also Medial.

Again,

a) C. 24. 10.

Again, (as in the 105 Proposition,) it shall be shewn that the Rectangle under A C and B C is commensurable to the Rectangle under D F and E F; therefore the one being proposed rational, [b] the other shall be also rational: And lastly, as is shewn in the precedent Proposition, D F and E F are incommensurable in power; Wherefore D F and E F being incommensurable in power, and making the compound of their squares medial, and the Rectangle under them rational: [c] D E shall make with a Space rational, a whole medial. Which was to be demonstrated.

b) 9. d.
c) 78. 10.

PROP. 108. THEOR. 84.

The right line D E, *commensurable to the line* A B, *the which with a Space medial, makes a whole medial, the same is also a line making with a Space medial a whole medial.*

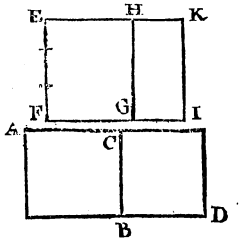
Demonstration. FOr having made the Construction as before, we shall shew as in the 106 Proposition, that the compound of the squares of A C and B C, is commensurable to the compound of the squares of D F and E F; therefore the compound of the squares of A C and B C being proposed Medial; [a] that of the squares of D F and E F shall be also medial.

Again, (as in the 105 Proposition,) it shall be shewn that the Rectangle under A C and B C is commensurable to the Rectangle under D F and E F; therefore the one being Medial, the other shall be also Medial. And as in the 106 Proposition, D F and E F shall be incommensurable in power.

Lastly, forasmuch as the compound of the squares of D F and E F is commensurable to the compound of the squares of A C and B C, as is said, and the Rectangle under D F and E F is commensurable to the Rectangle under A C and B C. But the compound of the squares of A C and B C, and the Rectangle under A C and B C are proposed incommensurable: Therefore the compound of the squares of D F and E F, and the Rectangle under D F and E F shall be incommensurable: Wherefore D F and E F being incommensurable in power, and the compound of their squares Medial, and incommensurable to the compound of their squares; [b] D E makes with a Space Medial a whole Medial: Therefore, &c. Which was to be demonstrated.

a) C. 24. 10.
b) 79. 10.

PROP. 109. THEOR. 85.



If a Space medial C D, *be cut from a rational* A D, *the right line which is equal in power to the rest of the Space* A B, *is one of these two Irrationals; to wit, a Residual, or Minor.*

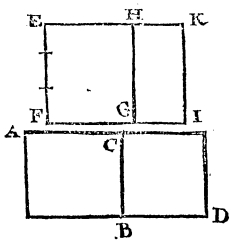
Demon-

Demonſtration FOr let the rational E F be expoſed, [a] to which let there be applyed the Rectangle E G equal to A B, and to G H the Rectangle H I, equal to A B, and to G H the Rectangle H I, equal to C D, in ſuch ſort as the whole E I may be equal to the whole A D; foraſmuch then as E I equal to A D rational is rational, [b] E K ſhall be rational, commenſurable in length to E F rational. a) 45. 1. b) 21. 10.

Again, foraſmuch as H I equal to C D medial is medial: H K ſhall be rational, incommenſurable in length to E F. Wherefore E K being commenſurable in length to E F: But H K incommenſurable in length to the ſame E F, [c] E K and H K ſhall be incommenſurable in length, and being rational, they ſhall be rational, commenſurable in power only; and therefore [d] the remainder E H is a Reſidual, and H K an agreeing line thereto: Therefore E K is more in power then H K, by the ſquare of a line which is thereto commenſurable in length, or incommenſurable. c) 23. 10. d) 74. 10.

If E K be more in power then H K, by the ſquare of a line which is commenſurable in length thereto, the whole E K being alſo ſhewn commenſurable in length to E F rational: by the Definition, E H ſhall be a firſt Reſidual: Wherefore the line equal in power to the Space E G, conteined under E F rational, and the firſt Reſidual E H; that is to ſay, the Space A B, equal to the ſame, [e] is a Reſidual: But if E K be more in power then H K, by the ſquare of a line which ſhall be incommenſurable thereto in length. The whole E K being ſhewn commenſurable in length to E F rational, according to the Definition, E H ſhall be a fourth Reſidual; Wherefore the line equal in power to the Space E G, conteined under E F rational, and E H a fourth Reſidual; that is to ſay, the Space A B equal thereto, [f] is a Minor: Therefore, &c. Which was to be demonſtrated. e) 92. 10. f) 95. 10.

PROP. 110. THEOR. 86.



If a Superficie rational C D, *be cut from a medial* A D, *they make two other irrationals; to wit, either a Reſidual firſt medial, or a line making with a Space rational a whole medial.*

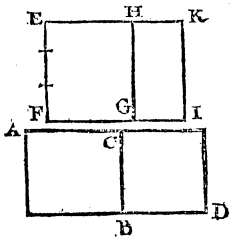
Demonſtration LEt be done the ſame Conſtruction as before: Foraſmuch as E I equal to the Medial A D, is medial, E K [a] ſhall be rational, incommenſurable in length to E F. a) 23. 10.

And foraſmuch as H I equal to the rational C D is rational: H K [b] ſhall be rational, commenſurable in length to E F: Therefore H K being commenſurable in length to E F: but E K incommenſurable in length to the ſame E F, H K and E K [c] ſhall be incommenſurable in length, and are rationals, they are then rational, commenſurable in power only: Therefore [d] the remainder E H is a Reſidual, and H K an agreeing line thereunto; b) 21. 10. c) 13. 10. d) 74. 10.

there-

therefore E K is more in power then H K, by the ſquare of a line commenſurable thereto in length, or incommenſurable, if commenſurable, and the agreeing line H K being ſhewn commenſurable in length to E F rational, by the Definition, E H ſhall be a ſecond Reſidual: Therefore the line equal in power to the Space E G, conteined under E F rational, and E H, a ſecond Reſidual, that is to ſay, A B equal thereto, [c] is a Reſidual firſt medial: But if E K be more in power then H K, by the ſquare of a line incommenſurable in length thereto, the agreeing line H K being ſhewn commenſurable in length to E F rational, E H ſhall be a fifth Reſidual, by the Definition: Therefore the line equal in power to the Space E G, conteined under E F rational, and the fifth Reſidual E H, that is to ſay A B, equal thereto, is a line [f] which with a rational makes a whole Medial: Therefore, &c, Which was to be demonſtrated. c) 95. 10. f) 96. 10.

PROP. 111. THEOR. 87.



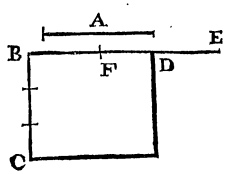
If a ſuperficies medial C D, *be cut from a ſuperficies medial* A D, *incommenſurable to the whole, there is made two other irrationals; to wit, a Reſidual ſecond medial, or a line making with a ſuperficies medial, a whole medial.*

Demonſtration FOr having made the ſame Conſtruction as before: Foraſmuch as E I and H I, equal to the Medials A D and C D are Medials; E K and H K [a] ſhall be rational, incommenſurable in length to E F; and A D and C D, that is to ſay, E I and H I being propoſed incommenſurable, [b] E K and H K having the ſame rate, ſhall be incommenſurable in length, and are rational; therefore they are rational, commenſurable in power only: therefore [c] E H remaining is a Reſidual, and H K an agreeing line thereto; therefore E K is more in power then H K, by the ſquare of a line commenſurable thereto in length, or incommenſurable. a) 23. 10. b) 10. 10. c) 74. 10.

If by the ſquare of a line which may be commenſurable thereto in length; both the one and the other E H and H K being ſhewn not commenſurable in length to E F rational, according to Definition, E H ſhall be a third Reſidual. Wherefore the line equal in power to the Space E G, conteined under the rational E F, and the third Reſidual E H; that is to ſay A B equal thereto, is a Reſidual ſecond Medial: But if E K be more in power then H K, by the ſquare of a line incommenſurable in length thereto, neither the one nor the other E K nor H K, being commenſurable in length to E F rational, as is demonſtrated by the Definition, E H ſhall be a ſixth Reſidual: Therefore the line equal in power to E G, conteined under E F rati-

onal, and E H a sixth Residual; that is to say, A B equal thereto, is that which with a Space Medial makes a whole Medial: Therefore, If, &c. Which was to be demonstrated.

PROP. 112. THEOR. 88.



The line A, *called* Apotome *or* Residual, *is not the same as the line of two names, or* Binomial.

Demonstration. FOr let A be a Binomial (if possible) and having exposed the rational B C, [a] let there be applyed to B C the Rectangle C D, equal to the square of A; Forasmuch then as A is a Residual, the breadth B D [b] shall be a first Residual: Let then D E be an agreeing line thereto; Therefore by the Definition of the first Residual, B E and D E shall be Rational, commensurable in power only, and B E shall be more in power then D E, by the square of a line which shall be commensurable in length thereto; and B E shall be commensurable in length to B C rational: Again A being also proposed to be a Binomial; [c] the same breadth B D shall be a first Binomial: Let B F be his greatest name: Therefore by the Definition of the first Binomial B F and F D shall be rational, commensurable in power only, and B F shall be more in power then F D, by the square of a line commensurable in length thereto, and B F shall be commensurable in length to the rational B C; therefore as well B E, as B F being commensurable in length to B C; [d] B E and B F shall be also commensurable in length to one another: Wherefore the whole B E, being commensurable in length to his part B F, the same B E shall be also commensurable in length to his part B F, the same B E shall be also commmensurable in length to the other part F E; and therefore B E being Rational, F E shall be also Rational.

And forasmuch as the two B E and F E, are commensurable in length, B E is incommensurable in length to D E (B E and D E being Rational, commensurable in power only,) [e] the remainder F E shall be incommensurable in length to the same D E: But as well F E as D E, is shewn Rational: Therefore F E and D E are Rational, commensurable in power only: Wherefore the remainder F D, [f] is a Residual, and therefore Irrational. But it is also shewn Rational, which is absurd: Therefore, &c. Which was to be demonstrated.

a) 45. 1.
b) 98. 10.
c) 61. 10.
d) 12. 10.
e) 14. 10.
f) 74. 10.

COROL-

COROLLARIE.

From these things demonstrated, may be gathered, that the Residual, and the other Irrationals following the same Residual, are neither Medials, nor the same to one another.

For the square of a Medial *applyed to a* Rational, *makes* [a] *the breadth rational, incommensurable in length to the* Rational *proposed.*

But the square of the Residual *applyed to a rational line, makes* [b] *the breadth, a first* Residual.

And the square of the Residual *first* Medial, *applyed to a rational line,* [c] *makes the breadth a second* Residual.

But the square of the Residual *second* Medial, *applyed to a rational line,* [d] *makes the breadth a third* Residual.

Furthermore the square of the Minor applyed to a rational line, [e] *makes the breadth a fourth* Residual.

But the square of a line, the which with a space rational, makes a whole Medial, *applyed to a rational line,* [f] *makes the breadth a fifth* Residual.

Lastly, the square of a line, which with a Space Medial *makes a whole* Medial, [g] *applyed to a rational line, makes the breadth a sixth* Residual.

Therefore these breadths being different from the breadth of the Medial, *and to one another; I say, from the breadth* Medial, *being the one is rational, and the others irrational: But to one another; forasmuch as they are not* Residuals *of the same order: it is manifest that the* Residual *and the other Irrationals following do differ among themselves, and from the* Medial.

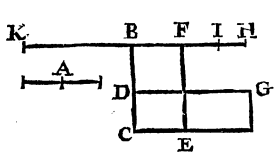
But forasmuch as it is shewn in this Theorem, that the Residual *is not the same as the* Binomial; *and that the squares of the* Residual, *and of the other five Irrationals following, applyed to a rational line, makes the breadths a First, Second, Third, Fourth, Fifth, and Sixth* Residual; *but the square of the* Binomial, *and the other five Irrationals following applyed to a rational line makes the breadth a First, Second, Third, Fourth, Fifth, and Sixth* Binomial. *It is manifest that the breadths of the* Residual, *and the other five Irrationals following, are not the same as the breadths of the* Binomials, *and the five other Irrationals following, forasmuch as not any*

a) 23. 10.
b) 98. 10.
c) 99. 10.
d) 100. 10.
e) 101. 10.
f) 102. 10.
g) 103. 10.

Reſidual, *is the ſame as any* Binomial: *Therefore the* Reſidual *and the others following, do differ from the* Binomial, *and the others following: Wherefore as well thoſe as the others, do differ from the* Medial, *in ſuch ſort as any whatſoever rational line being propoſed, there will be* 13 *Irrational lines differing from one another, of which we have diſcourſed hitherto, and they are theſe following.*

1 The Medial line.
2 The Binomial line, or line of two names.
3 The firſt Bimedial.
4 The ſecond Bimedial.
5 The line Major.
6 The line equal in power to a Rational and a Medial.
7 The line equal in power to two Medials.
8 The Reſidual, or Apotome.
9 The Reſidual firſt Medial.
10 The Reſidual ſecond Medial.
11 The line Minor.
12 The line making with a Superficie Rational, a whole Medial.
13 The line making with a Superficie Medial, a whole Medial.

PROP. 113. THEOR. 89.



The ſquare of a rational line A, *being applyed to a Binomial line* B C, *make the breadth* B F *Reſidual, of which the names are commenſurable, and proportional to the names* B D *and* D C, *of the Binomial* B C, *and over and above the Reſidual* B F, *is of the ſame order as the Binomial* B C.

Demonſtration FOr [a] to D C the leſſer name, let there be applyed C G, equal to the ſquare of A; and therefore to B E, making D G the breadth; and let B H be equal to D G and B E, C G being equal, as B C ſhall be to D C, ſo D G, or his equal B H, to B F, and in dividing, as

a) 45. 1.

as B D to D C, ſo H F to F B: But B D is greater then D C; therefore H F ſhall be greater then F B. Let F I be equal to F B, and let it be as H I to I F, ſo F B to B K: Therefore in compounding H F ſhall be to I F, or to his equal F B, as F K to B K: But as H F to B F, ſo B D is ſhewn to be to D C: Therefore alſo as B D ſhall be to D C, ſo F K to B K: But B D and D C (names of the Binomial B C) are rational, commenſurable in power only; Therefore [b] F K and B K are commenſurable in power only. b) 10. 10.

Again, H F being to B F, as F K to B K, [c] the Antecedents H F and F K together, to wit, H K the whole, ſhall be to F B and B K conſequents; that is to ſay, to F K the whole, as F K to B K: therefore F K is a mean proportional between H K and B K; therefore [d] as H K the firſt, to B K the third, ſo the ſquare of H K the firſt, to the ſquare of F K the ſecond: But foraſmuch as C G rational (being equal to the ſquare of A rational, applyed to D C rational,) makes [e] the breadth D G rational, commenſurable in length to D C; H B equal to D G, ſhall be alſo rational, commenſurable in length to D C, and it being ſhewn that as B D is to D C, ſo F K is to B K: But as F K is to B K, ſo H K to F K; alſo as B D ſhall be to D C, ſo H K to F K; therefore [f] as the ſquare of B D to the ſquare of D C, ſo the ſquare of H K to the ſquare of F K. But the ſquare of B D is commenſurable to the ſquare of D C: (B D & D C names of the Binomial B C rational, commenſurable in power;) Therefore [g] the ſquare of H K ſhall be alſo commenſurable to the ſquare of F K; but as the ſquare of H K to the ſquare of F K, ſo H K to B K: therefore [h] H K is commenſurable in length to B K: and therefore [i] to the reſt B H: But B H is ſhewn rational, therefore H K commenſurable thereto, is rational; and therefore B K commenſurable to H K, is rational: Therefore F K being ſhewn commenſurable in power only to B K, F K ſhall be alſo rational; therefore F K and B K being rationals, and ſhewn commenſurable in power only; [k] the remainder B F ſhall be a Reſidual; and B K ſhall be an agreeing line thereto, which is firſt propoſed. c) 12. 5. d) C. 20. 6. e) 21. 10. f) 22. 6. g) 10. 10. h) 10. 10. i) C. 16. 10. k) 74. 10.

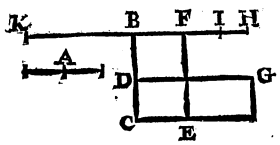
But foraſmuch as the whole H K is ſhewn commenſurable in length to his part B K, B K and B H [l] ſhall be commenſurable in length: Therefore B H being ſhewn commenſurable in length to D C, B K ſhall be alſo commenſurable in length to D C, as appeares by the 12 Propoſition of this Book. l) 16. 10.

And it being ſhewn that as B D is to D C, ſo F K to B K, and by permutation, as B D to F K, ſo D C to B K, but D C and B K are ſhewn commenſurable in length: Therefore [m] B D and F K ſhall be alſo commenſurable in length: Therefore F K being commenſurable in length to B D, and B K to D C, F K and B K names of the Reſidual B F, ſhall be commenſurable in length to B D and D C, names of the Binomial B C, which is in the ſecond place propoſed. m) 10. 10.

Alſo it being demonſtrated, that as B D is to D C, ſo F K is to B K: F K and B K, names of the Reſidual, ſhall be in the ſame rate as B D and D C, names of the Binomial B C, which is in the third place propoſed.

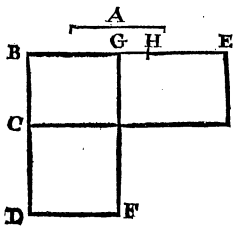
Laſtly, B D is more in power then D C, by the ſquare of a line commenſurable in length thereto, or incommenſurable; if it be more in power by the ſquare of a line commenſurable in length thereto; B D being to D C, as E K to B K, [n] F K ſhall be alſo more in power then B K, by the ſquare of a line which ſhall be commenſurable in length thereto, and if B D be more in power then D C, by the ſquare of a line commenſurable thereto in length, F K [o] ſhall be more in power then B K, by the ſquare of a line n) 15. 10. o) 15. 10.

line incommenſurable thereto in length; and if B D be commenſurable in length to the rational expoſed F K; ſhewn commenſurable in length to BD, it ſhall be alſo commenſurable in length to the ſame rational, as appears by the 12 Propoſition of this Book. But if D C be commenſurable in length to the rational B K; by the ſame reaſon it ſhall be commenſurable in length to the ſame rational; and laſtly, if neither BD nor DC



p) 14. 10. be commenſurable in length to the rational, neither F K p nor B K alſo ſhall be commenſurable in length to the rational propoſed: Therefore BF Reſidual is of the ſame order as B C Binomial, as appeares by the ſecond and third Definitions, which is in the fourth place propoſed: Therefore, &c. Which was to be demonſtrated.

PROP. 114. THEOR. 90.



The ſquare of a rational line A, *being applyed to a Reſidual* B C, *makes the breadth* B E *a Binomial, whoſe names are commenſurable to the names* B D *and* C D, *of the Reſidual* B C, *and in the ſame rate: and over and above, the Binomial* B E *is of the ſame order as the Reſidual* B C.

Demonſtration FOr let there be alſo applyed to the whole B D the Rectangle B F, equal to the ſquare of A; that is to ſay, to C E, making B G the breadth; foraſmuch as B F and C E are equal, a as B E, ſhall be to B G, ſo B D to B C: and by converſion of reaſon, as B E to G E, ſo B D ſhall be to C D.

a) 14, 16. 6.

Let E G alſo be divided in H, according to the rate of B E to G E, to the end that E A may be to H G as B E to G E: but as B E the whole is to G E the whole, ſo E H cut off from B E, to H G cut off from G E: B H the remainder of B E, ſhall be alſo to H E the remainder of G E, as the whole B E to the whole G E.

But as B E was to G E, ſo E H to H G; Therefore alſo as B H ſhall be to H E, ſo E H to G H; therefore H E is a mean proportional between B H and G H: Wherefore b as B H the firſt, to G H the third, ſo the ſquare of B H the firſt, to the ſquare of H E the ſecond: But as B D to C D, ſo B E to G E, that is to ſay, B H to H E, and B D to C D, names of the Reſidual B C, being rational, commenſurable in power only; c B H and H E are

b) C. 20. 6.

c) 10. 10.

are alſo commenſurable in power, and therefore the ſquares of B H and H E commenſurable: Therefore B H and G H being in the ſame rate as the ſquares of B H and H E, as hath been demonſtrated, are commenſurable in length; and the whole B H being commenſurable in length to its part G H; d it ſhall be alſo commenſurable in length to the other part B G.

d) C. 16. 10.

But B D being rational, to wit, the greateſt name of the Reſidual B C, and the Rectangle B F rational, being equal to the ſquare of A rational; e its breadth B G, ſhall be rational, commenſurable in length to B D: Therefore f B H is commenſurable in length to the ſame rational B D; and therefore is rational, B H and B G having been ſhewn commenſurable in length.

e) 21. 10.

f) 12. 10.

And ſeeing that B H and H E are ſhewn commenſurable in power only, and B E rational, H E commenſurable thereto, is alſo rational: Therefore B H and H E are rational, commenſurable in power only; Therefore g B E is a Binomial, which is firſt propoſed.

g) 37. 10.

But it being ſhewn that B H is to H E, as B D to C D, and alternately, B H to B D, as H E to C D; but B H being ſhewn commenſurable in length to B D, h H E ſhall be alſo commenſurable in length to C D: Wherefore B H and H E, names of the Binomial B E, are commenſurable in length to B D and C D, names of the Reſidual B C, which is in the ſecond place propoſed.

h) 10. 10.

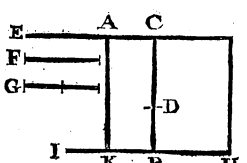
They are alſo in the ſame rate. It being demonſtrated that B H is to H E, as B D to C D, which is in the third place propoſed.

Laſtly, either B D is more in power then C D, by the ſquare of a line commenſurable in length thereto, or is incommenſurable: if commenſurable B H ſhall be alſo more in power then H E, by the ſquare of a line which ſhall be commenſurable in length thereto; and if incomenſurable, it ſhall be more in power, by the ſquare of a line which ſhall be incommenſurable thereto in length: and if B D be commenſurable in length to the rational expoſed, B H i ſhall be ſo alſo, being commenſurable in length to B D, commenſurable in length to the Rational. But if C D be commenſurable in length to the Rational; A E by the ſame reaſon, ſhall be ſo alſo. If neither the one nor the other, B D nor C D, be commenſurable in length to the Rational, alſo k neither B H nor H E, ſhall be commenſurable in length thereto: Therefore by the Second and the Third Definitions, B E is a Binomial of the ſame order as B C, the Reſidual which is in the fourth place propoſed: Therefore, The Square, &c. Which was to be demonſtrated.

i) 12. 10.

k) 14. 10.

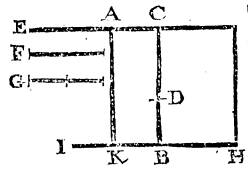
PROP. 115. THEOR. 91.



If a Space A B, *be conteined under a Reſidual* A C, *and under a line called a Binomial* C B, *whoſe names* C D *and* D B, *are commenſurable to the names* C E *and* A E,

A E, *of the Reſidual* A C, *and in the ſame rate* (C D *to* D B, *as* C E *to* A E,) *the right line* F, *equal in power to the ſame Space* A B, *is a Rational.*

Demonſtration FOr having expoſed the Rational G, let [a] there be applyed to the Binomial C B, the Rectangle C H, equal to the ſquare of G: Therefore [b] the breadth B H, ſhall be a Reſidual: Therefore the names H I and B I are commenſurable in length to the names C D and D B, and in the ſame reaſon; to wit, H I to B I, as C D to D B; and therefore as E to A E, and by permutation as the whole H I to the whole C E: ſo the part cut off B I, to the part cut off A E: Therefore [c] the remainder B H ſhall be to the remainder A C, as the whole H I is to the whole C E: But H I is commenſurable in length to C E, as well H I as C E being commenſurable in length to C D: Therefore [d] B H is commenſurable in length to A C, and therefore H C is commenſurable to B A: H C being to B A, as H B to A C. But H C being equal to the ſquare of the Rational G, is Rational: Therefore B A commenſurable thereto, ſhall be Rational: Therefore F equal in power to the ſame, ſhall be Rational: Therefore, &c. Which was to be demonſtrated.

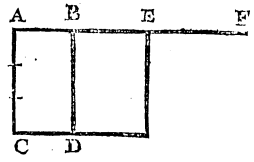


a) 45. 1.
b) 113. 10.
c) 19. 5.
d) 10. 10.

COROLLARIE.

From this it is manifeſt, that a Space rational, may be conteined under two Irrational lines: For A B *conteined under* A C *and* C B *a Reſidual and a Binomial, which are Irrationals, is ſhewn to be rational.*

PROP. 116. THEOR. 92.



Of the Medial line A B, *there are made an infinite number of Irrationals, and not one of them the ſame with any one of the Antecedents.*

Demonſtration FOr the Rational A C being propoſed, let the Space A D be conteined under A B Medial, and A C rational; Therefore A D conteined under A D rational, and A B irrational, is irrational:

Let

of A C and C G, [b] equal to the ſaid ſquare of A G, ſhall be alſo the quadruple of the ſquare of F G: Therefore the three ſquares of A C, C G, and G F, together, ſhall be the quintuple of the ſquare of G F; but the ſquares of C G and of G F together, ſides of the Hexagon and of the Decagon, are [c] equal to the ſquare of C H, the ſide of the Pentagon; therefore the ſquares of A C and of C H, are quintuples of the ſquare of G F. Which was propoſed. Now

b) 47. 1.

c) 10. 13.

Let I K be the diameter of the ſphere conteining the Dodecahedron and the Icoſahedron; and let A B C H E be one Pentagon of the ſame Dodecahedron, and L M N a triangle of the Icoſahedron: I ſay that one and the ſame circle conteineth the Pentagon A B C H E, and the triangle L M N; that is to ſay, that the circles which circumſcribe A B C H E and L M N, are equal, and that it is no other than one and the ſame; for having drawn A C ſubtending the angle B of the Pentagon, [d] A C ſhall be the ſide of the cube inſcribed in the ſame ſphere.

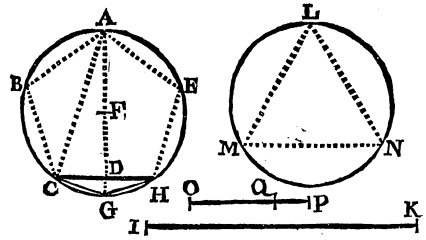
d) C. 17. 13

Let there be taken O P, ſo as that the ſquare of I K the diameter of the ſphere, may be quintuple the ſquare of O P, and O P equal to the ſemidiameter of the circle where the Icoſahedron is inſcribed: And let O P [e] be cut in extream and mean proportion at Q; O Q the greateſt ſegment ſhall be the ſide of the Decagon inſcribed in the ſame circle, whereof O P [f] is the ſemidiameter or ſide of the Hexagon. But [g] C A being divided by extream and mean proportion, his greateſt ſegment is

e) C. 16. 13.

f) C. 9. 13.

g) 7. 13.



A B, the ſide of the Pentagon: Therefore [h] as A C the whole, to O P the whole, ſo the greateſt ſegment A B, to the greateſt ſegment O Q: Therefore [i] as the ſquare of A C to the ſquare of O P, ſo the ſquare of A B to the ſquare of O Q, and [k] at the triple of the ſquare of A C to the quintuple of the ſquare of O P, ſo the triple of the ſquare of A B, to the quintuple of the ſquare of O Q. But the triple of the ſquare of A C is equal to the quintuple of the ſquare of O P ([l] the ſquare of I K the diameter of the ſphere being put equal, as well to the triple of the ſquare of A C the ſide of the cube, as to the quintuple of the ſquare of O P:) Wherefore the triple of the ſquare of A B ſhall be alſo equal to the quintuple of the ſquare of O Q: But [m] M L the ſide of the triangle of the Icoſahedron is equal to the ſide of the Pentagon inſcribed in the circle, whereof O P is the ſemidiameter: Therefore the ſquare of the ſaid ſide being [n] equal to the ſquares of O P and O Q, the ſides of the Decagon and of the Hexagon, five times the ſquare of M L ſhall be equal to five times the ſquare of O P, [o] five times that of O Q, or to three times the ſquare of B A, and [p] five times the ſquare of A C; therefore the ſquares of B A and A C

h) 2. 14.

i) 22. 6.

k) 4. 5.

l) 15. 13.

m) C. 16. 13

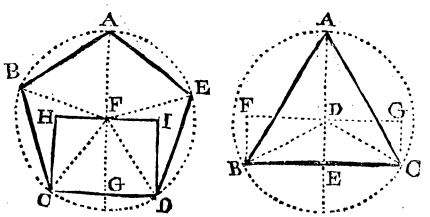
n) 10. 13.

being quintuples of the ſquare of the ſemidiameter F A (as is demonſtrated) three times the ſquare of AB and three times the ſquare of AC ſhall be equal to fifteen times the ſquare of FA the ſemidiameter: But five times the ſquare of ML the ſide of the equilateral triangle, is equal to fifteen times the ſquare of the ſemidiameter of the circle L M N (each ſquare [o] of ML being triple the ſquare of the ſemidiameter.) Wherefore the three ſquares of B A and A C being equal to five of M L, fifteen of F A the ſemidiameter, ſhall be equal to fifteen of the ſemidiameter of the circle L M N: Wherefore each ſquare of them ſhall be equal to each ſquare, and the ſemidiameter to the ſemidiameter: Therefore the circles A B C and L M N ſhall be equal: Which was to be demonſtrated.

o) 12. 13.

PROP. 4. THEOR. 4.

If from the center F *of the circle, circumſcribing the Pentagon of the Dodecahedron* A B C D E, *be drawn a right line* F G, *perpendicular to one ſide* C D, *of the ſame pentagon, thirty times the rectangle conteined under the ſaid ſide and the the perpendicular, ſhall be equal to the Superficies of the Dodecahedron.*



Demonſtration FOr from the center F, having drawn the right lines F A, F B, F C, F D, and F E, to all the angles of the Pentagon, the Pentagon ſhall be divided into five equal triangles, as [a] it appeares, all the ſides being equal, and all the baſes; let there be made the rectangle C I, conteined under C D and F G: Foraſmuch [b] as the rectangle C I is the double of the triangle F C D, and by conſequents the double of each of the four remaining triangles, equal to F C D; the rectangle C I taken five times, ſhall be equal to ten ſuch triangles; that is to ſay, to two Pentagons of the Dodecahedron (five triangles being equal to one Pentagon;) therefore the ſextuple of the rectangle C I (that is to ſay, ſix rectangles, ſuch as is C I,) taken five times, to wit, thirty times the rectangle C I ſhall be alſo equal to the ſextuple of the two Pentagons (that is to ſay, to twelve Pentagons) to wit, to the whole ſuperficies of the Dodecahedron: Therefore, If from the center, &c. Which was to be demonſtrated.

a) Cor. 8. 1.
b) 4. 1 1.

In like manner may be ſhewn, that if from the center of the circle circumſcribing the triangle of the Icoſahedron A B C, be drawn a [illegible]

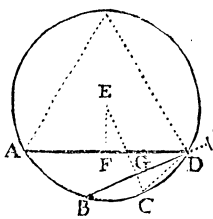
dicular D E on one ſide, to wit, on B C; the rectangle B G conteined under B C and D E taken thirty times, ſhall be equal to the ſuperficies of the Icoſahedron.

For the rectangle B G being the double of the triangle D B C, that is to ſay, equal to the ſaid triangle taken two times, the rectangle B G taken thirty times ſhall be equal to the triangle D B C taken ſixty times; but ſixty ſuch triangles as D B C make twenty triangles of the Icoſahedron, ſeeing that three of them make one, as it appeares in the triangle A B C, where the ſides D A, D B, and D C, are equal, and the three baſes A B, B C, and C A, equal by ſuppoſition: Therefore B G taken thirty times, ſhall be equal to the whole ſuperficies of the Icoſahedron.

COROLLARIE.

Whence it followes, that as the rectangle C I *is to the rectangle* B G, *ſo the whole ſuperficies of the Dodecahedron, to the ſuperficies of the Icoſahedron.*

PROP. 5. THEOR. 5.



As the Superficies of the Dodecahedron is to the Superficies of the Icoſahedron, ſo the ſide of the cube is to the ſide of the Icoſahedron inſcribed in one and the ſame ſphere.

Conſtruction IN the circle A B C D, [a] enclosing the Pentagon of the Dodecahedron, and the triangle of the Icoſahedron; let B D be the ſide of the Pentagon, and A D the ſide of the triangle; and from the center E let there be drawn E F and E G, perpendicular to A D and B D, and let F G be prolonged to the circumference in the point C, and let C D be drawn, E C ſhall cut the arch B C D in two equal parts, and C D ſhall be the ſide of the Decagon, as [b] we have ſhewn.

a) 13. 14.
b) 1. 14.

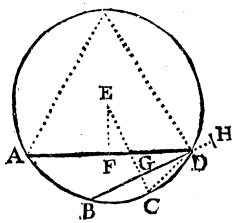
And let B D be prolonged to H, in ſuch manner as that B H be equal to the ſide of the cube inſcribed in the ſame ſphere.

I ſay now that the ſuperficies of the Dodecahedron is to the ſuperfices of the Icoſahedron, as B H the ſide of the cube, is to A D the ſide of the Icoſahedron.

Demonſtration FOr E C being the ſide of the Hexagon, and D C the ſide of the Decagon, inſcribed in one and the ſame circle, E C and C D, taken as one only line, [c] ſhall be divided by extream and mean proportion, and E C the greateſt ſegment, and [d] E G is the half of the whole compounded of E C and C D, and E F is the half of the greateſt ſegment E C ([e] the ſemidiameter being cut in two equal parts by A D) and E C being divided by extream and mean proportion, the half E F ſhall be the greateſt ſegment, as is manifeſt; for the half of the greateſt ſegment and the half of the leſſer, makes the half of the whole, [f] their whole being to one another as their parts: But B H the ſide of the cube, being divided by extream and mean proportion, [g] B D the ſide of the Dode-

c) 9. 13.
d) 1. 14.
e) C. 12. 13.
f) 15. 5.
g) C. 17. 13.

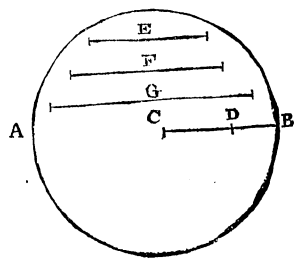
Dodecahedron shall be his greateſt ſegment; therefore [b] as the whole B H to B D his greateſt ſegment, ſo the whole E G ſhall be to E F his greateſt ſegment; therefore [i] the rectangle under B H and E F extreams, ſhall be equal to the rectangle under B D and E G means, and the rectangles under B H and E F [k] being to the rectangle under A D and E F, as B H is to A D; and as B H ſhall be to A D, ſo the rectangle under B D and E G (ſhewn equal to the rectangle under B H and E F) ſhall be to the rectangle under A D and E F; therefore [l] the rectangle under B D and E G being to the rectangle under A D and E F, as the ſuperficies of the Dodecahedron to that of the Icoſahedron, alſo as B H the ſide of the cube, ſhall be to A D the ſide of the Icoſahedron, ſo the Superficies of the Dodecahedron ſhall be to the Superficies of the Icoſahedron. Which was to be demonſtrated.



b) 2. 14.
i) 16. 6.
k) 1. 6.
l) Cor. 4. 14.

PROP. 6. THEOR. 6.

If a right line C D, *be divided by extream and mean proportion at* D, *as the ſquare of the whole* C B, *and the ſquare of the greateſt ſegment* C D, *are to the ſquares of the whole* C B, *and the leaſt ſegment* D B, *ſo the ſquare of the ſide of the cube* G, *is to the ſquare of the ſide of the Icoſahedron* F, *inſcribed in one and the ſame ſphere with the cube.*



Demonſtration FOr E being the ſide of the Pentagon, and F the ſide of the triangle inſcribed in the circle propoſed: If G the ſide of the cube be divided by extream and mean proportion, [a] E ſhall be his greateſt ſegment: Wherefore F being the ſide of the triangle inſcribed in the given circle, [b] his ſquare ſhall be triple the ſquare of the ſemidiameter C B, and [c] the ſquares of C B and D B are triple the ſquare of C D: Therefore the ſquare of F is to the ſquare of C B, as the two ſquares of C B and D B are to the ſquare of C D; and by permutation, the ſquare of F ſhall be to the two ſquares of C B and D B, as the ſquare of C B to the ſquare of C D: But as the ſquare of C B is to the ſquare of C D, ſo the ſquare of G to the ſquare of E (ſeeing that as [d] C B the whole is to his greateſt ſegment C D, ſo G the whole, is to E his greateſt ſegment: Therefore [e] as the ſquare of C B ſhall be to the ſquare of C D, ſo the ſquare G to the ſquare of E;) therefore [f] as the ſquare of F is to the ſquares of C B and D B, ſo the ſquare of G to the ſquare of E; and alternately, as the ſquare of F to the ſquare of G, ſo the ſquares of C B and D B are to the ſquare of E, which is the ſide of the Pentagon, [g] equal to the ſquares of C B and C D, the ſides of the Hexagon and of the Decagon which are inſcribed in one and the ſame circle: Therefore the ſquare of G the ſide of the cube, is to the ſquare of F the ſide of the Icoſahedron, as the ſquares of C B and C D are to the ſquares of C B and D B: Wherefore, If a right line, &c. Which was to be demonſtrated.

a) C. 17. 13.
b) 12. 13.
c) 4. 13.
d) 2. 14.
e) 22. 6.
f) 11. 5.
g) 10. 13.

PROP. 7. THEOR. 7.

As the ſide of the Cube is to the ſide of the Icoſahedron, ſo the ſolid of the Dodecahedron is to the ſolid of the Icoſahedron inſcribed in one and the ſame ſphere.

Demonſtration FOraſmuch as the Pentagon of the Dodecahedron, and the triangle of the Icoſahedron inſcribed in one and the ſame ſphere, are [a] inſcribed in one and the ſame circle or equal circles; and that in the ſphere the equal circles are equally diſtant from the center; for the perpendiculars drawn from the center of the ſphere, to the plain of circles, are equal, and fall in the centers of the ſame circles: Therefore the perpendiculars drawn from the center of the ſphere to the center of the circle conteining the Pentagon of the Dodecahedron, and the triangle of the Icoſahedron, are equal: Wherefore the pyramids having for baſes the Pentagons of the Dodecahedron, and the triangles of the Icoſahedron, are of the ſame height: But [b] the pyramids of the ſame height are to one another as their baſes: Therefore as the Pentagon to the triangle, ſo the pyramid which hath for baſe the Pentagon, to the Dodecahedron, and the center of the ſphere for its top; is to the pyramid, whereof the baſe is the triangle of the Icoſahedron, and the center of the ſphere the top.

Therefore alſo as 12 Pentagons are to 20 triangles, ſo 12 Pyramids having the baſes Pentagonal, are to 20 Pyramids having the baſes triangular: But 12 Pentagons are the ſuperficies of the Dodecahedron, and 20 triangles are the ſuperficies of the Icoſahedron: Therefore as the ſuperficies of the Dodecahedron to the ſuperficies of the Icoſahedron, ſo 12 Pyramids having the baſes pentagonal, to 20 Pyramids, having the baſes triangular, and 12 Pyramids having the baſes Pentagonal, are the Solid of the Dodecahedron, and 20 pyramids having the baſes triangular, are the ſolid of the Icoſahedron.

Therefore as the ſuperficies of the Dodecahedron to the ſuperficies of the Icoſahedron, ſo the Solid of the Dodecahedron to the Solid of the Icoſahedron: But as the ſuperficies of the Dodecahedron to the ſuperficies of the Icoſahedron, ſo is ſhewn the ſide of the cube to the ſide of the Icoſahedron; ſo the Solid of the Dodecahedron to the Solid of the Icoſahedron. Which was to be demonſtrated.

a) 5. 14.
b) 5, 6. 12.

COROLLARIE.

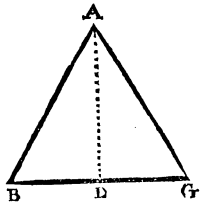
The Solid of a Dodecahedron is to the Solid of an Icoſahedron, as the ſuperficies of

of the one are to the superficies of the other, being described in one and the self same sphere.

Namely, as the side of the cube is to the side of the Icosahedron, as was before manifest, for they are resolved into Pyramids of one and the self same altitude.

PROP. 8. THEOR. 8.

If the side AB *of an equilater triangle* ABG, *be rational, the superficies* ABG *shall be irrational, called Medial.*



Demonstration. Forasmuch as the line AB is in power sesquitertia to the line AD, [a] of what parts the line AB conteineth in power 12 of the same parts, the line AD conteineth in power 9: Wherefore the residue BD conteineth in power of the same parts 3. (For the line AB conteineth in power the lines AD and BD [b]: Wherefore the lines AD and DB are rational and commensurable to the rational line AB [c]: But forasmuch as the power of the line AD is to the power of the line DB in proportion as 9 a square number, is to 3 a number not square: Therefore they are not in the proportion of square numbers [d], and therefore they are not commensurable in length [e]. Wherefore that which is conteined under the lines AD and DB, which are rational lines commensurable in power only, is Medial [f]: But that which is conteined under the lines AD and DB is double to the triangle ABD [g]. Wherefore that which is conteined under the lines AD and DB, is equal to the whole triangle ABG (which is double to the triangle ABD [h].) Wherefore the the triangle ABG is Medial. If therefore the side, &c. Which was required to be proved.

a) C. 12. 13.
b) 47. 1.
c) 6. 10.
d) C. 25. 8.
e) 9. 10.
f) 22. 10.
g) 41. 1.
h) 1. 6.

COROLLARIE.

If an Octohedron and a Tetrahedron be inscribed in a Sphere, whose Diameter is Rational, their Superficies shall be Medial.

For those Superficies consisting of equilater triangles, whose sides are commensurable to the diameter which is rational [i]; and therefore they are rational. But they are commensurable in power onely to the perpendicular line, and therefore they contein a Medial triangle as was before manifested.

i) 13, 14. 13.

PROP. 9. THEOR. 9.

If a Tetrahedron and an Octohedron be inscribed in one and the self same sphere, the base of the Tetrahedron shall be sesquitertia to the base of the Octohedron, and the Superficies of the Octohedron shall be sesquialtera to the Superficies of the Tetrahedron.

Demon-

Demonstration. Forasmuch as the diameter of the sphere is in power sesquialtera to the side of the Tetrahedron [a], and the same diameter is in power double to the side of the Octohedron [b]: Therefore of what parts the diameter conteineth in power 6, of the same the side of the Tetrahedron conteineth in power 4, and of the same the side of the Octohedron conteineth in power 3. Wherefore the power of the side of the Tetrahedron is to the power of the side of the Octohedron in the same proportion that 4 is to 3, which is sesquitertia. And like triangles (which are the bases of the Solids) described of those sides, shall have the one to the other the same proportion that the squares made of those sides shall have. For both the triangles are the one to the other, and also the squares are the one to the other in double proportion of that in which the sides are [c]: Wherefore of what parts one base of the Tetrahedron was 4, of the same are four bases of the Tetrahedron 16. Likewise of what parts of the same one base of the Octohedron was 3, of the same are eight bases of the Octohedron 24. Wherefore the bases of the Octohedron are to the bases of the Tetrahedron, in that proportion that 24 is to 16, which is sesquialtera. If therefore a Tetrahedron and an Octohedron be inscribed in one and the self same sphere, the base of the Tetrahedron shall be sesquitertia to the base of the Octohedron, and the superficies of the Octohedron shall be sesquialtera to the superficies of the Tetrahedron: Which was required to be proved.

a) 13. 13.
b) 14. 13.
c) 20. 6.

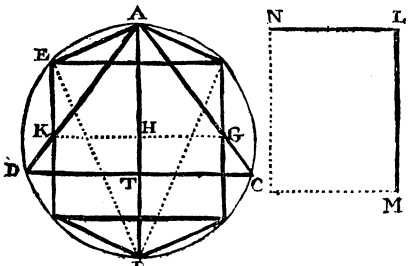
PROP. 10. THEOR. 10.

A Tetrahedron AD C, *is to an Octohedron* AEKBG, *inscribed in one and the self same sphere* ADBC, *in proportion as the rectangle Parallelogram conteined under the line* NL, *which conteineth in power* $\frac{27}{64}$ *parts of the side of the Tetrahedron* AC, *and under the line* ML, *which is subsesquioctava to the same side of the Tetrahedron, is to the square of the diameter of the sphere* AB.

Demonstration 1. Forasmuch as the line drawn from the angle A, by the center H, perpendicular upon the base of the Tetrahedron, falleth upon the center I of the circle which conteineth that base, and maketh the right line HT the sixth part of the diameter AB [a]; therefore the line HA (which is drawn from the center to the circumference) is triple to the line HT, and therefore the whole line AT is to the line AH as 4 is to 3. Let the Tetrahedron ADC be cut by a plain GHK, passing by the center H, and being parallel to the base DTC [b]. Now then the triangle ADC of the Tetrahedron shall be cut by the right line KG, which is parallel to the line DC [c]: Wherefore as the line AT is to the line AH, so is the line AC to the line AG [d]: Wherefore the line AC is to the line AG sesquitertia, that is, as 4 to 3. And forasmuch as the triangles ADC, AKG, and the rest which are cut by the plain KHG, are like the one to the other [e]: The pyramids AD and AKG shall be like

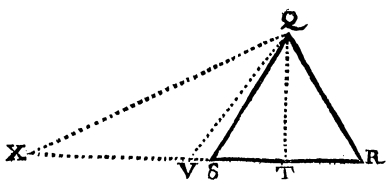
a) 13. 13.
b) C. 15. 11.
c) 16. 11.
d) 2. 6.
e) 5. C.

like the one to the other [f]: Wherefore they are in triple proportion of that in which the ſides of A C and A G are [g]. But the proportion of the ſides A C to A G is as the proportion of 4 to 3. Now then if [h] you find out four of the leaſt numbers in continual proportion, and in that proportion that 4 is to 3, which ſhall be 64, 48, 36, and 27, it is manifeſt [i] that the extreams 64 to 27 are in triple proportion of that in which the proportion given 4 to 3 is: Or the quantity of the proportion of 4 to 3, which is 1 and $\frac{1}{3}$) being twice multiplyed into it ſelf, there ſhall be produced the proportion



of 64 to 27: Wherefore the Pyramis or Tetrahedron A D C is to the Pyramis A K G as 64 is to 27, which is triple to the proportion of 4 to 3. And foraſmuch as the line A C is to the line A G in length ſeſquitertia of what parts the line A C conteineth in power 64, of the ſame parts doth the line A G contein in power 36. For [k] the proportion of the powers or ſquares, is duple to the proportion of the ſides which are as 64 to 48.

Demonſtration 2. NOw then upon the line R S, which let be equal to the line A G, let there be deſcribed an equilateral triangle Q R S [l], and from the angle Q drawn to the baſe R S, a perpendicular line Q T, and extend the line R S to the point X. And as 27 is to 64, ſo [m] let the line R S be to the line R X, and divide the line R V into two equal parts in the point X, and draw the line Q V. And foraſmuch as the line R S is equal to the line A G, of what parts the line A C conteineth in power 64, of the ſame parts the line R S conteineth in power 36, for it is proved that the line A G conteineth in power 36 of thoſe parts: And of what parts the line R S conteineth in power 36, of the ſame parts the line Q T conteineth in power 27 [n]. Wherefore of what parts the line A C conteineth in power 64, of the ſame parts the line Q T conteineth in power 27. Wherefore the right line Q T ſhall be equal to the right line L N by ſuppoſition. Again, foraſmuch as the line R S is put equal to the line A G, and of



f) 7. def. 11.
g) 8. 12.
h) 2. 8.
i) 15. def. 1.
k) 2. 6.
l) 1. 1.
m) C. 6. 10.
n) C. 12. 13.

and of what parts the line R S conteineth in length 27, of the ſame parts is the line R X put to contein in length 64, and of what parts the line R X conteineth in length 64, of the ſame the line A C (which is in length ſeſquitertia to the line A G or R S) conteineth 36. Wherefore the line R V (which is the half of the line R X) conteineth in length of the ſame parts 32, of which the line A C conteined in length 36. Wherefore the line R V is to the line A C ſubſeſquioctava, and therefore the line R V is equal to the line L M, which is alſo ſubſeſquioctava to the ſame line A C. And foraſmuch as the line N L is equal to the line Q T, and the line L M to the line R V (as before hath been proved) the rectangle Parallelogram conteined under the lines Q T and K V, ſhall be equal to the rectangle conteined under the line N L, which is in power $\frac{27}{64}$ to the ſide A C, and under the line L M, which is in length ſubſeſquioctava to the ſame ſide A C. But that which is conteined under the lines Q T and R V is double to the triangle Q V R [o], and to the ſame triangle Q V R is the triangle Q X R double [p]. Wherefore the whole triangle Q X R is equal to that which is conteined under the lines Q T and R V, and therefore is equal to the Parallelogram M N. And foraſmuch as the line R X by ſuppoſition conteineth in length 64 of thoſe parts of which the line R S conteineth 27, and the triangles Q R X and Q R S are [q] in the proportion of their baſes; that is as 64 to 27: But as 64 is to 27, ſo is the Pyramis or Tetrahedron A D C to the Pyramis A K G: Wherefore as the Parallelogram N M or the triangle Q R X is to the triangle Q R S, ſo is the Pyramis A D C to the Pyramis A K G: And foraſmuch as the ſemidiameter A H is the altitude of the Pyramis A K G, and alſo of the two equal and like pyramids of the Octohedron which have their common baſe in the ſquare of the Octohedron [r]: Therefore as the baſe of the Pyramis A K G (which is the triangle Q R S) is to two ſquares of the Octohedron, that is, to the ſquare of the diameter A B, which is equal to thoſe ſquares [s], ſo is the Pyramis A K G to the Octohedron A E B [t]. And foraſmuch as the Parallelogram M N is to the baſe Q R S, as the Pyramis A D C is to the Pyramis A K G, and the baſe Q R S is to the ſquare of the line B E, as the Pyramis A K G is to the Octohedron A E B: Therefore by proportion of equality, taking away the meanes [v], as the Parallelogram N M is to the ſquare of the line B E, ſo is the Pyramis A D C to the Octohedron A E B, inſcribed in one and the ſame ſphere. But the parallelogram N M is conteined under the line N L, which by ſuppoſition is in power $\frac{27}{64}$ to L M, which is alſo by ſuppoſition in length ſubſeſquioctava to the ſame line A C: Wherefore, &c. Which was required to be proved.

o) 41. 1.
p) 1. 6.
q) 1. 6.
r) C. 14. 13.
s) 47. 1.
t) 6. 12.
v) 22. 5.

PROP. 11. THEOR. 11.

If a Cube be conteined in a ſphere, the ſquare of the diameter doubled, is equal to all the Superficies of the Cube taken together. And a perpendicular line drawn from the center of the Sphere to any baſe of the Cube, is equal to half the ſide of the Cube.

Demonſtration FOraſmuch [a] as the diameter of the ſphere is in power triple to the ſide of the Cube: Therefore the ſquare of the diameter doubled is ſextuple to the baſe of the ſame Cube. But the ſextuple of

a) 15. 13.

the power of one of the ſides conteineth the whole Superficies of the Cube, for the Cube is compoſed of ſix ſquare Superficies,[b] whoſe ſides therefore are equal: Wherefore the ſquare of the diameter doubled is equal to the whole Superficies of the cube. And foraſmuch as the diameter of the Cube, and the line which falleth perpendicularly upon the oppoſite baſes of the Cube do cut the one the other into two equal parts, in the center of the Sphere which conteineth the Cube [c], and the whole right line which coupleth the centers of the oppoſite baſes, is equal to the ſide of the Cube [d]; for it coupleth the equal and parallel ſemidiameters of the baſes: Therefore the half thereof ſhall be equal to the half of the ſide of the Cube [e]: Wherefore a Cube, &c. Which was required to be proved.

b) 21. def. 11.
c) 2 C. 15. 13
d) 33. 1.
e) 15. 5.

COROLLARIE.

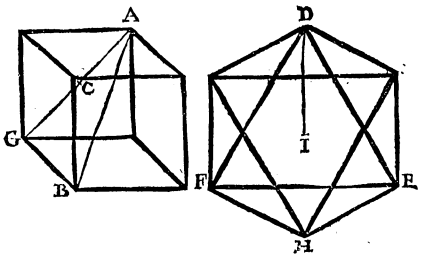
If two thirds of the power of the diameter of the ſphere be multiplyed into the perpendicular line equal to half the ſide of the Cube, there ſhall be produced a Solid equal to the Solid of the Cube.

For it is before manifeſt that two third parts of the power of the diameter of the ſphere are equal to two baſes of the Cube: If therefore unto each of thoſe two thiads be applyed half the altitude of the Cube, they ſhall make each of thoſe Solids equal to half of the Cube [f]; for they have equal baſes: Wherefore two of thoſe Solids are equal to the whole Cube.

f) 31. 11.

PROP. 12. THEOR. 12.

One and the ſelf ſame circle conteineth both the ſquare of a Cube ABG, *and the triangle of an Octobedron deſcribed in one and the ſelf ſame Sphere.*



Conſtruction LEt the diameter of the Sphere be A B or D H, and let the lines drawn from the Centers (that is, the ſemidiameters of the circles which contein the baſes of thoſe Solids) be CA and ID. Then I ſay that the lines C A and I D are equal.

Demonſtration FOraſmuch as A B the diameter of the Sphere which conteineth the Cube, is in power triple to B G the ſide of the Cube [a], unto which ſide A G the diameter of the baſe of the Cube is in power

a) 15. 13.

power double [b], conteineth the baſe [c]. Therefore A B the diameter of the ſphere is in power ſeſquialter to the line A G: namely, of what parts the line A B conteineth in power 12 of the ſame line A G, ſhall contein in power 8. And therefore the right line A C which is drawn from the center of the Circle to the circumference, conteineth in power of the ſame parts 2. Wherefore the diameter of the Sphere is in power ſextuple to the line which is drawn from the center to the circumference of the circle which conteineth the ſquare of the Cube. But the diameter of the ſame Sphere which conteineth the Octohedron is one and the ſame with the diameter of the Cube, namely, D H is equal to A B, and the ſame diameter is alſo the diameter of the ſquare, which is made of the ſides of the Octohedron: Wherefore the ſaid diameter is in power double to the ſide of the ſame Octohedron [d]. But the ſide D F is in power triple to the line drawn from the center to the circumference of the circle which conteineth the triangle of the Octohedron (namely, to the line I D) [e]. Wherefore the ſame diameter A B or D H, which was in power ſextuple to the line drawn from the center to the circumference of the circle which conteineth the ſquare of the Cube, is alſo ſextuple to the line I D, drawn from the center to the circumference of the circle which conteineth the triangle of the Octohedron. Wherefore the lines drawn from the centers of the Circles to the circumferences which contein the baſes of the Cube and of the Octohedron are equal, and therefore the Circles are equal [f]. Wherefore one and the ſelf ſame Circle, &c. Which was required to be proved.

b) 47. 1.
c) 9. 4.
d) 14. 13.
e) 12. 13.
f) 1. def. 3.

COROLLARIE.

Hereby it is manifeſt, that Perpendiculars coupling together in a Spere, the Centers of the Circles which contein the oppoſite baſes of the Cube and of the Octohedron, are equal.

For the Circles are equal, and the lines which paſſing by the center of the Sphere, do couple together the centers of the baſes, are alſo equal. Wherefore the Perpendicular which coupleth together the oppoſite baſes of the Octohedron, is equal to the ſide of the Cube, for either of them is the altitude erected.

PROP. 13. THEOR. 13.

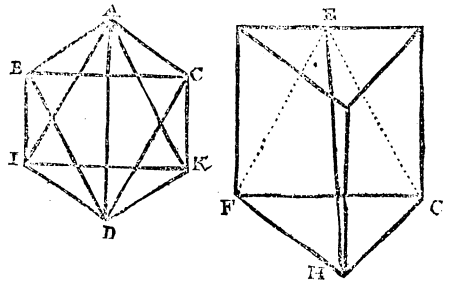
An Octohedron ABCD, *is to the triple of a Tetrahedron* EFGH, *conteined in one and the ſame Sphere, in that proportion as their ſides are.*

Conſtruction UPon the baſe F G H erect a Priſme, which is done by erecting from the angles of the baſe perpendicular lines, equal to the altitude of the Tetrahedron, which Priſme ſhall be triple to the Tetrahedron E F G H [a]. Then I ſay that the Octohedron is to the Priſme which is triple to the Tetrahedron, as the ſide B C is to the ſide F G.

a) 1 C. 7. 12.

Demonſtration FOr, foraſmuch as the ſides of the oppoſite baſes of the Octohedron are right lines touching one another, and are parallels to other right lines which touch one another, for the ſides of the ſquares which are compoſed of the ſides of the Octohedron, are oppoſite:

Wherefore the oppoſite plain triangles, namely, A B C and K I D, ſhall
b) 15. 11. be parallels, and ſo the reſt [b]. Let the diameter of the Octohedron be the line A D. Now then the whole Octohedron is cut into four equal and like Pyramids, ſet upon the baſes of the Octohedron, and having the ſame altitude with it, and being about the diameter A D, namely the Pyramis ſet upon the baſe B I D, and having his top the point A, and alſo the Pyramis ſet upon the baſe B C D, having his top the ſame point A. Likewiſe the Pyramis ſet upon the baſe I K D, and having his top the ſame point A: and moreover the Pyramis ſet upon the baſe C K D, and
c) 8. def. 11. having his top the former point A, which Pyramids ſhall be equal [c] (for they each conſiſt of two baſes of the Octohedron, and of two triangles conteined under the diameter A D, and two ſides of the Octohedron.) Wherefore the Priſme which is ſet upon the baſe of the Octohedron, and having the ſame altitude with it, namely, the altitude of the parallel baſes, as it is manifeſt by the former, is equal to three of thoſe Pyramids
d) C. 7. 12. of the Octohedron [d]. Wherefore that priſme ſhall have to the other Priſme under the ſame altitude compoſed of the four Pyramids of the
c) Co. 7. 12. whole Octohedron, the proportion of the triangular baſes [e]: And foraſmuch as 4 Pyramids are unto 3 Pyramids in ſeſquitertia proportion to the



baſe of the Priſme which conteineth three Pyramids of the ſame Octohedron and are ſet upon the baſe of the Octohedron; and under the altitude thereof, that is in ſubſeſquitertia proportion to the baſe of the Octohedron. But the baſe of the ſame Octohedron is in ſeſquitertia proportion
f) 10. 14. to the baſe of the pyramis [f]: Wherefore the triangular baſes, namely, of the Priſme which conteineth four pyramids of the Octohedron, and is under the altitude thereof, are equal to the triangular baſes of the Priſme, which conteineth three Pyramids under the altitude of the Pyramis E F G H. But the Priſme of the Octohedron is equal to the Octohedron, and the Priſme of the Pyramis E F G H is proved triple to the ſame Pyramis E F G H. Now then the Priſmes ſet upon equal baſes, are the one to
g) C. 25. 11. the other as their altitudes are [g]: namely, as are the Parallelepipedons
h) 31. 11. their doubles [h]. But the altitude of the Octohedron is equal to the ſide of
i) C. 13. 14. the Cube conteined in the ſame ſphere [i], and the ſide of the Cube is in power to the altitude of the Tetrahedron in that proportion that 12 is to
k) 18. 13. 16 [k]. And the ſide of the Octohedron is to the ſide of the Pyramis in that
l) 18. 13. proportion that 18 is to 24 [l], which proportion is one and the ſame with the proportion of 12 to 16. Wherefore that Priſme which is equal to the Octo-

Octohedron, is to the Priſme which is triple to the Tetrahedron in that proportion that the altitudes, or that the ſides are. Wherefore an Octohedron is to the triple of a Tetrahedron, &c. Which was required to be demonſtrated.

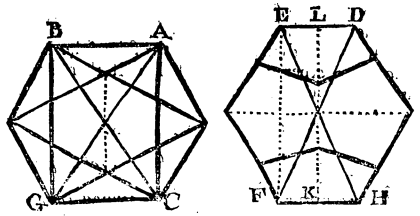
COROLLARIE.

The ſides of a Tetrahedron and of an Octohedron are proportionall with their altitudes.

For the ſides and altitudes were in power ſeſquitertia. Moreover the diameter of the Sphere is to the ſide of the Tetrahedron as the ſide of the Octohedron is to the ſide of the Cube, namely, the powers of each is in ſeſquialter proportion [m]. m) 18. 13.

PROP. 14. THEOR. 14.

If a rational line A G *or* D F, *conteining in power two lines* A B *and* B G, *make the whole and the greater ſegment, and again conteining in power two lines* E F *and* E D, *make the whole and the leſſer ſegment, the greater ſegment* A B *ſhall be the ſide of the Icoſahedron* A B G C, *and the leſſer ſegment* E D *ſhall be the ſide of the Dodecahedron* D E F H, *conteined in one and the ſame Sphere.*

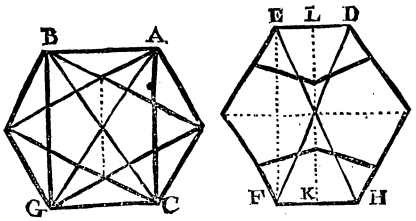


Conſtruction Suppoſe that A G be the diameter of the ſphere which conteineth the Icoſahedron A B G C; and let B G ſubtend the ſides of the Pentagon deſcribed of the ſides of the Icoſahedron [a]. Moreover, upon the ſame diameter A G, or D F equal unto it, let there be deſcribed a Dodecahedron [b], whoſe oppoſite ſides E D and F H, let be cut into two equal parts in the points I and K, and draw a line from I to K. And let the line E F couple two of the oppoſite angles of the baſes which are joyned together. Then I ſay that A B the ſide of the Icoſahedron, is the greater ſegment which the diameter A G conteineth in power together with the whole line, and the line E D is the leſſer ſegment which the ſame diameter A G or D F conteineth in power together with the whole. a) 16. 13. b) 17. 13.

Demonſtration For ſeeing that the oppoſite ſides A B and G C of the Icoſahedron being coupled by the diameters A G and B C, are equal and parallels, [c] the right lines B G and A C which couple them c) 2 C. 16. 13

d) 33. 1. e) 8. 1. f) 29. 1. g) 47. 1. h) 8. 13.

them together are equal and parallel [d]: Moreover, the angles B A C and A B G being ſubtended of equal diameters, ſhall [e] be equal, and [f] they ſhall be right angles. Wherefore the right line A G conteineth in power the two lines A B and B G [g]. And foraſmuch as the line B G ſubtendeth the angle of the Pentagon compoſed of the ſides of the Icoſahedron, the greater ſegment of the right line B G ſhall be the right line A B [h], which line A B together with the whole line B G, the line A G conteineth in power. And foraſmuch as the line I K coupling the oppoſite and parallel



ſides E D and F H of the Dodecahedron, maketh at thoſe points right angles [i], the right line E F which coupleth together equal and parallel lines E I and F K, ſhall be equal to the ſame line I K [k]. Wherefore the angle D E F ſhall be a right angle [l]: Wherefore the diameter D F conteineth in power the two lines E D and E F. But the leſſer ſegment of the line I K is E D the ſide of the Dodecahedron [m]. Wherefore the ſame line E D is alſo the leſſer ſegment of the line E F (which is equal to the line I K:) Wherefore the diameter D F conteining in power the two lines E D and E F [n], conteineth in power E D the ſide of the Dodecahedron the leſſer ſegment, together with the whole: If therefore, &c. Which was required to be demonſtrated.

i) C. 17. 13. k) 33. 1. l) 29. 1. m) C. 17. 13. n) 47. 1.

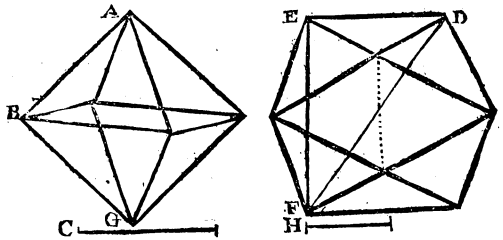
PROP. 15. THEOR. 15.

If the power of the ſide A B *of an Octohedron* A B G, *be expreſſed by two right lines* C *and* H *joyned together by an extream and mean proportion, the ſide* D E *of the Icoſahedron* D E F, *conteined in the ſame Sphere, ſhall be duple to the leſſer ſegment* H.

Demonſtration FOraſmuch as [a] it was manifeſt that E D the ſide of the Icoſahedron is the greater ſegment of the line E F, and that the diameter D F conteineth in power the two lines E D and E F, namely, the whole and the greater ſegment; but by ſuppoſition, the ſide A B conteineth in power the two lines C and H joyned together in the ſame proportion. Wherefore the line E F is to the line E D, as the line C is to the line H [b]. And alternately, [c] the line E F is to the line C, as the line E D is to the line H; and foraſmuch as the line D F conteineth in power the two lines E D and E F, and the line A B conteineth in power the two lines

a) 15. 14. b) 2. 14. c) 16. 5.

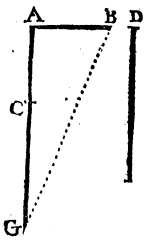
lines C and H. Therefore the ſquares of the lines E F and E D are to the ſquare of the line D F, as the ſquares of the lines C and H to the ſquare



of the line A B. And alternately, the ſquares of the lines E F and E D are to the ſquares of the lines C and H, as the ſquare of the line D F is to the ſquare of the line A B. But D F the diameter is [d] in power double to A B the ſide of the Octohedron inſcribed (by ſuppoſition) in the ſame Sphere. Wherefore the ſquares of the lines E F and E D are double to the ſquares of the lines C and H. And therefore one ſquare of the line E D is double to one ſquare of the line H [e]. Wherefore E D the ſide of the Icoſahedron, is in power dupla to the line H, which is the leſſer ſegment. Therefore, &c. Which was required to be demonſtrated.

d) 14. 13. e) 12. 5.

PROP. 16. THEOR. 16.



If the ſide A B *of a Dodecahedron, and the right line of which the ſaid ſide is the leſſer ſegment* A G *be ſo ſet, that they make a right angle (as in the point* A*) the right line* D *which conteineth in power half the line* B G, *ſubtending the angle, is the ſide of an Octohedron conteined in the ſame Sphere.*

Demonſtration FOraſmuch as the line A G maketh the greater ſegment G C the ſide of the Cube conteined in the ſame ſphere [a], and the ſquares of the whole line A G, and of the leſſer ſegment A B, are triple to the ſquare of the greater ſegment G C [b]: Moreover, the diameter of the Sphere is in power triple to the ſame line G C, the ſide of the Cube [c]. Wherefore the line B G is equal to the diameter, for it conteineth in power the two lines A B and A G [d]; and therefore it conteineth in power the triple of the line G C. But the ſide of the Octohedron conteine in the ſame Sphere, is in power triple to halt the diameter of the Sphere [e]. And by ſuppoſition, the line D conteineth in power the half of the

a) C. 17. 13. b) 4. 13. c) 15. 13. d) 47. 1. e) 14. 13.

the line B G. Wherefore the line D (conteining in power the half of the ſame diameter) is the ſide of an Octohedron. If therefore, &c. Which was required to be proved.

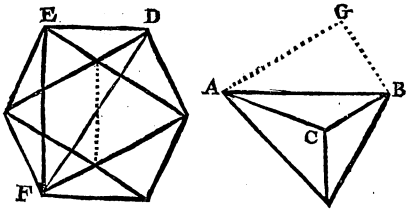
COROLLARIE.

Unto what right line the ſide of the Octohedron is in power ſeſquialter, unto the ſame line the ſide of the Dodecahedron inſcribed in the ſame Sphere, is the greater ſegment.

For the ſide of the Dodecahedron is the greater ſegment of the ſegment C G, unto which D the ſide of the Octohedron is in power ſeſquialtera, that is, is half of the power of the line E G, which was triple to the line C G.

PROP. 17. THEOR. 17.

If the ſide A B *of a Tetrahedron* A B C, *contein in power two right lines* A G *and* G B, *joyned together by an extream and mean proportion, the ſide* E D *of a Icoſahedron* E D F, *deſcribed in the ſame Sphere, is in power ſeſquialter to the leſſer right line* G B.



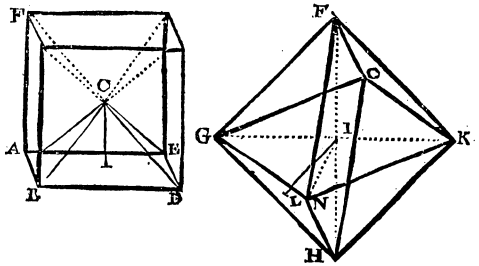
Demonſtration FOraſmuch as [a] the ſide E D is the greater ſegment of the line E F, which ſubtendeth the angle of the Pentagon. But as the whole line E F is to the greater ſegment E D, ſo is the ſame greater ſegment to the leſſer [b]; and by ſuppoſition A G was the whole line, and G B the greater ſegment: Wherefore as E F is to E D, ſo is A G to G B [c]. And alternately, the line E F is to the line A G, as the line E D is to the line G B. And foraſmuch as (by ſuppoſition) the line A B conteineth in power the two lines A G and G B, therefore [d] the angle A G B is a right angle: But the angle D E F is a right angle [e]. Wherefore the triangles A G B and F E D are equiangled [f]: Wherefore their ſides are proportional, namely, as the line E D is to the line G B, ſo is the line F D to the line A B [g]. But by what hath been demonſtrated, F D is the diameter of the Sphere which conteineth the Icoſahedron, which diameter is in power ſeſquialter to A B the ſide of the Tetrahedron inſcribed in the ſame Sphere [h]. Wherefore the line E D the ſide of the Icoſahedron, is in power ſeſquialter to G B the greater ſegment, or leſſer line. If therefore, &c. Which was required to be done.

a) 15. 14.
b) 30. 6.
c) 2. 14.
d) 48. 1.
e) 15. 14.
f) 6. 6.
g) 4. 6.
h) 13. 13.

PROP.

PROP. 18. THEOR. 18.

The Superficies of a Cube A B C D, *is to the Superficies of an Octohedron* F G H K, *inſcribed in one and the ſame Sphere, in that proportion that the Solids are.*



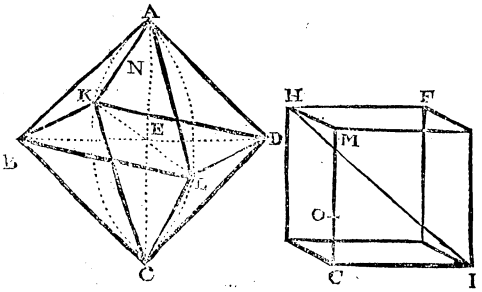
Conſtruction THe four diameters of the Cube are A C, B C, D C, and E C, produced on each ſide, and the three diameters of the Octohedron are F H, G K, and O N. Draw from the center of the Cube to the baſe A B E D, a perpendicular line C R, and from the center of the Octohedron draw to the baſe G N H, a perpendicular line I L.

Demonſtration ANd foraſmuch as the three diameters of the Cube do paſſe by the center C. Therefore [a] there ſhall be made of the Cube ſix Pyramids, as the Pyramis A B D E C, equal to the whole Cube; for there are in the Cube ſix baſes, upon which fall equal perpendiculars from the center, for the baſes are conteined in equal circles of the Sphere. But in the Octohedron the three diameters do make upon the ten baſes eight Pyramids; having their tops in the center [b]. Now the baſes of the Cube and of the Octohedron are conteined in equal circles of the Sphere [c]. Wherefore they ſhall be equally diſtant from the center, and the perpendicular lines C R and I L ſhall be equal. Wherefore the Pyramids of the Cube ſhall be under one and the ſame altitude with the Pyramids of the Octohedron, namely, under the perpendicular line, drawn from the center to the baſes. Wherefore ſix Pyramids of the Cube are to eight Pyramids of the Octohedron, being under one and the ſame altitude, in that proportion that their baſes are [d]; that is, one Pyramis ſet upon ſix baſes of the cube, and having to his altitude the perpendicular line, which Pyramis is equal to the ſix Pyramids, [e] is to one Pyramis ſet upon the eight baſes of the Octohedron, being equal to the Octohedron, and alſo under one and the ſame altitude, in that proportion that ſix baſes of the Cube, which contein the whole Superficies of the Cube are to eight baſes of the Octohedron, which contein the whole Superficies of the Octohedron, For the Solids of thoſe Pyramids are in proportion to one another as their baſes are [f]. Wherefore, &c. Which was required to be proved.

a) 2 Cor. of 15. 13.
b) 3 Cor. of 14. 13.
c) 13. 14.
d) 6. 12.
e) 6. 12.
f) 6. 12.

PROP. 19. THEOR. 19.

If a Cube FGHIM, *and an Octohedron* AECDB, *be conteined in one and the same Sphere* ABCD, *they shall be in proportion to one another as the side* MG *of the Cube, is to the semidiameter* AE *of the Sphere.*



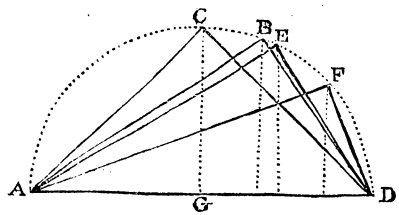
Demonstration. FOrasmuch as the diameter AC is in power double to BK the side of the Octohedron [a], and is in power triple to MG the side of the Cube [b]; therefore the square BKDL shall be sesquialter to FM the square of the Cube. From the line AE cut off a third part AN, and from the line MG cut off likewise a third part GO [c]. Now then the line EN shall be two third parts of the line AE, and so also shall the line MO be of the line MG. Wherefore the Parallelepipedon set upon the same base BKDL, and having his altitude the line EA, is triple to the Parallelepipedon set upon the same base, and having his altitude the line AN [d]. But it is also triple to the Pyramis ABKDL, which is set upon the same base, and is under the same altitude [e]. Wherefore the Pyramis ABKDL is equal to the Parallelepipedon which is set upon the base BKDL, and hath to his altitude the line AN. But unto that Parallelepipedon is double the Parallelepidedon which is set upon the same base BKDL, and hath to his altitude a line double to the line AN [f]; and unto the Pyramis is double the Octohedron ABKLDC [g]. Wherefore the Octohedron ABKDLC is equal to the Parallelepipedon set upon the same base BKLD, and having his altitude the line EN [h]. But the Parallelepipedon set upon the base BKDL, which is sesquialter to the base FM, and having to his altitude the line MO, which is two third parts of the side of the Cube MG, is equal to the Cube FGI [i]. (For it was before proved that the base BKDL is sesquialter to the base FM.) Now then these two Parallelepipedons, namely, the Parallelepipedon which is set upon the base BKDL (which is sesquialter to the base of the Cube) and hath to his altitude the line MO (which is two third parts of MG the side of the Cube) which Parallelepipedon is proved equal to the Cube, and the Parallelepipedon set upon the same base BKDL, and having his altitude the line EN, which Parallelepipedon is proved equal to the Octohedron) these two parallelepipedons (I say) are to one another as the altitude MO is to the altitude EN [k]. Wherefore also as the altitude MO is to the altitude EN, so is the Cube FGHIM to the Octohedron ABKDLC [l]. But as the line MO is to the line EN, so is the whole line MG to the whole line EA [m]. Wherefore as MG the side of the Cube is to EA the semidiameter, so is the Cube FGHIM to the Octohedron ABKDLC inscribed in one and the same Sphere. If therefore, &c. Which was required to be demonstrated.

a) 14. 13.
b) 15. 13.
c) 9. 6.
d) C 31. 11.
e) 2 C. 7. 12.
f) C. 31. 1.
g) 2 Cor. of 14. 13.
h) 15. 5.
i) 2 part of 34. 11.
k) C. 31. 11.
l) 7. 5.
m) 18. 5.

COROLLARIE.

Distinctly to notifie the powers of the sides of the five Solids by the power of the Diameter of the Sphere.

The sides of the Tetrahedron and of the Cube do cut the power of the Diameter of the Sphere into two squares, which are in proportion double to one another. The Octohedron cutteth the power of the Diameter into two equal squares. The Icosahedron into two squares, whose proportion is double to the proportion of a line divided by an extream and mean proportion, whose lesser segment is the side of the Icosahedron. And the Dodecahedron into two squares, whose proportion is quadruple to the proportion of a line divided by an extream and mean proportion, whose lesser segment is the side of the Dodecahedron. For AD the Diameter of the Sphere conteineth in power AB the side of the Tetrahedron, and BD the side of the Cube, which BD is in power half of the side AB. The Diameter also of the Sphere conteineth also in power AC and CD, two equal sides of the Octohedron. But the Diameter conteineth in power the whole line AE, and the greater segment thereof ED, which is the side of the Icosahedron [n]. Wherefore their powers being in dupla proportion of that in which the sides are [o], have their proportion dupla to the proportion of an extream and mean proportion. Again the Diameter conteineth in power the whole line AF, and his lesser segment FD, which is the side of the Dodecahedron [p]. Wherefore the whole having to the lesser a double proportion of that which the extream hath to the mean, namely, of the whole to the greater segment [q], it followes that the proportion of the power is double to the doubled proportion of the sides [r]; that is, is quadruple to the proportion of the extream and of the mean [s].



n) 15. 14.
o) 1 C. 20. 6.
p) 15. 14.
q) 10. de. 5.
r) 1 C. 20. 6.
s) def. 6.

By this meanes therefore the Diameter of a Sphere being given, there shall be given the side of every one of the bodies inscribed. And forasmuch as three of those bodies have their sides commensurable in power only, and not in length unto the Diameter given (for their powers are in the proportion of a square number to a number not square. Wherefore they have not the proportion of a square number to a square number [t].

t) C. 25. 8.

v) 9. 10.

Wherefore alſo their ſides are incommenſurable in length [v]: Therefore it is ſufficient to compare the powers and not the lengths of thoſe ſides to one another; which powers are conteined in the power of the Diameter, namely, from the power of the Diameter let there be taken away the power of the Cube, and there ſhall remain the power of the Tetrahedron, and taking away the power of the Tetrahedron, there remaineth the power of the Cube; and taking away from the power of the Diameter half the power thereof, there ſhall be left the power of the ſide of the Octohedron. But foraſmuch as the ſides of the Dodedecahedron and of the Icoſahedron are proved to be irrational, for the ſide of the Icoſahedron is a leſſer line [w], and the ſide of the Dodecahedron is a Reſidual line [x]; therefore thoſe ſides are unto the Diameter which is a Rational line, incommenſurable both in length and power. Wherefore their compariſon cannot be defined or deſcribed by any proportion expreſſed by numbers [y], neither can they be compared to one another; for irrational lines of divers kinds are incommenſurable to one another; for if they ſhould be commenſurable, they ſhould be of one and the ſame kind [z], which is impoſſible. Wherefore we ſeeking to compare them to the power of the diameter, though they could not be more aptly expreſſed then by ſuch proportions, which cut that rational power of the Diameter according to their ſides, dividing the power of the Diameter by lines in that proportion, as the greater to the leſſer ſegment, to put the leſſer ſegment to be the ſide of the Icoſahedron, and dividing the ſaid power of the Diameter by lines in proportion as the whole to the leſſer ſegment, to expreſſe the ſide of the Dodecahedron by the leſſer ſegment, which may well be done between Magnitudes incommenſurable.

w) 16. 13.

x) 17. 13.

y) 8. 10.

z) 103, 105. 10.

The End of the Fourteenth Element of EUCLIDE.

THE

THE FIFTEENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

THis Fifteenth and laſt Book of *EUCLIDE*, or rather the Second Book of *Appollonius* or *Hypſicles*, teacheth the inſcription and circumſcription of the five regular bodies one within & about another, a thing undoubtedly pleaſant and delectable in mind to contemplate, and alſo profitable and neceſſary to practice. For without practice in act it is very hard to ſee and conceive the conſtructions and demonſtrations of the Propoſitions of this Book, unleſſe a man have a very deep, ſharp, and fine imagination. Wherefore I would wiſh the diligent ſtudent in this Book (to make the ſtudy thereof more pleaſant unto him) to have preſently before his eyes, the bodies formed and framed of paſted paper: And then to draw and deſcribe the Lines and Diviſions, and Superficies, according to the conſtructions of the Propoſitions. In which Deſcriptions if he be careful and diligent, he ſhall find all things in theſe Solid matters, as cleer and manifeſt to the eye, as were things before taught only in plain or ſuperficial figures.

DE-

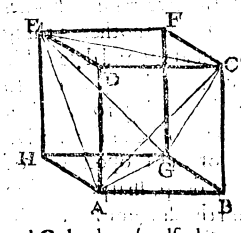
DEFINITIONS.

1 *A solid figure is then said to be inscribed in a solid figure, when the angles of the figure inscribed touch together at one time, either the angles of the figure circumscribed, or the superficies, or the sides.*

2 *A solid figure is then said to be circumscribed about a solid figure, when together at one time either the angles, or the superficies, or the sides of the figure circumscribed, touch the angles of the figure inscribed.*

PROPOSITIONS, and PROBLEMES.

PROPOSITION 1. PROBLEM 1.



In a given Cube ABCDEFGH, *to inscribe a Pyramid.*

Construction LEt the given Cube be ABCDEFGH, to inscribe therein a Pyramid or Tetrahedron; from one angle thereof E, let there be drawn to the bases constituting it three diameters EA, EG, and EC, from the extremities of which A, G, and C, let there be also drawn the diameters AG, GC, and CA, to the other three bases, which joyne the extremities of the three first Diameters.

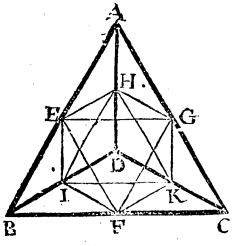
Demonstration FOrasmuch as the diameters of the equal squares are equal to one another, being double in power [a] to the sides of the equal squares; It is manifest, that the four triangles ACE, GAC, GAE, and GCE compounded of the said diameters, are equilateral and equal to one another; therefore do constitute a Pyramid; and all the angles of the Pyramid being compared to the angles of the Cube, [b] it is manifest that it is inscribed in the Cube. Therefore in a given Cube we have inscribed a Pyramid. Which was to be done.

a) 47. 1.

b) 3. def. 11.

PROP.

PROP. 2. PROBL. 2.



In a given Pyramid to inscribe an Octohedron.

Construction LEt the given Pyramid be, ABCD, in which an Octohedron ought to be inscribed. Having divided all the sides in two equal parts at E, F, G, H, I, and K, and drawn EF, FG, GE, HI, IK, KH, EI, IF, FK, KG. GH, and HE, there shall be constituted eight triangles, four of which EHI, IHK, KHG, and HGE, are on the plain EIKG: But the four others IFE, EFG, GFK, and KFI, below the same plain.

Demonstration BUt forasmuch as the sides AE and AH of the triangle AEH, are equal to AG and AH of the triangle AGH, being the halves of the equal lines AB, AD, and AC, and the angles contained of them EAH and GAH equal, being angles of the equilateral triangles ABD and ACD, [a] the bases EH and GH shall be equal, and by the same reason EH and IH and the others, in like manner: Wherefore these eight triangles are equilateral and equal to one another; and therefore do compound the Octohedron EIGHF, [b] the which is inscribed in the Pyramid, seeing that all the angles do touch all the sides of the Pyramids, by the Construction. Therefore in a Pyramid we have inscribed an Octohedron. Which was to be done.

a) 4. 1.

b) 31. dec. 11

COROLLARIE.

Then forasmuch as the three plains EIKG, GHIF, *and* FKHE, [c] *are equal squares, cuting one another at right angles each of which cutteth the Pyramid* ABCD *in two equal parts (for the square* EK *divideth it in the Pyramid* HIKD, *and in the Prisme* HIKGAE, *and also in the Pyramid* EBFI, *and in the Prisme* EFCGKI: *But it appeares that the Pyramid is equal to the Pyramid, and the Prisme to the Prisme: In like manner the other squares* [d] GI *and* FH *do divide the same Pyramid* ABCD *in two equal parts:) It is manifest that if in the Tetrahedron there be inscribed an Octohedron, that the Tetrahedron shall be cut in two equal parts, by three equal squares, the which do cut one another at right angles, and the Tetrahedron in two equal parts.*

c) Cor. 1. of 14. 13.

d) 8. 12.

PROP. 3. PROBL. 3.

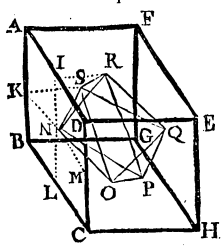
In a given Cube to inscribe an Octohedron.

Construction LEt the given Cube be AH, in which an Octohedron ought to be inscribed. Let the sides of the base AD and CD, be divided in two equal parts by I, K, L, and M, which let be joyned by IL and KM, cutting one another in N, which is the center of the square BD, as appeares by the demonstration of the 8th. of the 4th. and so the centers of the other bases, O, P, Q, R, and S, shall be found. Therefore all the right lines drawn from the said centers to the middle of the bases,

baſes, as are N I, R I, N K, S K, &c. are equal to the halves of the ſides of the Cube, or the ſquares, as appeares by the ſaid Demonſtration of the eighth of the fourth.

Laſtly, if the ſaid centers be joyned by N O, O P, P Q, Q R, R S, S N, N P, P R, R N, S O, O Q, and Q S, there will be conſtituted eight triangles, four of which N S R, R S Q, Q S O, and O S N, are on the plain N O Q R, and the other four O P Q, Q P R, R P N, and N P O, below the ſame Plain.

Demonſtration But foraſmuch as the ſides I N and N R, of the triangle I N R, are equal to K N and R S of the triangle K N S, being all halves of the equal ſides of the Cube as is ſaid, alſo the angles conteined of them are equal; for N I and I R being parallel to B A and A F, [a] the angle N I R is equal to the right angle K A F of the ſquare A G; by the ſame reaſon N R S being equal to the right angle I A F of the ſquare A E, [b] the baſes N R and N S ſhall be equal; ſo we ſhall demonſtrate all the other lines equal both to one another, and to theſe two: If from the centers be drawn right lines to the middle of the ſides, in ſuch manner as that each two be drawn to the middle of the ſide common to two ſquares of the Cube, from which theſe two points, from whence come the right lines, are the centers; ſo as N I and R I which are drawn to I, halfe of the ſide A D, common to the ſquares A C and A E, whoſe centers are N and R: In like manner N K and S K are drawn to K, the middle of A B, common to the ſquares of A C and A G, whoſe centers are N and S, &c. Therefore we have conſtituted eight equilateral triangles, and equiangled to one another, which do conſtitute the Octohedron N O Q R S P, which [c] is inſcribed in the Cube, ſeeing that all his ſix angles touch all the ſix baſes of the Cube at their centers. Therefore in the given Cube we have inſcribed an Octohedron, &c. Which was to be done.



a) 10. 11.
b) 4. 1.
c) d. 31. 11.

Otherwiſe, [d] in the Cube let there be inſcribed a Pyramid, and [e] in that Pyramid let there be inſcribed an Octohedron, and what was propoſed is done. For ſeeing that according to the Demonſtration of the ſecond Propoſition of this Book, the angles of the Octohedron do touch the ſides of the Pyramid; but the ſides of the Pyramid (by the Demonſtration of the firſt Propoſition of this Book) are in the plains of the baſes of the Cube, being the diameters of the ſame baſes: It is manifeſt that the angles of the Octohedron do touch in like manner the baſes of the Cube; and therefore that the Octohedron is inſcribed in the Cube.

d) 1. 15.
e) 2. 15.

COROLLARIE.

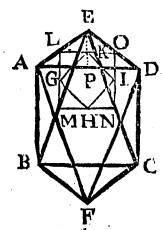
Then foraſmuch as the diameters of the Octohedron inſcribed in the Cube, N Q *and* R O *(if they were drawn)* [f] *cut one another at right angles, and joyn the centers of the oppoſite baſes of the Cube. It is manifeſt that the right lines which joyn together the centers of the oppoſite baſes of the Cube divide themſelves not only in two equal parts,* [g] *as is ſhewn; but alſo at right angles, as we have here ſhewn.*

f) 1 C. 14. 13
g) 1 C. 15. 13

PROP.

PROP. 4. PROBL. 4.

In a given Octohedron ABCDEF, *to inſcribe a Cube.*

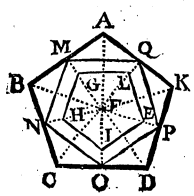


Demonſtration FOraſmuch as ſix Pyramids quadrangular are conteined in the Octohedron whoſe tops are the ſix angles of the Octohedron, and each two conſtitute an Octohedron: Let one of them, to wit, the Pyramid A B C D E, whoſe baſe is the ſquare A B C D, and the equilateral triangles A B E, E B C, C E D, and D E A, whoſe centers are G, H, I, and K, by the which let there be drawn L M, M N, N O, and O L parallel to A B, B C, C D, and D A, the triangles L M E, E M N, N E O, and O E L, ſhall be equilateral, [a] being alike to the triangles equilateral aboveſaid, and therefore equal to one another, having the ſides common, and the baſes L M, M N, and N O, by conſequence equal, and if from E by the center K, be drawn E P, it will divide the angle at E in two equal parts; therefore L K ſhall be equal to K O; that is to ſay, that L is divided into two equal parts by K the center; and in like manner L M, M N, and N O, ſhall be cut each in two equal parts by the centers G, H, and I; therefore the four triangles G L K, K O I, G M H, and H N I, are Iſoſceles triangles ([b] the angles G L K, K O I, I N H, and H M G, being equal to the right angles of the ſquare G K I H) therefore the baſes G K, K I, I H, and H G, ſhall be equal, and ſhall contein right angles. Wherefore G H I K is a ſquare, and if in the ſame manner, in the other five Pyramids of the Octohedron, the centers of the triangles are joyned by right lines, there ſhall be deſcribed in like manner five ſquares, which ſhall be equal to one another, having the ſides common. Wherefore ſix ſuch ſquares ſhall compound a Cube, [c] which ſhall be deſcribed in the Octohedron, ſeeing that the eight angles thereof do touch the eight baſes of the Octohedron at their centers. Therefore in the Octohedron given we have inſcribed a Cube. Which was to be done.

a) Cor. 4. 6.
b) 10. 11.
c) 31. de. 11.

PROP. 5. PROBL. 5.

In a given Icoſahedron to inſcribe a Dodecahedron.



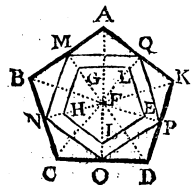
Conſtruction LEt one of the twelve Pyramids of the Icoſahedron, whoſe baſes are Pentagons; A B C D E F, and the baſe the Pentagon A B C D E, and the equilateral triangles A B F, B C F, C D F, D E F, and E A F, and their centers G, H, I, K, and L, joyned, by G H, H I, I K, K L, and L G. Again, from the top F, by the centers of the triangles let there be drawn F M, F N, F O, F P, and F Q, cutting [a] A B, B C, C D, D E, and E A, at M, N, O, P, and Q, in ſuch ſort as M A, M B, N B, N C, O C, O D, P D, P E, Q E, and Q A, may be equal and comprehend equal angles, to wit, of

a) C. 10. 13.

the Pentagon [b] MN, NO, OP, P Q, and Q M, joyning the points M, N, O, P, and Q, are alſo equal; therefore [c] the angles MF N, NF O, OF P, PF Q, and Q FM, are equal, the ſides FM, FN, FO, FP, and FQ, being equal, to wit, perpendiculars of the angles of equilateral triangles drawn to the oppoſite baſes.

b) 4. 1.
c) 8. 1.

Demonſtration FOr FB and BM of the triangle FBM, being equal to FB and BN of the triangle FBN, and conteining equal angles, to wit, of the equilateral triangles, the baſes [d] FN and FM ſhall be equal, and ſo of the others. But foraſmuch as FG, FH, FI, FK, and FL, ſemidiameters of the equal circles circumſcribing the equilateral triangles are equal, and contein equal angles as is demonſtrated; [e] GH, HI, IK, KL, and LG, ſhall be equal; and ſeeing that as well AMQ, Q MN, MNB, as BNM, NMO, and ONC, are equal to two right angles. But AMQ and NMB are equal to BNM and ONC, the remainders Q MN and MNO ſhall be equal. By the ſame reaſon the other angles NOP, OPQ, and PQM, ſhall be equal both to one another and to theſe: Wherefore the Pentagon MNOPQ ſhall be equilateral and equiangled to the plain of the Pentagon ABCDE. Laſtly, FM, FN, FO, FP, and F Q, being cut proportionally at G, H, I, K, and L: Foraſmuch alſo as FG, FH, FI, FK, and FL, are equal lines; drawn from the center; [f] GH, HI, IK, KL, and LG, ſhall be parallel to MN, NO, OP, PQ, and QM; and [g] therefore the angles IGH, GHI, HIK, IKL, and LKG, are equal to the equal angles Q MN, MNO, NOP, OP Q, PQM, therefore alſo equal to one another, and [h] the plains drawn by GH, HI, and HI, IK, being parallels to the plain of the Pentagon MNOP Q, drawn by MN, NO, and OP, and meeting one another at HI, ſhall make one only plain. In like manner may be ſhewn that the plains IK, KL, and KL, LG, and LG, GE, will make one and the ſame plain with the plain drawn by GH, HI, and IK. Wherefore GHIKL is an equilateral Pentagon and equiangled, having the angles and the ſides equal; and if in the ſame manner to the other eleven Pyramids of the Icoſahedron, there be joyned right lines to the centers of the triangles, there will in like manner be deſcribed the equilateral triangles equiangled, the which having the ſides common, ſhall be equal to one another: Wherefore twelve ſuch Pentagons ſhall conſtitute a Dodecahedron, the which [i] ſhall be inſcribed in the Icoſahedron, the twenty angles of the Dodecahedron being conſtituted at the centers of the twenty baſes of the Icoſahedron. Therefore in the Icoſahedron given we have inſcribed a Dodecahedron. Which was to be done.



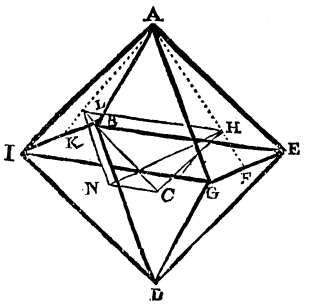
d) 4. 1.
c) 4. 1.
f) 2. 6.
g) 10. 11.
h) 15. 11.
i) 31. d. 11.

PROP. 6. PROBL. 6.

In an Octohedron given ABGDEI, *to inſcribe a trilateral equilateral Pyramis.*

Conſtruction TAke four baſes of the Octohedron, that is, three which cloſe in the loweſt triangle EGD, namely, AEG, BED, IGD,

IGD, and let the fourth be AIB, which is oppoſite to the loweſt triangle before put, namely, to EGD. And take the centers of thoſe four baſes, which let be the points H, C, N, and L, and upon the triangle HCN erect a Pyramis HCNL. Now foraſmuch as theſe two baſes of the Octohedron, to wit, AGE and ABI, are ſet upon the right lines EG and BI, which are oppoſite to one another in the ſquare GEBI of the Octohedron; from the point A draw by the centers of the baſes, to wit, by the centers H and L, perpendicular lines AHF and ALK, cutting the lines EG and BI into two equal parts in the points F and K [a]: Wherefore a right line drawn from the point F to the point K, ſhall be a parallel, and equal to the ſides of the Octohedron, to wit, to EB and GI [b]. And the right line HL, which cutteth the equal ſides AF and AK proportionally (for AH and AL are drawn from the centers of equal circles, to the circumferences) is a parallel to the right line FK [c], and alſo to the ſides of the Octohedron, to wit, to EB and IG [d]. Wherefore as the line AF is to the line AH, ſo is the line FK to the line HL [e].



a) C. 12. 13.
b) 33. 1.
c) 2. 6.
d) 9. 11.
e) 4. 6.

Demonſtration FOr the triangles AFK and AHL are like [f]: But the line AF is in ſeſquialter proportion to the line AH: (for the ſide EG maketh HF the half of the right line AH [g]:) Wherefore FK or GI the ſide of the Octohedron is ſeſquialter to the right line HL. And by the ſame reaſon may we prove that the ſides of the Octohedron are ſeſquialter to the reſt of the right lines which make the pyramis HNCL, to wit, to the right lines HN, NC, CL, LN, and CH: Wherefore thoſe right lines are equal, and therefore the triangles which are deſcribed of them, to wit, the triangles HCN, HNL, NCL, and CHL, which make the Pyramis HNCL, are equal and equilateral. And for aſmuch as the angles of the ſame Pyramis, to wit, the angles H, C, N, and L, do end in the centers of the baſes of the Octohedron: Therefore it is inſcribed in the ſame Octohedron [h]. Wherefore, &c. Which was required to be done.

f) Cor. 2. 6.
g) C. 12. 13.
h) 1 def. 15.

COROLLARIE I.

The baſes of a Pyramis inſcribed in an Octohedron, are parallels to the baſes of the Octohedron.

For ſeeing the ſides of the baſes of the Pyramis touching one another, are parallels to the ſides of the Octohedron, which alſo touch one another: As for example, HL was proved to be a parallel to GI, and LC to DI. Therefore [i] the plain Superficies which is drawn by the lines HL and LC, is a parallel to the plain Superficies drawn by the lines GI and DI, and ſo alſo of the reſt.

i) 15. 11.

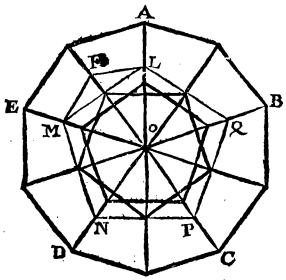
COROLLARIE II.

A right line joyning together the centers of the oppoſite baſes of the Octohedron, is ſeſquialter to the perpendicular line drawn from the angle of the inſcribed Pyramis, to the baſe thereof.

For ſeeing the Pyramis and the Cube which conteineth it, do in the ſame points end their angles [k]: Therefore they ſhall both be incloſed in one and the ſame Octohedron [l]. But the diameter of the Cube joyneth together the centers of the oppoſite baſes of the Octohedron, and therefore is the diameter of the ſphere which conteineth the Cube and the Pyramis inſcribed in the Cube [m]; which diameter is ſeſquialter to the perpendicular which is doawn from the angle of the Pyramis to the baſe thereof; for the line which is drawn from the center of the ſphere to the baſe of the Pyramis is the ſixth part of the diameter [n].

k) 1. 15.
l) 4. 15.
m) 13, 14. 13
n) 2 C. 13. 13

PROP. 7. PROBL. 7.



In a Dodecahedron given ABCDE, *to inſcribe an Icoſahedron.*

Conſtruction LEt the centers of the Circles, which contein ſix baſes of the Dodecahedron, be the points L, M, N, P, Q, and O. And draw thoſe right lines O L, O M, O P, and O Q, as alſo the right lines L M, M N, N P, P Q, and Q L.

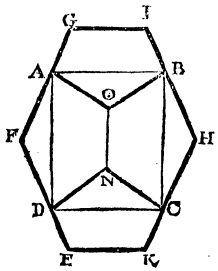
Demonſtration AND now foraſmuch as equal and equilateral Pentagons are conteined in equal Circles: Therefore perpendicular lines drawn from their centers to the ſides, ſhall be equal [a], and ſhall divide the ſides of the Dodecahedron into two equal parts [b]: Wherefore the foreſaid perpendicular ſhall concurre in the point of the Section, wherein the ſides are divided into two equal parts, as L F and M F do. And they alſo contein equal angles, to wit, the inclination of the baſes of the Dodecahedron [c]: Wherefore the right lines L M, M N, N P, P Q, and Q L, and the reſt of the right lines which joyn together two centers of the baſes, and which ſubtend the equal angles conteined under the ſaid equal perpendicular lines, are equal to one another [d]: Wherefore the triangles O L M, O M N, O N P, O P Q, O Q L, and the reſt of the triangles which are ſet at the centers of the Pentagons are equilateral and equal. Now foraſmuch as the 12 Pentagons of a Dodecahedron contein 60 plain ſuperficial angles, of which 60 every 3 make one ſolid angle of the Dodecahedron: It followes that a Dodecahedron hath 20 ſolid angles, but each of theſe ſolid angles is ſubtended of each of the triangles of the Icoſahedron, to wit, of each of thoſe triangles which joyn together the centers of the Pentagons which make the ſolid angle, as hath been proved. Wherefore

a) 14. 3.
b) 3. 3.
c) 3 C. 18. 11
d) 4. 1.

20 equal and equilateral triangles, which ſubtend the 20 ſolid angles of the Dodecahedron, and have their ſides which are drawn from the centers of the Pentagons common, do make an Icoſahedron [e]; and it is inſcribed in the Dodecahedron given [f], for that the angles thereof do all at one time touch the baſes of the Dodecahedron. Wherefore, &c. Which was required to be done.

e) 25. de. 11.
f) 1. def. 15.

PROP. 8. PROBL. 8.



In a Dodecahedron given, to include a Cube.

Conſtruction DEſcribe [a] a Dodecahedron: And take the 12 ſides of the Cube, each of which ſubtend one angle of each of the 12 baſes of the Dodecahedron; for the ſide of the Cube ſubtendeth the angle of the Pentagon of the Dodecaheron [b]: If therefore in the Dodecahedron deſcribed we draw the twelve right lines ſubtended under the foreſaid 12 angles, and ending in 8 angles of the Dodecahedron, and concurring together in ſuch ſort that they be in like ſort ſituate (as was plainly proved [c]) then ſhall it be manifeſt that the right lines drawn in this Dodecahedron from the foreſaid 8 angles thereof, do make the foreſaid Cube, which therefore is included in the Dodecahedron, for that the ſides of the Cube are drawn in the ſides of the Dodecahedron.

a) 17. 13.
b) 2 Cor. of 17. 13.
c) 2 Cor. of 17. 13.

Demonſtration AS for Example, take four Pentagons of a Dodecahedron, to wit, A G I B O, B H C N O, C K E D N, and D F A O N; and draw theſe right lines A B, B C, C D, and D A, which four right lines make a ſquare, for that each of thoſe right lines do ſubtend equal angles of equal Pentagons, and the angles which thoſe four right lines contein are right angles: Wherefore the ſix baſes being ſquares, do make a Cube [d]; and for that the eight angles of the ſaid Cube are ſet in eight angles of the Dodecahedron: Therefore is the ſaid Cube inſcribed in the Dodecahedron [e]: Wherefore, &c. Which was required to be done.

d) 21. d. 11.
e) 1. def. 15.

PROP. 9. PROBL. 9.

In a Dodecahedron given ABGD, *to include an Octohedron* AGBDCI.

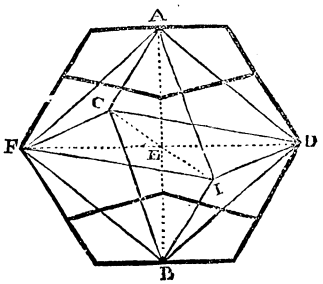
Conſtruction TAke [a] the 6 ſides which are oppoſite to one another, thoſe 6 ſides I ſay whoſe ſections wherein they are divided into two equal parts, are coupled by three right lines within the center of the ſphere wherein the Dodecahedron is conteined, do cut one another perpendicularly. And let the points wherein the foreſaid ſides are cut into two equal parts be A, B, G, D, C, and I. And let the foreſaid three right lines joyning together the ſaid ſections, be A B, G D, and C I, and let the center of the ſphere be E.

a) 3 C. 17. 13

Demon-

c) Cor. 3. of 17. 13.
c) 4. 1.

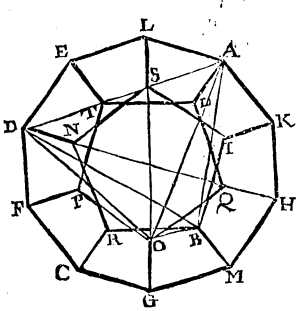
Demonſtration NOw foraſmuch [b] as thoſe three right lines are equal, it followeth [c] that the right lines ſubtending the right angles which they make at the center of the ſphere, which right angles are conteined under the halves of the ſaid three right lines, are equal to one another, that is, the right lines A G, G B, B D, D A, C A, C G, C B, C D, and I A, I G, I B, and I D, are equal to one another. Wherefore alſo the eight triangles C A G, C G B, C B D, C D A, I A G, I G B, I B D, and I D A, are equal and equilateral. And therefore A G B D C I is an Octohedron [d]. And the ſaid Octohedron is included in the Dodecahedron [e]; for that all the angles thereof do at one time touch the ſides of the Dodecahedron. Wherefore, In the, &c. Which was required to be done.



d) 23. de. 11
e) 1. def. 15.

PROP. 10. PROBL. 10.

In a Dodecahedron given ABCD, *to inſcribe an equilateral trilateral Pyramis.*



Conſtruction TAke three baſes of the Dodecahedron, meeting at the point S, to wit, the baſes A L S I K, D N S L E, and S I B R N, and of thoſe three baſes take the three angles at the points A, B, and D, and draw theſe right lines A B, B D, and D A; and let the diameter of the ſphere conteining the Dodecahedron be S O, and there draw theſe right lines A O, B O, and D O.

a) 17. 13.

Demonſtration NOw foraſmuch as [a] the angles of the Dodecahedron are ſet in the Superficies of the Sphere deſcribed about the Dodecahedron: Therefore if upon the diameter S O, and by the point A, be deſcribed a ſemicircle, it ſhall make the angles S B O and S D O right angles. Wherefore the diameter S O conteineth in power both the lines S A and A O, or the lines S B and B O, or elſe S D and D O; but the lines S A, S D, and S B, are equal to one another; for they each ſubtend one of the angles of equal Pentagons: Wherefore the other lines remaining, to wit, A O, B O, and D O, are equal to one another, and by the ſame reaſon may be proved that the diameter H D, which ſubtendeth the two right lines H A and A D, conteineth in power both the ſaid two right lines, and alſo conteineth in power both the right lines H B and B D, which two right lines it alſo ſubtendeth. And moreover, by the ſame reaſon, the diameter A C, which ſubtendeth the right lines C B and B A, conteineth in power both the ſaid right lines C B and B A. But the right lines H A, H B, and C B, are equal to one another, for that each of them alſo ſubtendeth one of the angles of equal Pentagons: Wherefore the right lines remaining, to wit, A D, B D, and B A, are equal to one another. And by the ſame reaſon may be proved that each of theſe right lines A D, B D, and B A, is equal to each of the right lines A O, B O, and D O: Wherefore the ſix right lines A B, B D, D A, A O, B O, and D O, are equal to one another: And therefore the triangles which are made of them, to wit, the triangles A B D, A O B, A O D, and B O D, are equal and equilateral; which triangles therefore do make a Pyramis A B D O, whoſe baſe is A B D, and tops the point O, each of the angles of which Pyramis, to wit, the angles at the points A, B, D, and O, do in the ſame points touch the angles of the Dodecahedron. Wherefore the ſaid Pyramis is inſcribed in the Dodecahedron [b]. Wherefore, In a Dodecahedron, &c. Which was required to be done.

b) 1. def. 15.

PROP. 11. PROBL. 11.

In an Icoſahedron given, to inſcribe a Cube.

IT was manifeſt [a] that the angles of a Dodecahedron are ſet in the centers of the baſes of the Icoſahedron. And [b] it was proved that the angles of a Cube are ſet in the angles of a Dodecahedron. Wherefore the ſame angles of the Cube ſhall of neceſſity be ſet in the centers of the baſes of the Icoſahedron. Wherefore the Cube ſhall be inſcribed in the Icoſahedron [c]. Wherfore, &c. Which was required to be done.

a) 7. 15.
b) 8. 15.
c) 1. def. 15.

PROP. 12. PROBL. 12.

An Icoſahedron given, to inſcribe a trilateral equilateral Pyramis.

IT was manifeſted [a] that the angles of a Cube are ſet in the centers of the baſes of the Icoſahedron: And [b] it was plain that the four angles of a Pyramis are ſet in four angles of a Cube. Wherefore it is evident [c] that a Pyramis deſcribed of right lines joyning together theſe four centers of the baſes of the Icoſahedron, ſhall be inſcribed in the ſame Icoſahedron. Wherefore, In an Icoſahedron, &c. Which was required to be done.

a) 11. 15.
b) 1. 15.
c) 1. def. 15.

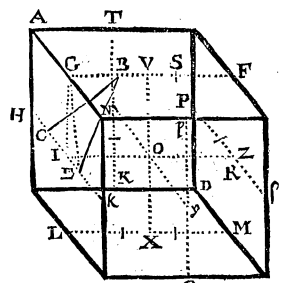
PROP. 13. PROBL. 13.

In a Cube ADFL, *to inſcribe a Dodecahedron.*

Conſtruction 1. DIvide each of the ſides of the Cube into two equal parts in the points T, H, K, P, G, L, M, F, and *p k* Q, ſ. And draw theſe right lines T K, G F, *p* Q, H *k*, P ſ, and L M, which lines again

again divide into two equal parts in the points N, V, Y, I, Z, and X, and draw the lines N Y, V X, and I Z. Now the three lines N Y, V X, and I Z, together with the diameter of the Cube, ſhall cut one another into
a) 39. 11. two equal parts in the center of the Cube [a]: Let that center be the point O. And not to ſtand long about the Demonſtration, underſtand all theſe right lines to be equal and parallels to the ſides of the Cube, and to cut
b) 29. 1. one another at right angles [b]. Let their halves, to wit, F V, G V, H I and K I, and the reſt ſuch like, be divided by an extream and mean proportion
c) 30. 6. [c], whoſe greater ſegments let be the lines F S, G B, H C, and *k* E, &c. and draw the lines G I, G E, B C, and B E.

Demonſtration NOw foraſmuch as the line G I is equal to the whole line G V, which is the half of the ſide of the Cube; and the line I E is equal to the line B V, that is, to the leſſer ſegment: Therefore the ſquares of the lines C I and I E, are
d) 4. 13. triple to the ſquare of the line G B [d]. But unto the ſquares of



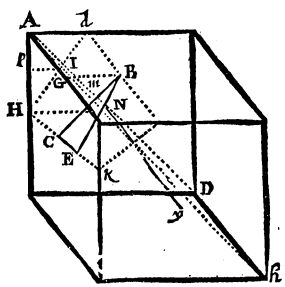
c) 47. 1. the lines G I and I E, the ſquare of the line G E is equal [c]; for the angle G I E is a right angle: Wherefore the ſquare of the line G E is triple to the ſquare of the line G B; and foraſmuch as the line F G is erected perpendicularly to the plain A G K L
f) 4. 11. [f], for it is erected perpendicularly to the two lines A G and G I; therefore the angle B G E is a right angle, for the line G E is drawn in the plain A G *k* L. Wherefore the line B E containing in power
g) 47. 1. the two lines B G and G E [g], is in power quadruple to the line G B (for the line G E was proved to be in power triple to the ſame line G B:)
h) 20. 6. Wherefore the line B F is in length double to the B G [h]. But (by Conſtruction) the line C E is double to the line I E: Wherefore the halves G B and I E are in proportion the one to the other as their doubles B E and C E:
i) 15. 5. Wherefore [i] the line C E is the greater ſegment of the line B E, divided by an extream and mean proportion. And foraſmuch as the ſame thing may be proved touching the line B C: Therefore the lines B E and B C are equal, making an iſoſceles triangle.

Conſtruction 2. NOw let us prove that three angles of the Pentagon of the Dodecahedron are ſet at the points B, C, and E, and the other two angles are ſet between the lines B C and B E.

Foraſmuch as the circle which conteineth the triangle B C E circumſcribeth the Pentagon, whoſe ſide is the line C E [k]: Extend the plain
k) 11. 4. of the triangle B C E, by the parallel lines *d* B and H E, cutting the line A D, to wit, the diameter A D, the baſe of the Cube in the point I; and let it cut the liue A H the diameter of the Cube in the point M. And by the point I, draw in the baſe A D a parallel to the line A *d*, which let be I *l*.

Demon-

Demonſtration FOraſmuch as from the triangle A H N, there is by the parallel line *l* I, taken away the triangle A *l* I, like unto the whole triangle A H N [l], the lines A I and *l* I ſhall be equal. But as the
l) Cor. 2. 6. line H A is to the line A *d*, ſo [m] is the line H *l* to the line *l* I, or to the line *l* A, which is equal to the line *l* I; and the greater ſegment of the line H A, (which is half the ſide of the Cube) is as before hath been proved, the
m) 2. 6. line A *d*, that is, the line G B, which is equal to the line A *d* [n]: Wherefore the greater ſegment of the line H *l* is the line *l* A, and as the whole
n) 33. 1. line H *l* is to the greater ſegment, ſo ſhall the ſame greater ſegment H *l*, be to the leſſer ſegment *l* A [o]. Wherefore the line H A is divided by an extream and mean proportion in the point *l*. But in the triangle A H N, the line N A which is drawn from the center of the baſe A D, is in the point I, cut like unto the line A H, by the parallel line *l* I [p], for the lines H N and I *l* are parallels by conſtruction: Wherefore the line N A is in the point I divided by an extream and mean proportion by the Superficies *d* B E H. And foraſmuch as the line Y O N, which coupleth the centers of the oppoſite baſes, is a parallel to the line H E; a plain Superficies extended by the line Y O N, parallel to the plain *d* B E H, the two plains ſhall cut the lines A O and A N (the ſemidiameter of the Cube, and the ſemidiameter of the baſe A D) into the ſame proportions in the points *m* and I [q]. But the line A N is in the point I divided by an extream and mean proportion: Wherefore the ſemidiameter of the Cube is in the point *m* divided by an extream and mean proportion by the plain of the triangle B C E. And foraſmuch as the reſt of the triangles deſcribed in the Cube after the like manner, may by the ſame reaſons be proved to be in a plain, which cutteth the ſemidiameter of the Cube by an extream and mean proportion, it is manifeſt that three plains of the Dodecahedron ſhall under every angle of the Cube concurre in one and the ſame point of the ſemidiameter, being cut by an extream and mean proportion.



o) 5. 13.

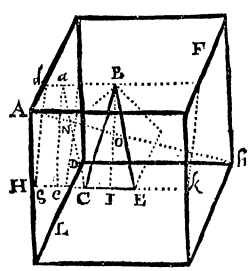
p) 2. 6.

q) 17. 11.

Conſtruction 3. NOw reſteth to prove that the right lines which couple that point of the ſemidiameter, with the angles of the triangle B E C are equal; whereby may be proved that the Pentagons are equilateral and equiangled.

Take the two baſes of the Cube, whereon are ſet the triangle B C E, to wit, the baſes A F and A *k*, take alſo the ſame diameter of the Cube, that was before, to wit, A *h*, and let the ſide ſet at the point *n* of the ſection of the diameter by an extream and mean proportion, be the line C *n* or B *n*, and let the center of the Cube be as before the point O, and extend the line C *n* to the line B *d*, and let it concurre with it in the point *a*.

Demonſtration FOraſmuch as the plain which paſſeth by the line H C E, and the center O (cutting the Cube into two equal parts) is parallel to A F the baſe of the Cube by Conſtruction: Imagine that by the point *n* be extended a plain Superficies parallel to the former parallel plains which ſhall cut the ſemidiameter O A, and the line C *a* proportionally in the point *n* r: For thoſe lines do touch the extream parallel plains extended by the lines H E and E O, and by the lines A *d* and *d* B; but it is proved that the line O A is divided by an extream and mean proportion in the point *n*: Wherefore the line C *a* is divided alſo by an extream and mean proportion in the point *n*. Again, foraſmuch as B C E is an Iſoſceles triangle, and it is proved that the line B I cutteth the baſe C E into two equal parts in the point I, the angles B I C and B I E ſhall be right angles. Imagine by the line B I and the center O, a plain to paſſe (cutting the Cube into two equal parts) parallel to the baſe A D: And unto thoſe plains let there be imagined another parallel plain paſſing by the point *e*, which let be *n e*, which ſhall cut the ſemidiameter A O, and the half ſide of the Cube, to wit, the line I H, like, in the points *n* and *e*. Wherefore the line I H is in the point *e* divided by an extream and mean proportion:



Wherefore the line H *e* is equal to the line C I or I E, to wit, each are leſſer ſegments. And foraſmuch as the line I *e* is to the line I C (which is equal to the line *e* H) as the whole is to the greater ſegment, take away from the whole line I *e* the greater ſegment I C; there ſhall remain the leſſer ſegment C *e* s: Wherefore the line I *e* is divided by an extream and mean proportion in the point C. Again, unto the ſame plaines imagine another plain to paſſe by the point *a*, parallel to them, and let the ſame be *a g*. Now then t the lines C *a* and C *g* are in like ſort cut in the points *n* and *e*. But the line C *a* was in the point *n* cut by an extream and mean proportion. But the line I C is to the line C *e* as the greater ſegment is to the leſſer: Wherefore the line C *e* is to the line *e g*, as the greater ſegment is to the leſſer; and therefore their proportion is as the whole line I C to the greater ſegment C *e*; and as the greater ſegment C *e* is to the leſſer ſegment *e g*: Wherefore the whole line C *e g* which maketh the greater ſegment and the leſſer, is equal to the whole line I C or I E. And foraſmuch as two parallel plain ſuperficies (to wit, that which is extended by I O B, and that which is extended by the line *a g*) are cut by the plain of the triangle B C E, which paſſeth by the lines *a g* and I B, their common ſections *a g* and I B ſhall be parallels v. But the angle B I E or B I C is a right angle: Wherefore the angle *a g* C is alſo a right angle w, and thoſe right angles are conteined under equal ſides, to wit, the line *g* C is equal to the line C I, and the line *a g* to the line B I x: Wherefore the baſes C *a* and C B are equal y: But of the line C B the line C E was proved to be the greater ſegment; wherefore the ſame line C E is alſo the greater ſegment of the line C *a*: But *c n* was alſo the greater ſegment of the ſame line C *a*: Wherefore unto the line C E

r) 17. 11.
17. 11.
s) 5. 13.
t) 17. 11.
v) 16. 11.
w) 29. 1.
x) 33. 1.
y) 4. 1.

C E the line *c n*, which is the ſide of the Dodecahedron, and is ſet at the Diameter, may be proved equal to lines equal to the line C E. Wherefore the Pentagon inſcribed in the circle, wherein is conteined the triangle B C E, z is equiangled and equilateral. And foraſmuch as two Pentagons ſet upon every one of the baſes of the Cube, do make a Dodecahedron, and ſix baſes of the Cube do receive twelve angles of the Dodecahedron, and the eight ſemidiameters do in the points where they are cut by an extream and mean proportion, receive the reſt: Therefore the twelve Pentagon baſes conteining 20 ſolid angles do inſcribe the Dodecahedron in the Cube a. Wherefore, In a Cube, &c. Which was required to be done.

z) 11. 4.
a) 1. def. 15.

COROLLARIE I.

The Diameter of the Sphere which conteineth the Dodecahedron, conteineth in power theſe two ſides, to wit, the ſide of the Dodecahedron, and the ſide of the Cube wherein the Dodecahedron is inſcribed.

For in the firſt figure, a line drawn from the center O to the point B the angle of the Dodecahedron, to wit, the line O B, conteineth in power the two lines O V the half ſide of the Cube, and V B the half ſide of the Dodecahedron b. Wherefore c the double of the line O B, which is the Diameter of the Sphere conteining the Dodecahedron, conteineth in power the double of the other lines O V and V B, which are the ſides of the Cube and of the Dodecahedron.

b) 47. 1.
c) 15. 5.

COROLLARIE II.

The ſide of a Cube divided by an extream and mean proportion, maketh the leſſer ſegment the ſide of the Dodecahedron inſcribed in it; and the greater ſegment the ſide of the Cube inſcribed in the ſame Dodecahedron.

For it was before proved that the ſide of the Dodecahedron is the greater ſegment of B E the ſide of the triangle B E C; but the ſide B E (which is equal to the lines G B and S F) is the greater ſegment of G F, the ſide of the Cube; which line B E (ſubtending the angle of the Pentagon) was d the ſide of the Cube inſcribed in the Dodecahedron.

d) 8. 15.

COROLLARIE III.

The ſide of a Cube is equal to the ſides of a Dodecahedron inſcribed in it, and circumſcribed about it.

For it was manifeſt by this Propoſition that the ſide of a Cube maketh the leſſer ſegment the ſide of the Dodecahedron inſcribed in it, to wit, as in the firſt figure, the line B S the ſide of the Dodecahedron, is the leſſer ſegment of the line G F the ſide of the Cube, and it was proved e that the ſame ſide of the Cube ſubtendeth the angle of the Pentagon of the Dodecahedron circumſcribed, and therefore it maketh the greater ſegment the ſide of the Dodecahedron, or of the Pentagon f. Wherefore it is equal to both thoſe ſegments.

e) 17. 13.
f) 1 Cor. of 17. 13.

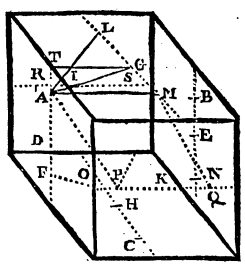
PROP. 14. PROBL. 14.

In a Cube given A B C, *to inſcribe an Icoſahedron.*

Conſtruction LEt the centers of the baſes of the given Cube be the points D, E, G, H, I, and K, by which points, draw in the baſes

unto the other sides parallels not touching one another: And divide the lines drawn from the centers, as the line D T, &c. by an extream and mean proportion in the points A, F, L, M, N, B, P, Q, R, S, C, and O, a) 30. 6. a and let the greater segments be about the centers: And draw the lines A L, A G, A M, and T G.

Demonstration FOrasmuch as the lines cut are parallels to the sides of the Cube, they shall make right angles to one another b: b) 29. 1. And forasmuch as they are equal, their sections shall be equal, for that c) 2. 14. the sections are like c: Wherefore the line T G is equal to the line D T, for they are each half sides of the Cube. Wherefore the square of the whole line T G, and of the lesser segment T A, is triple to the square d) 4. 13. of the line A D the greater segment d: But the line A G conteineth in power the lines A T and T G, for the angle A T G is a right angle. Wherefore the square of the line A G is triple to the square of the line A D: And forasmuch as the line M G L is erected perpendicularly to the plain passing by the lines A T and T G, which is parallel to the bases of the Cube e; therefore the angle A G L is a right angle. But the line L G is equal to the line A D, for they are the greater segments of equal lines: Wherefore the line A G (which is in power triple to the line A D) is in power triple to the line L G: Wherefore adding to the same square of the line A G the square of the line L G, the square of the line A L, which f conteineth in power the two lines A G and G L, shall be quadruple to the line A D or L G. Wherefore the line A L is double to the line A D g; and therefore is equal to the line A F, or to the line L M. And by the same reason may we prove that every one of the other lines which couple the next sections of the lines cut, as the lines A M, P F, P M, M Q, and the rest, are equal: Wherefore the triangles A L M, A P F, A M P, P M Q, and the rest such like are equal, equiangled, and equilateral h. And forasmuch as upon every one of the lines cut of the Cube are set two triangles, as the triangles A L M and B L M, there shall be made 12 triangles. And forasmuch as under every one of the eight angles of the Cube are subtended the other eight triangles, as the triangle A M P, &c. of twelve and eight triangles shall be produced twenty triangles, equal and equilateral, containing the solid of an Icosahedron i, which shall be inscribed in the given Cube A B C k, the invention of the demonstration of this dependeth of the ground of the former. Wherefore, In a given Cube, &c. Which was required to be done.



c) C. 14. 11.

f) 47. 1.

g) 20. 6.

h) 4, 8. 1.

i) 25. de. 11.

k) 1. def. 15.

COROLLARIE I.

The Diameter of a Sphere which conteineth an Icosahedron, conteineth two sides,

sides, to wit, the side of the Icosahedron, and the side of the Cube which conteineth the Icosahedron.

For if we draw the line A B, it shall make the angles at the point A right angles; for that it is a parallel to the sides of the Cube: Wherefore the line which coupleth the opposite angles of the Icosahedron at the poines F and B, conteineth in power the line A B (the side of the Cube) and the line A F (the side of the Icosahedron) l which line F B is equal to the diameter of the sphere, which conteineth the Icosahedron m. l) 47. 1. m) 16. 13.

COROLLARIE II.

The six opposite sides of the Icosahedron divided into two equal parts, their sections are coupled by three equal right lines, cutting one another into equal parts, and perpendicularly in the center of the Sphere which conteineth the Icosahedron.

For these three lines are the three which couple the centers of the bases of the Cube, which do in such sort in the center of the Cube cut one another n, and therefore are equal to the sides of the Cube. But right lines drawn from the center of the Cube to the angles of the Icosahedron, every one of them shall subtend the half side of the Cube, and the half side of the Icosahedron (which half sides contein a right angle:) Wherefore those lines are equal. Whereby it is manifest that the foresaid center is the center of the sphere which conteineth the Icosahedron. n) Co. 3. 15.

COROLLARIE III.

The side of a Cube divided by an extream and mean proportion, maketh the greater segment the side of an Icosahedron described in it.

For the half side of the Cube maketh the half of the side of the Icosahedron the greater segment: Wherefore also the whole side of the Cube maketh the whole side of the Icosahedron the greater segment o, for the sections are like p. o) 15. 5. p) 2. 14.

COROLLARIE IV.

The sides and bases of the Icosahedron, which are opposite to one another, are parallels.

Forasmuch as every one of the opposite sides of the Icosahedron, may be in the parallel lines of the Cube, to wit, in those parallels which are opposite in the Cube, and the triangles which are made of parallel lines, are parallels q. Therefore the opposite triangles of the Icosahedron, as also the sides, are parallels to one another. q) 15. 11.

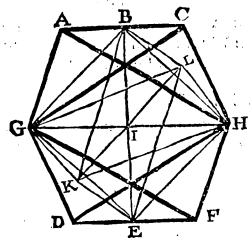
PROP. 15. PROBL. 15.

In an Icosahedron given A C D F, *to inscribe an Octohedron* B G E H K L.

Construction LEt a there be taken the three right lines which cut one another into two equal parts perpendicularly, and which couple the sections into two equal parts of the sides of the Icosahedron; which let be B E, G H, and K L, cutting one another in the point I, and draw the lines B G, G E, E H, and H B. a) 2 C. 14. 15

Demon-

Demonſtration FOraſmuch as the angles at the point I are (by Conſtruction) right angles, and are conteined under equal lines; the
b) 4. 1. baſes G B and H E ſhall make a ſquare [b]. Likewiſe unto thoſe baſes ſhall be equal the lines drawn from the points K and L, to every one of the points B, G, E, and H; and therefore the triangles which make the Pyramis B G E H K, ſhall be equal and equilateral. And by the ſame reaſon ſhall the reſt of the triangles which make the other Pyramis B G E H L upon the ſame baſe B G E H, be equal and equilateral. Wherefore B G E H K L ſhall be an Octohedron [c], and ſhall be inſcribed in the Icoſahedron [d]: Wherefore, In an Icoſahedron, &c.

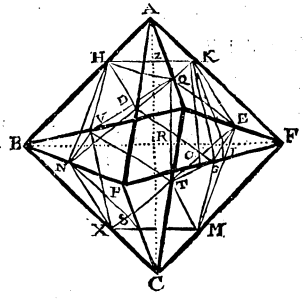


c) 23. d. 11.
d) 1. de. 15.

Which was required to be done.

PROP. 16. PROBL. 16.

In an Octohedron given A B C F P L, *to inſcribe an Icoſahedron.*



Conſtruction LEt the ſix angles of the Octohedron be A, B, C, F, P, and L, and draw the lines A C, B F, and P L, cutting one another perpendicularly in the point R [a]; and let every one of the twelve ſides of the Octohedron be divided by an extream and mean proportion in the points H, X, M, K, D, S, N, G, V, E, Q, and T; and let the greater ſegments be the lines B H, B X, F M, F K, A D, A Q, C S, C I, P N, P G, L V, and L E, and draw the lines H K, X M, G E, N V, D S, and Q T.

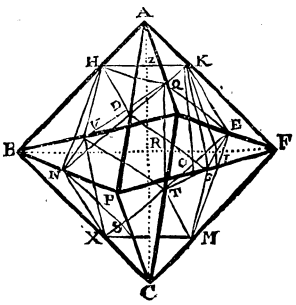
a) 2. Cor. of 14. 13.

Demonſtration FOraſmuch as in the triangle A B F, the ſides are cut proportionally, to wit, as the line B H is to the line H A, ſo
b) 2. 14. is the line F K to the line K A [b]; therefore the line H K ſhall be a parallel
c) 2. 6. to the line B F [c]. And foraſmuch as the line A C cutteth the line H K in the point Z, and the line Z K is a parallel to the line R F, the line R A
d) 2. 6. ſhall be cut by an extream and mean proportion in the point Z [d]; to wit, ſhall be cut like unto the line F A; and the greater ſegment thereof
e) 3. 1. ſhall be the line Z R. Unto the line Z R put the line R O equal [e], and
f) 33. 1. draw the line K O. Now then the line K O ſhall be equal to the line Z R [f]. Draw the lines K G, K E, and K I. And foraſmuch as the triangles A R F
g) 6. 6. and A Z K, are equiangled [g], the ſides A Z and Z K ſhall be equal to
h) 4. 6. one another [h]; for the ſides A R and R F are equal: Wherefore the line Z K

Z K ſhall be the leſſer ſegment of the line R A. But if the greater ſegment R Z be divided by an extream and mean proportion, the greater ſegment thereof ſhall be the line Z K, which was the leſſer ſegment of the whole line R A i. And foraſmuch as the two lines F E and F G, are equal to the two lines A H and A K, to wit, each are leſſer ſegments of equal ſides of the Octohedron, and the angles H A K and E F G are equal, namely, are right angles [k], the baſes H K and G E ſhall be equal [l]. And by the ſame reaſon the other lines X M, N V, D S, and Q T, may be proved equal. And foraſmuch as the lines A C, B F, and P L, do cut one another in two equal parts, and perpendicularly by Conſtruction; the lines H K and G E (which ſubtend angles of triangles like unto the triangles whoſe angles the lines A C, B F, and P L, ſubtend,) are cut in two equal parts in the points Z and I [m]; ſo alſo are the other lines N V, X M, D S, and Q T, (which are equal to the lines H K and G E) cut in like ſort, and they ſhall cut the lines A C, B F, and P L, like. Wherefore the line K O (which is equal to R Z) ſhall make the greater ſegment the line R O, which is equal to the line Z K, (for the greater ſegment of R Z was the line Z K) and therefore the line O I ſhall be the leſſer ſegment: When as the whole line R I is equal to the whole line R Z. Wherefore the ſquares of the whole line K O and of the leſſer ſegment O I, are triple to the ſquare of the greater ſegment R O [n]: Wherefore the line K I, which conteineth in power the two lines K O and O I, is in power triple to the line R O [o]; for the angle K O I is a right angle. And foraſmuch as the lines F E and F G (which are the leſſer ſegments of the ſides of the Octohedron) are equal; and the line F K is common to them both, and the angles K F G and K F E (of the triangles of the Octohedron) are equal, the baſes K G and K E ſhall [p] be equal: And therefore the angles K I E and K I G, which they ſubtend, are equal [q]: Wherefore they are right angles [r]. Wherefore the right line K E (which conteineth in power the two lines K I and I E [s], is in power quadruple to the line R O (or I E); for the line K I is proved to be in power triple to the ſame line R O: But the line G E is double to the line I E. Wherefore the line G E is alſo in power quadruple to the line I E [t]. Wherefore the two lines K E and G E are equal. And by the ſame reaſon may the reſt of the lines, to wit, H K, H N, N V, V X, and X S, and the other lines which couple the ſections of the ſides of the Octohedron, be proved equal to the ſame lines K E and G E. Wherefore the triangles deſcribed of them, to wit, G E K, G K D, G D S, G S M, and G M E, ſhall be equal and equilateral [v], making a ſolid angle at the point G, which is therefore the angle of an Icoſahedron [w], and is ſet in the ſection G of the ſide P F. And by the ſame reaſon may be proved that the reſt of the eleven ſolid angles of the Icoſahedron, are ſet in the ſections of every one of the ſides of the Octohedron, to wit, in the points E, N, V, H, K, M, X, D, S, Q, and T. Wherefore there are twelve angles of the Icoſahedron. Moreover, foraſmuch as every one of the baſes of the Octohedron do each contein triangles of the Icoſahedron, as in the Pramis A B C F P (which is the half of the Octohedron) the triangle F C P receiveth in the ſections of his ſides the triangle G M S, and the triangle C P B conteineth the triangle N X S, and the triangle B A P conteineth the triangle H N D: And moreover, the triangle A P F conteineth the triangle K D G, and the ſame may be proved in the oppoſite Pyramis A B C F L: Wherefore there ſhall be eight triangles. And foraſmuch

i) 5. 13.
k) 14. 13.
l) 4. 1.
m) 4. 6.
n) 4. 13.
o) 47. 1.
p) 4. 1.
q) 8. 1.
r) 13. 1.
s) 47. 1.
t) 20. 6.
v) 8. 1.
w) 16. 13.

asmuch as besides these triangles, to every one of the solid angles of the Octohedron, are subtended two triangles, as the triangles K E G and M E G, to the angle F, and the triangles H N V and X N V to the angle B, also the triangles N D S and G D S to the angle P; likewise the triangles D H K and Q H K to the angle A: Moreover the triangles E Q T and V Q T to the angle L, and finally, the triangles S X M and T X M to the angle C, these twelve triangles being added to the eight former triangles, shall produce twenty triangles equal and equilateral coupled together; which shall make an Icosahedron x. And it shall be inscribed in the Octohedron given A B C F P L y; for the twelve angles thereof are set in twelve like sections of the sides of the Octohedron. Wherefore, In an, &c. Which was required to be done.



x) 25. d. 11.

y) 1. dc. 15.

COROLLARIE I.

The side of an equilateral triangle being divided by an extream and mean proportion, a right line subtending within the triangle, the angle which is conteined under the greatest segment and and the lesser, is in power duple to the lesser segment of the same side.

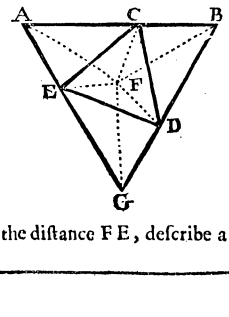
For the line K E which subtendeth the angle K F E of the triangle A F L, which angle K F E is conteined under the two segments K F and F E, was proved equal to the line H K, which conteineth in power the two lesser segments H A and A K z; for the angle H A K is a right angle: Wherefore the line K E or H K is in proportion duple to the line A K.

z) 47. 1.

COROLLARIE II.

The bases of the Icosahedron are concentrical (that is, have one and the same center) with the bases of the Octohedron which conteineth it.

For suppose that A B G be the base of an Octohedron, conteining E C D the base of an Icosahedron; and let the center of the base A B G be the point F, and draw the lines F A, F B, F C, and F E. Now then the two lines F A and A E shall be equal to the two lines F B and B C; for they are lines drawn from the center, and are also lesser segments, and they contein the halves of equal angles. Wherefore a the bases F C and F E are equal, and by the same reason unto them shall be equal the other line F D. Wherefore making the center the point F, with the distance F E, describe a circle, and it shall be circumscribed about the

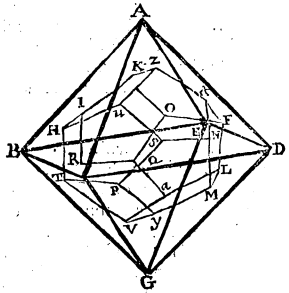


a) 4. 1.

the triangle C E D, and so shall the point F the center of the base of the Octohedron, be the center of C E D, the base of the Icosahedron.

PROP. 17. PROBL. 17.

In an Octohedron given ABGDEC, *to inscribe a Dodecahedron.*



Construction LEt the twelve sides of the Octohedron be cut by an extream and mean proportion a. It was manifest that of the right lines which couple these sections are made twenty triangles, of which eight are concentrical with the bases of the Octohedron b.

a) 16. 15.

b) 3 Cor. of 16. 15.

Demonstration IF therefore in every one of the centers of the twenty triangles be inscribed c every one of the twelve angles of the Dodecahedron, we shall find that eight angles of the Dodecahedron are set in the eight centers of the bases of the Octohedron, to wit, those angles I, u, S, O, M, a, P, and X, and of the other twelve solid angles, there are two in the centers of the two triangles which have one side common under every one of the solid angles of the Octohedron, to wit, under the solid angle A, the two solid angles K and Z; under the solid angle B, the two solid angles H and T; under the solid angle G, the two solid angles Y and V; under the solid angle D, the two solid angles F and L; under the solid angle E, the two solid angles S and N; and under the solid angles C, the two solid angles Q and R. And forasmuch as in the Octohedron are six solid angles, under them shall be subtended twelve solid angles of the Dodecahedron; and so are made twenty solid angles, composed of twelve equal and equilateral superficial Pentagons d; which therefore contein a Dodecahedron e. And it is inscribed in the Octohedron f, for that every one of the bases of the Octohedron do receive angles thereof. Wherefore, &c. Which was required to be done.

c) 5. 15.

d) 5. 15.

e) 14. d. 11.

f) 1. def. 15.

PROP. 18. PROBL. 18.

In a trilateral and equilateral Pyramis, to inscribe a Cube.

Construction LEt the base of the given Pyramis be A B C, and his top the point D; and let it be comprehended in a Sphere a; and let the center of that sphere be the point E, and from the solid angles A, B, C, and D, draw right lines passing by the center E, unto the opposite bases of the Pyramis, and they shall fall perpendicularly upon the bases, and shall also fall upon the centers of the circles which contein the bases b. Let the center of the triangle A B C be the point G; and let the center of the triangle A D C be the point H, and of the triangle A D B, let the point N be the center; and lastly, let the point F be the center of the other tri-

a) 13. 13.

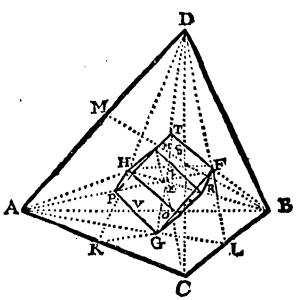
b) C. 13. 13.

triangle DBC. And let the right lines falling upon thoſe centers be DEG, BEH, CEN, and AEF. And by thoſe centers G, H, N, and F, let there be drawn from the angles to the oppoſite ſides, theſe right lines AGL, DHK, BNM, and DFL, which ſhall fall perpendicularly upon the ſides BC, CA, AD, and CB [c]; and therefore they ſhall cut them into two equal parts in the points K, L, and M [d]. Again let the lines which were drawn from the ſolid angles to the oppoſite baſes be divided into two equal parts, to wit, the line DG in the point T, the line CN in the point O, the line AF in the point P, and the line BH in the point R, and draw the lines HT, FT, HO, and FO.

c) C. 12. 13.
d) 3. 3.

Demonſtration FOraſmuch as the lines GK and GL, which are drawn from the center of one and the ſame triangle ABC, to the ſides, are equal, and the lines DK and DL are equal, for they are the perpendiculars of equal and like triangles; and the line DG is common to them. Wherefore [e] the angles KDG and LDG are equal. And foraſmuch as the lines HD and DF are drawn from the center of equal circles, which contein the equal triangles ADC and DBC, therefore they are equal; and the line DT is common to them both, and they contein equal angles, as before hath been proved. Wherefore the baſes HT and FT are equal [f]. And by the ſame reaſon if we draw the lines CF and CH, may we prove that the other lines HO and FO are equal to the ſame lines HT and FT, and alſo to one another. Wherefore alſo after the ſame manner may be proved that the reſt of the lines which couple the centers of the triangles, and the ſections of the perpendicular into two equal parts, as the lines NP, GR, GP, RN, NT, PH, GO, and RF, are equal. And foraſmuch as from every one of the centers of the baſes are drawn three right lines to the ſections into two equal parts of the perpendiculars, and there are four centers, it followeth that thoſe equal right lines ſo drawn are twelve in number, of which every three and three make a ſolid angle in the four centers of the baſes, and in the four ſections, into two equal parts of the perpendiculars: Wherefore that ſolid hath eight angles, conteined under twelve equal ſides, which make ſix quadrangled figures, to wit, HOFT, PGRN, PHOG, GOFR, FRNT, INPH. Now we will prove that thoſe quadrangled figures are rectangled.

e) 8. 1.
f) 4. 1.



Foraſmuch as upon DC the common baſe of the triangles ADC and BDC, falleth the perpendiculars AS and BS, which are drawn by the centers H and F, either of theſe lines HS and SF ſhall be the third part of either of theſe lines AS and SB; for the line AH is double to the line HS, and divideth the baſe DC into two equal parts [g]: Wherefore in the triangle ABS, the ſides AS and BS are cut proportionally in the points

g) C. 12. 13.

H

H and F; and therefore the line HF is a parallel to the ſide AB [h]. Wherefore the triangles ASB and HSF are equiangled [i]: Wherefore the baſe HF ſhall be the third part of the baſe AB [k]. We may alſo prove that the line TO is the third part of the line DC; for the lines EC and ED, which are drawn from the center of the Sphere which conteineth the Pyramis are equal; and the line EN (which is drawn from the center to the baſe) is the third part of the line EC, ſo alſo is the line GE the third part of the line ED [l]; for it is the ſixth part of the diameter of the Sphere which conteineth the Pyramis. And the line ON is the half of the whole line NC. Wherefore the reſidue EO is the third part of the line EC, and ſo alſo is the line ET the third part of the line ED. Wherefore the line TO in the triangle DEC is a parallel to the line DC, and is a third part of the ſame [m]; as the line HF was proved the third part of the line AB. But AB and DC being ſides of the Pyramis, are equal: Wherefore the lines HF and TO being the third parts of equal lines, are equal [n]. Wherefore [o] the angles HTF and TFO are equal; and by the ſame reaſon, the angles oppoſite to them, to wit, the angles FOH and OHT are equal to one another; and alſo are equal to the ſaid angles HTF and TFO; but theſe four angles are equal to four right angles [p]: Wherefore the angles of the quadrangle HOFT are right angles. And by the ſame reaſon may the angles of the other five quadrangled figures be proved right angles. Now reſteth to prove that the foreſaid quadrangles are each of them in one and the ſame plain.

h) 2. 6.
i) 6. 6.
k) 4. 6.
l) C. 13. 13.
m) 2, 4. 6.
n) 15. 5.
o) 8. 1.
p) C. 32. 1.

Take the quadrangle HOFT, and foraſmuch as in the triangle ASB, the line HF is proved a parallel to the line AB; therefore it cutteth the lines SV and SB proportionally, in the points I and F [q]. Now then foraſmuch as SF was proved the third part of the line SB, the line SI ſhall alſo be the third part of the line SV. Moreover, foraſmuch as the line VS which coupleth the Sections into two equal parts of the oppoſite ſides of the Pyramis, to wit, of the ſides AB and DC, is by the center E divided into two equal parts [r] (for it is the diameter of the Octohedron inſcribed in the Pyramis) therefore the line ST is two third parts of the half line SE. And by the ſame reaſon, Foraſmuch as in the triangle DEC, the line TO is proved to be a parallel to the ſide DC, it ſhall in the ſame triangle cut the lines CE and SE proportionally, in the points O and I [ſ]. But the line EO is proved to be a third part of the line EC: Wherefore the line EI is alſo a third part of the line ES. Wherefore the reſidue IS ſhall be two third parts of the whole line ES. Wherefore the point I cutteth either of the lines TO and HF. Wherefore the two lines HIF and TIO, cutting one another, are in one and the ſame plain [t]. And therefore the points H, T, F, and O, are in one and the ſame plain. Wherefore the rectangled figure HOFT being quadrilateral and equilateral, and in one and the ſame plain, is a ſquare, by the definition of a ſquare. And by the ſame reaſon may the reſt of the baſes of the ſolid be proved to be ſquares equal and plain, or ſuperficial: Now then the ſolid is comprehended of ſix equal ſquares (which are conteined of twelve equal ſides) which ſquares make eight ſolid angles, of which four are in the centers of the baſes of the Pyramis, and the other four are in the middle ſections of the four perpendiculars. Wherefore the ſolid HOFTPGRN is a Cube [v], and is inſcribed in the Pyramis [w]. Wherefore, In a trilateral, &c. Which was required to be done.

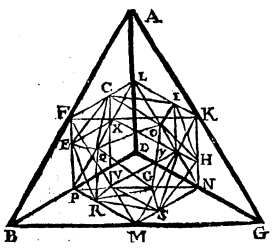
q) 2. 6.
r) Co. 2. 15.
ſ) 2. 6.
t) 2. 11.
v) 21. d. 11.
w) 1. d. 15.

COROLLARIE

The line which cutteth into two equal parts the oppoſite ſides of the Pyramis, is triple to the ſide of the Cube inſcribed in the Pyramis, and paſſeth by the center of the Cube.

For the line S E V, whoſe third part the line S I is, cutteth the oppoſite ſides C D and A B into two equal parts: But the line E I (which is drawn from the center of the Cube to the baſe) is proved to be a third part of the line E S: Wherefore the ſide of the Cube which is double to the line E I, ſhall be a third part of the whole line V S, which is (as hath been proved) double to the line E S.

PROP. 19. PROBL. 19.



In a trilateral equilateral Pyramis given A B G D, *to inſcribe an Icoſahedron.*

Conſtruction LEt each of the ſides of the given Pyramis be divided into two equal parts in the points F, M, K, L, P, and N, and in every one of the baſes of that Pyramis, deſcribe the triangles L F P, P M N, N K L, and F M K, which triangles ſhall be equilateral [a]; for the ſides ſubtend equal angles of the Pyramis, conteined under the halves of the ſides of the ſame Pyramis: Wherefore the ſides of the ſaid triangles are equal: Let thoſe ſides be divided by an extream and mean proportion [b], in the points C, E, Q, R, S, T, H, I, O, V, Y, and X. Now then, thoſe ſides are cut into the ſame proportion [c]; and therefore they make the like Sections equal [d]. Now I ſay that the foreſaid points do receive the angles of the Icoſahedron inſcribed in the Pyramis A B G D.

In the foreſaid triangles, Let there again be made other triangles, by coupling the ſections, and let thoſe triangles be T R S, I O H, C E Q, and V X Y, which ſhall be equilateral; for every one of their ſides do ſubtend equal angles of equilateral triangles, and thoſe ſaid equal angles are conteined under equal ſides (to wit, under the greater ſegment and the leſſer.) And therefore the ſides which ſubtend thoſe angles are equal [e].

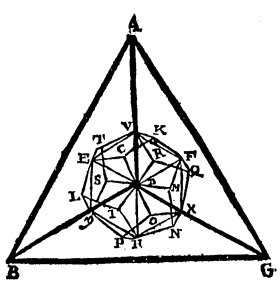
Demonſtration NOw let us prove that at each of the foreſaid points, as for example at T, is ſet the ſolid angle of an Icoſahedron. Foraſmuch as the triangles T R S and T Q O are equilateral and equal, the four right lines T R, T S, T Q, and T O, ſhall be equal. And foraſmuch as F P N K is a ſquare, cutting the Pyramis A B G D into two equal parts [f], the line T H ſhall be in power double to the line T N or N H [g]. For the lines T N or N H are equal, for that by Conſtruction they are each of them leſſer ſegments, and the line R T or T S is in power double to the ſame line T N or N H [h]; for it ſubtendeth the angle of the triangle conteined under the two ſegments. Wherefore the lines T H, T S,

a) 4. 1.
b) 30. 6.
c) 2. 4.
d) 9. 5.
e) 4. 1.
f) Co. 2. 15.
g) 47. 1.
h) C. 16. 15.

T S, T R, T Q, and T O, are equal, and ſo alſo are the lines H S, S R, R Q, Q O, and O H, which ſubtend the angles at the point T equal. For the line Q R conteineth in power the two lines P Q and P R, the leſſer ſegments, which two lines the line T H alſo conteined in power. And the reſt of the lines do ſubtend angles (of equilateral triangles) conteined under the greater and the leſſer ſegments. Wherefore the five triangles T R S, T S H, T H O, T O Q, and T Q R, are equilateral and equal, making the ſolid angle of the Icoſahedron at the point T, [i] in the ſide P N of the triangle P N M. And by the ſame reaſon, in the other ſides of the four triangles P N M, N K L, F M K, and L F P, (which are inſcribed in the baſes of the Pyramis) (which ſides are twelve in number, ſhall be ſet twelve angles of the Icoſahedron, conteined under twenty equal and equilateral triangles, of which four are ſet in the four baſes of the Pyramis, to wit, thoſe four triangles T R S, H O I, C E Q, and V X Y, four triangles are under four angles of the Pyramis; that is, the four triangles C I X, Y S H, E R V, and T Q O, and under every one of the ſix ſides of the Pyramis are ſet two triangles, to wit, under the ſide D G, the triangle T H S and T H O; under the ſide D B, the triangles R Q E and R Q T; under the ſide D A the triangles C O Q and C O I; under the ſide A B the triangles E X C and E X V; under the ſide B G the triangles S V R and S V Y; and under the ſide A G, the triangles I Y H and I Y X. Wherefore the ſolid being conteined under twenty equilateral and equal triangles, ſhall be an Icoſahedron [k], and ſhall be inſcribed in the Pyramis A B G D [l]; for all his angles do at one time touch the baſes of the Pyramid. Wherefore, In a trilateral, &c. Which was required to be done.

i) 16. 13.
k) 23. d. 11.
l) 1. def. 15.

PROP. 20. PROBL. 20.



In a trilateral equilateral Pyramis given A B G D, *to inſcribe a Dodecahedron.*

Conſtruction LEt each of the ſides of the given Pyramis be cut into two equal parts, and draw the lines which couple the ſections, which being divided by an extream and mean proportion, and right lines being drawn by the ſections, ſhall receive twenty triangles, making an Icoſahedron [a].

Now then if we take the centers of thoſe triangles, we ſhall there find the twenty angles of the Dodecahedron inſcribed in it [b]. And foraſmuch as four baſes of the foreſaid Icoſahedron are concentrical with the baſes of the Pyramis [c], there ſhall be placed four angles of the Dodecahedron, to wit, the four angles E, F, H, and D, in the four centers of the baſes; and of the other ſixteen angles, under every one of the ſix ſides of the Pyramis are ſubtended two, to wit, under

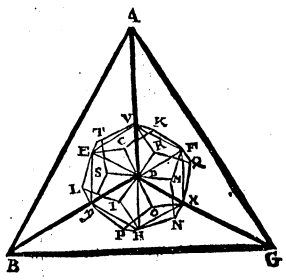
a) 19. 15.
b) 5. 15.
c) 2 Cor. of 6. 15.

under the side A D, the angles C and K; under the side B D, the angles L and I; under the side G D, the angles M and N; under the side A B, the angles T and S; under the side B G, the angles P and O; and under the side A G, the angles R and Q; so there rests four angles, whose true places we will now appoint.

Demonstration FOrasmuch as a Cube conteined in one and the same Sphere with the Dodecahedron, is inscribed in the same Dodecahedron, it followes that a Cube and a Dodecahedron circumscribed about it, are conteined in one and the same bodies, for that their angles meet in the same points. And as was proved [c] that four angles of the Cube inscribed in the Pyramis are set in the middle sections of the perpendiculars which are drawn from the solid angles of the Pyramis to the opposite bases. Wherefore the other four angles of the Dodecahedron are also as the angles of the Cube set in those middle sections of the perpendiculars, to wit, the angle V, is set in the middest of the perpendicular A H; the angle Y in the middest of the perpendicular B F; the angle X in the middest of the perpendicular G E; and lastly, the angle D in the middest of the perpendicular D, which is drawn from the top of the Pyramis to the opposite base. Wherefore those four angles of the Dodecahedron may be said to be directly under the solid angles of the Pyramis, or they may be said to be set at the perpendiculars. Wherefore the Dodecahedron after this manner set, is inscribed in the Pyramis given [f]; for that upon each of the bases of the Pyramis is set an angle of the Dodecahedron inscribed. Wherefore, In a, &c. Which was required to be done.

d) 17. 13. and 8. 15.

c) 18. 15.



f) 1. def. 15.

PROP. 21. PROBL. 21.

In every one of the regular Solids to inscribe a Sphere.

Demonstration IT was declared [a] that the five regular Solids are so conteined in a sphere that right lines drawn from the center of the sphere, or of the solid inscribed, to every one of the angles of the solid inscribed, are equal, which right lines therefore make Pyramids, whose tops are in the center of the Sphere, or of the Solid, and the bases are every one of the bases of those Solids. And forasmuch as those bases are in every Solid equal and like to one another, and described in equal circles, those circles shall cut the Sphere; for the angles which touch the circumference of the circle, touch also the Superficies of the Sphere: Wherefore perpendiculars drawn from the center of the Sphere to the bases, or to the plain Superficies of the equal circles, are equal: Wherefore making the center the center of the Sphere which conteineth the Solid, and the distance some

a) 13, 14, 15, 16, and 17. of the 13.

some one of the equal perpendiculars, describe a Sphere, and it shall touch each of the bases of the Solid, neither shall the Superficies of the Sphere passe beyond those bases, when as those perpendiculars are the least lines which are drawn from the center to the bases. Wherefore, We have [b], &c. Which was required to be done.

b) C. 18. 13.

COROLLARIE.

The regular figures inscribed in Spheres, and also the Spheres circumscribed about them, or conteining them, have one and the same center.

Namely, their Pyramids, the angles of whose bases touch the Superficies of the Sphere, do from those angles cause equal right lines to be drawn to one and the same point, making the tops of the Pyramids in the same point; and therefore they make the centers of the Spheres in the same tops, when as the right lines drawn from those angles to the crooked Superficies, wherein are set the angles of the bases of the Pyramids are equal.

An Advertisement.

Of these Solids, only the Octohedron receiveth the other Solids inscribed one within another. For the Octohedron conteineth the Icosahedron inscribed in it, and the same Icosahedron conteineth the Dodecahedron inscribed in the same Icosahedron; and the same Dodecahedron conteineth the Cube inscribed in the same Octohedron. And lastly, the same Cube circumscribeth the Pyramis inscribed in the said Octohedron. But this happeneth not in the other Solids.

The End of the Fifteenth Element of EUCLIDE, *after* CAMPANE *and* FLUSSAS.

THE

THE SIXTEENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.



HAving in the Fifteenth Book shewed how to inscribe the five regular Solids one within another, we shall here in this Sixteenth Book compare those Solids so inscribed one with another, and declare their Passions and Properties; which in this Book (aacording to *Flussas*) is excellently well performed, for the which he deserveth perpetual praise and commendations: In the perusal and practice whereof the Studious Reader shall take much delight, and have occasion offered him to invent and contrive greater variery of Properties and Passions incident to the said Regular Bodies.

PROPOSITIONS, and THEOREMES.

PROPOSITION 1. THEOREM 1.

A Dodecahedron and a Cube inscribed in it, and a Pyramis inscribed in the same Cube, are conteined in one and the same Sphere.

Demon-

Demonstration FOr the angles of the Pyramis are set in the angles of the Cube wherein it is inscribed [a]; and all the angles of the Cube are set in the angles of the Dodecahedron circumscribed about it [b]. And all the angles of the Dodecahedron are set in the Superficies of the Sphere [c]. Wherefore those three Solids inscribed one within another, are conteined in one and the same Sphere [d]: Therefore, A Dodecahedron, &c. Which was to be demonstrated.

a) 1. 15.
b) 8. 15.
c) 17. 13.
d) 1. def. 15.

COROLLARIE.

These three Solids likewise are set in one and the self-same Icosahedron, or Octohedron, or Pyramis.

For they are inscribed in one and the same Icosahedron [e], and they are inscribed in one and the same Octohedron [f]. Lastly, they are inscribed in one and the same Pyramis [g]; for the angles of all those Solids are set in the centers of the bases of the circumscribed Icosahedron, or Octohedron, or Pyramis.

e) 5, 11, & 12. of the 15.
f) 4, 6, & 16. of the 15.
g) 18, & 19. of the 15.

PROP. 2. THEOR. 2.

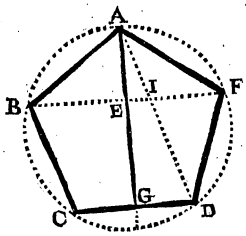
The proportion of a Dodecahedron circumscribed about a Cube, to a Dodecahedron inscribed in the same Cube, is triple to an extream and mean proportion.

Demonstration FOr asmuch as [a] it was proved that the side of a Dodecahedron inscribed in a Cube, is the lesser segment of the side of that Cube divided by an extream and mean proportion; and the side of the Dodecahedron circumscribed about the same Cube is the greater segment of the side of the same Cube [b], the side of the Dodecahedron circumscribed shall be to the side of the Dodecahedron inscribed; as the greater segment of a right line divided by an extream and mean proportion, is to the lesser segment of the same, which proportion is called an extream and mean proportion [c]. But the proportion of like Solid Polihedrons is triple to the proportion of the sides of like proportion [d]: Wherefore the proportion of the Dodecahedron circumscribed about the Cube, is to the Dodecahedron inscribed in the same Cube in triple proportion of the sides joyned together by an extream and mean proportion: Therefore, The proportion, &c. Which was to be demonstrated.

a) 2 Cor. of 13. 15.
b) 13. 15.
c) 30. 6.
d) C. 17. 12.

PROP. 3. THEOR. 3.

In every equiangled and equilateral Pentagon ABCDF, *a perpendicular* AG *drawn from one of the angles* A, *to the base* CD, *is divided in the point* I *by an extream and mean proportion, by a right line* BF, *subtending the same angle* BAF.



Demonſtration FOraſmuch as the angles GAF and GAB are equal [a], and the angles ABF and AFB are equal [b], therefore the angles remaining at the point E, of the triangles AEB and AEF, are equal; for that they are the reſidues of two right angles [c]. But the angle EGC is by conſtruction a right angle. Wherefore the lines BF and CD are parallels [d]. Wherefore as the line DI is to the line IA, ſo is the line GE to the line EA [e]. But the line DA is in the point I divided by an extream and mean Proportion [f]. Wherefore the line GA is in the point E divided by an extream and mean proportion [g]. Wherefore, In every equiangled and equilateral Pentagon, &c. Which was to be demonſtrated.

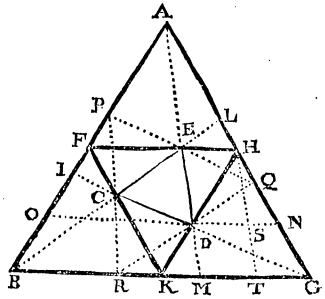
a) 27. 3.
b) 5. 1.
c) C. 32. 1.
d) 28. 1.
e) 2. 6.
f) 8. 13.
g) 2. 14.

COROLLARIE.

The line which ſubtendeth the angle of a Pentagon, is a parallel to the ſide oppoſite to the angle.

As was manifeſt in the lines BF and CD.

PROP. 4. THEOR. 4.



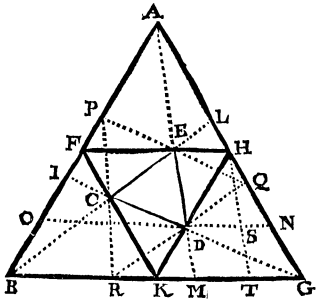
If from the angles A, B, *and* G, *of the baſe of a Pyramis* ABG, *be drawn to the oppoſite ſides* AB, BG, *and* GA, *the right lines* GI, AM, *and* BL, *cutting the ſaid ſides by an extream and mean proportion, they ſhall contein the baſe of the Icoſahedron* CDE; *inſcribed in the Pyramis, which baſe ſhall be inſcribed in an equilateral triangle* FKH, *whoſe angles* C, D, *and* E, *cut the ſides of the baſe of the Pyramis by an extream and mean proportion.*

Conſtruction THe equilateral triangle is deſcribed by dividing the ſides of the baſe of the Pyramis into two equal parts: And the baſe of the Icoſahedron is inſcribed in the Pyramis, by dividing the ſides FK, KH, and HF, by an extream and mean proportion in the points C, D, and E [a]. Again, Let the ſides AB, BG, and GA, be divided by an extream and mean proportion in the points I, M, and L [b], and draw AM, BL, and GI: I ſay thoſe lines deſcribe the triangle CDE, of the Icoſahedron.

a) 19. 5.
b) 30. 6.

Demonſtration FOr foraſmuch as the lines BG and FH are parallels [c], by the point D, let the line ODN be drawn parallel to either of the lines BG and FH. Wherefore the triangle HDN ſhall be like to the triangle HKG [d]: Wherefore either of theſe lines DN and NH, ſhall be equal to the line DH, the greater ſegment of the line KH or FH. And foraſmuch as the line FO is a parallel to the line HK, and the line OD to the line FH; the line OD ſhall be equal to the whole line FH, in the parallelogram FODH [e]. Wherefore as the whole line FH is to the greater ſegment FE, ſo ſhall the lines equal to them be, to wit, OD and DN [f]. Wherefore the line ON is divided by an extream and mean proportion in the point D [g]. But the triangles AOD, AFE, and ABM, are like to one another; and ſo alſo are the triangles ADN, AEH, and AMG [h]. Wherefore as FE is to EH, ſo is OD to DN, and BM to MG. Wherefore the line AM cutting the lines FH and ON, like to the line BG, in the points E, D, and M, deſcribeth ED the ſide of the triangle of the Icoſahedron ECD, which is deſcribed in the ſections E, C, and D, by ſuppoſition. And by the ſame reaſon, the lines BL and GI, ſhall deſcribe the other ſides EC and CD, of the ſame triangle. By the point E, let there be drawn to GI, a parallel line PEQ. Now foraſmuch as the lines BM and FE are parallels, the line AM is in the point E, cut like to the line AB in the point F [i]: Wherefore the line AE is equal to the line EM, and unto the line EM are alſo equal either of the lines GD and DI, which are cut like unto the foreſaid lines. Again, foraſmuch as in the triangle ADI, the lines DI and EP are parallels, as the line DI is to the line EP, ſo is the line AD to the line AE: But as the line AD is to the line AE, ſo is the line DG to the line EQ [k]: Wherefore as the line DI is to the line EP, ſo is the line DG to the line EQ; and alternately, as the line DI is to the line DG, ſo is the line EP to the line EQ: But the lines DI and IG are equal; wherefore alſo the lines EP and EQ are equal. And foraſmuch as the line AH is equal to the line FH, whoſe greater ſegment is the line HN; therefore the whole line AN is divided by an extream and mean proportion in the point H [l]. But as the line AN is to the line AH, ſo is the line AD to the line AE [m] (for the lines FH and ON are parallels) and again, as the line AD is to the line AE, ſo [n] is the line AG to the line AQ, and the line AI to the line AP; for the lines PQ and GI are parallels: Wherefore the lines AG and AI are divided by an extream and mean proportion in the points Q and P, and the line AQ ſhall be the greater ſegment of the line AG or AB. And foraſmuch as the whole line AG is to the greater ſegment AQ, as the greater ſegment AI is to the reſidue AP, the line AP ſhall be the leſſer ſegment of the whole line AB or AG: Wherefore the line PEQ (which by the point E paſſeth parallel to the line GI) cutteth the lines AG and BA by an extream and mean proportion in the points Q and P. And by the ſame reaſon, the line PR (which by the point C paſſeth paralelly to the line AM) ſhall fall upon the ſections P and R; ſo alſo ſhall the

c) 2. 6.
d) Cor. 2. 6.
e) 34. 1.
f) 7. 5.
g) 2. 14.
h) Cor. 2. 6.
i) 2. 6.
k) 2. 6.
l) 5. 13.
m) 2. 6.
n) 2. 6.



the line R Q (which by the point D, passeth parallelly to the line B L) fall upon the sections R and Q: Wherefore either of the lines P E and E Q shall be equal to the line C D, in the Paralellograms P D and Q C [o]. And forasmuch as the lines P E and E Q are equal, the lines P C, C R, R D, and D Q, shall be likewise equal. Wherefore the triangle P R Q is equilateral, and cutteth the sides of the base of the Pyramis in the points P, Q, and R, by an extream and mean proportion. And in it is inscribed the base E C D of the Icosahedron conteined in the foresaid Pyramis: If therefore from the angles, &c. Which was to be demonstrated.

o) 34. 1.

COROLLARIE

The side of an Icosahedron inscribed in an Octohedron, is the greater segment of the line, which being drawn from the angle of the base of the Octohedron, cutteth the opposite side by an extream and mean proportion.

For P F K H is the base of the Octohedron, which conteineth the base of the Icosahedron C D E, unto which triangle F K H, the triangle H K G is equal, as hath been proved. By the point H draw to the line M E a parallel H T, cutting the line D N in the point S. Wherefore E S, D T, and E T, are Paralellograms; and therefore the lines E H and M T, are equal; and the lines E M and H T are like cut in the points D and S [q]. Wherefore the greater segment of the line H T, is the line H S, which is equal to E D, the side of an Icosahedron. But [r] the line T K is cut like to the line H K, by the parallel D M. And therefore [s] it is divided by an extream and mean proportion. But the line T M is equal to the line E H. Wherefore also the line T K is equal to the line E F or D H: Wherefore the residues E H and T G are equal. For the whole lines F H and K C are equal. Wherefore K G the side of the triangle H K G, is in the point T divided by an extream and mean proportion, by the right line H T; and the greater segment thereof is the line E D, the side of the Icosahedron inscribed in the Octohedron, whose base is the triangle H K G (or the triangle F K H) which is equal to the triangle H K G [t].

p) 16. 15.
q) 34. 1.
r) 2. 6.
s) 2. 14.
t) 16. 15.

PROP. 5. THEOR. 5.

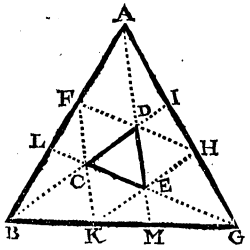
The side A G, *of a Pyramis divided by an extream and mean proportion, maketh the lesser segment* A I *in power double to the side* C E, *of the Icosahedron inscribed in it.*

Construction LEt A B G be the base of a Pyramis, and let the base of the Icosahedron inscribed in it be C D E, described of three right

right lines, which being drawn from the angles of the base A B G, cut the opposite sides by an extream and mean proportion [a]; to wit, of the three lines A M, B I, and G L.

a) 4. 16.

Demonstration FOrasmuch as [b] it was proved that the triangle C D E is inscribed in an equilateral triangle, whose angles cut the sides of A B G, the base of the Pyramis, by an extream and mean proportion; let that triangle be F H K, cutting the line A B in the point F. Wherefore the lesser segment F A is equal to the segment A I [c] (for the lines A B and A G are cut like:) Moreover the side F H of the triangle F H K, is in the point D cut into two equal parts [d]; and F C E D is a Paralellogram: Wherefore the lines C E and F D are equal [e]. And forasmuch as the line F H subtendeth the angle B A G, of an equilateral triangle, which angle is conteined under the greater segment A H, and the lesser segment A F; therefore the line F H is in power double to the line A F, or to the line A I, the lesser segment [f]. But the same line F H is in power quadruple to the line C E [g] (for the line F H is double to the line C E:) Wherefore the square of the line A I being the half of the square of the line F H, is in power double to the line C E, to which the line F H was in power quadruple. Wherefore, The side of a Pyramis divided, &c. Which was to be demonstrated.



b) 4. 16.
c) 2. 14.
d) 4. 16.
e) 33. 1.
f) C. 16. 15.
g) 4. 2.

COROLLARIE.

The side of an Icosahedron inscribed in a Pyramis, is a Residual line.

For the diameter of the Sphere which conteineth the five regular bodies, being rational, is in power sesquialtera to the side of the Pyramis [h]. And therefore the side of the Pyramis is rational, by the definition, which side being divided by an extream and mean proportion, maketh the lesser segment a Residual line [i]: Wherefore the side of the Icosahedron being commensurable to the same lesser segment (for the square of the Icosahedron is the half of the square of the said lesser segment) is a Residual line.

h) 13. 13.
i) 6. 13.

PROP. 6. THEOR. 6.

The side of a cube conteineth in power half the side of an equilateral triangular Pyramis inscribed in the said cube.

Demonstration FOrasmuch as the side of the Pyramis inscribed in the Cube, subtendeth two sides of the Cube which contein a right angle [a]; it is manifest [b] that the side of the Pyramis subtending the said sides, is in power double to the side of the Cube. Wherefore also the square of the side of the Cube is the halfe of the square of the side of the Pyramis: Therefore, &c. Which was to be demonstrated.

a) 1. 15.
b) 47. 1.

PROP.

PROP. 7. THEOR. 7.

The side of a Pyramis is double to the side of an Octohedron inscribed in it.

Demonstration FOrasmuch as [a] it was proved that the side of the Octohedron inscribed in a Pyramis, coupleth the middle sections of the sides of the Pyramis: Wherefore the sides of the Pyramis and of the Octohedron, are parallels [b]; and therefore [c] they subtend like triangles: Wherefore [d] the side of the Pyramis is double to the side of the Octohedron, to wit, in the proportion of the sides. Therefore, The side, &c. Which was to be demonstrated.

a) 2. 15.
b) Co. 39. 1.
c) Cor. 2. 6.
d) 4. 6.

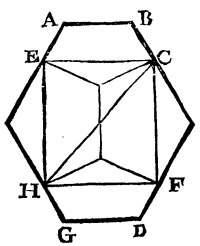
PROP. 8. THEOR. 8.

The side of a cube is in power double to the side of an Octohedron inscribed in it.

Demonstration IT was proved [a] that the diameter of the Octohedron inscribed in the Cube, coupleth the centers of the opposite bases of the Cube: Wherefore the said diameter is equal to the side of the Cube. But the same is also the diameter of the square made of the sides of the Octohedron, to wit, is the diameter of the sphere which conteineth it [b]: Wherefore that diameter being equal to the side of the Cube, is in power double to the side of that square, or to the side of the Octohedron inscribed in it [c]. The side therefore, &c. Which was to be demonstrated.

a) 3. 15.
b) 14. 13.
c) 47. 1.

PROP. 9. THEOR. 9.



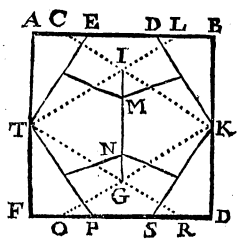
The side AB, *of a Dodecahedron* ABGD, *is the greater segment of the line which conteineth in power half the side* CH, *of the Pyramis inscribed in the said Dodecahedron.*

Demonstration FOr seeing that EC, the side of the Cube, being divided by an extream and mean proportion, maketh the greater segment the line AB, the side of the Dodecahedron [a] (for they are conteined in one and the same sphere [b]) and the line EC the side of the Cube, conteineth in power the half of the side CH [c]. Wherefore AB the side of the Dodecahedron, is the greater segment of the line EC, which conteineth in power the half of the line CH, which is the side of the Dodecahedron inscribed in the Pyramis. The side therefore, &c. Which was required to be proved.

b) 1 Cor. of 17. 13.
b) 1. 16.
c) 6. 16.

PROP.

PROP. 10. THEOR. 10.



The side CL, *of an Icosahedron, is the mean proportional between the side* AB, *of the Cube circumscribed about the Icosahedron* CLIGOR, *and the side* ED, *of the Dodecahedron* EDMNPS, *inscribed in the same Cube.*

Demonstration FOrasmuch as CL is the greater segment of AB [a], and the side ED is the lesser segment of the same side AB [b]: It followeth that AB the side of the Cube, being divided by an extream and mean proportion, maketh the greater segment CL, the side of the Icosahedron inscribed in it, and the lesser segment ED the side of the Dodecahedron likewise inscribed in it. Wherefore as the whole line AB, the side of the Cube, is to the greater segment CL, the side of the Icosahedron; so is the greater segment CL, the side of the Icosahedron, to the lesser segment ED, the side of the Dodecahedron [c]. Wherefore, The side, &c. Which was required to be proved.

a) 3. C. 14. 15
b) 2 C. 13. 15
c) 3. def. 6.

PROP. 11. THEOR. 11.

The side of a Pyramis, is in power Octodecuple (that is, as 18 to 1,) to the side of the Cube inscribed in it.

Demonstration FOr by what hath been demonstrated [a], the side of the Pyramis is triple to the diameter of the base of the Cube inscribed in it; and therefore it is in power noncuple to the same diameter [b]. But the diameter is in power double to the side of the Cube [c]. And the double of noncuple maketh Octodecuple. Wherefore, The side, &c. Which was required to be proved.

a) 18. 15.
b) 20. 6.
c) 47. 1.

PROP. 11. THEOR. 11.

The side of a Pyramis is in power Octodecuple to that right line, whose greater segment is the side of the Dodecahedron inscribed in the Pyramis.

Demonstration FOrasmuch as the Dodecahedron and the Cube inscribed in it, are set in one and the same Pyramis [a], and the side of the Pyramis circumscribed about the Cube, is in power Octodecuple to the side of the Cube inscribed [b]. But the greater segment of the same side of the Cube is the side of the Dodecahedron which conteineth the Cube [c]. Wherefore, The side, &c. Which was required to be proved.

a) Co. 1. 16.
b) 11. 16.
c) C. 17. 13.

PROP.

PROP. 13. THEOR. 13.

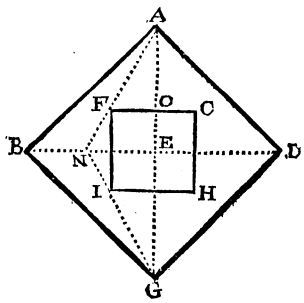
The side of an Icosahedron inscribed in an Octohedron, is in power double to the lesser segment of the side of the same Octohedron.

Demonstration FOrasmuch as [a] it was proved, that the side of an Icosahedron inscribed in an Octohedron, coupleth together the two sections (which are produced by an extream and mean proportion) of the side of the Octohedron, which make a right angle, and that right angle is conteined under the lesser segments of the sides of the Octohedron, and is subtended of the side of the Icosahedron inscribed: It followeth therefore that the side of the Icosahedron which subtendeth the right angle, being in power equal to the two lines which contein the said angle [b], is in power double to each of the lesser segments of the Octohedron which contein a right angle. Wherefore, The side of an Icosahedron, &c. Which was to be demonstrated.

a) 17. 15.

b) 47. 1.

PROP. 14. THEOR. 14.



The sides AB, *of the Octohedron* ABGDE, *and* FI, *of the Cube* FCHI, *inscribed in it, are in power to one another in quadrupla sesquialtera proportion; (that is as* 9 *to* 2.)

Construction LEt there be drawn to BE, the base of the triangle ABE, a perpendicular AN; and again, let there be drawn to the same base in the triangle GBE, the perpendicular GN, which AN and GN shall passe by the centers F and I.

Demonstration THe line AF is double to the line FN [a]: Wherefore the line AO is double to the line OE [b]; for the lines FO and NE are parallels; and therefore the diameter AG is triple to the line FI. Wherefore the power of AG is noncuple (that is, as 18 to 2, or 9 to 1,) to the power of FI. But the line AG is in power double to the side AB [c]. Wherefore the square of the line AB being the half of the square of the line AG, which is noncuple to the square of the line FI, is quadruple sesquialter to the square of the line FI. The sides therefore, &c. Which was required to be proved.

a) C. 12. 13. b) 2. 6.

c) 14. 13.

PROP.

PROP. 15. THEOR. 15.

The side of the Octohedron is in power quadruple sesquialter to that right line, whose greater segment is the side of the Dodecahedron inscribed in the same Octohedron.

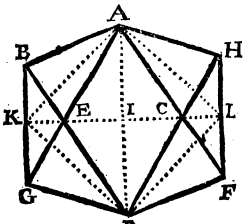
Demonstration FOrasmuch as [a] it was proved, that the side of the Octohedron is in power quadruple sesquialter to the side of the Cube inscribed in it; but the side of the Cube being cut by an extream and mean proportion, maketh the greater segment the side of the Dodecahedron circumscribed about it [b]: Therefore the side of the Octohedron is in power quadruple sesquialter to that right line (to wit, to the side of the Cube) whose greater segment is the side of the Dodecahedron inscribed in the Cube. But the Dodecahedron and the Cube inscribed one within another, are inscribed in one and the same Octohedron [c]. Therefore, The side, &c. Which was required to be proved.

a) 14. 16.

b) 3 C. 13. 15

c) Co. 1. 16.

PROP. 16. THEOR. 16.



The side BG, *or* EC, *of an Icosahedron* ABGDFHEC, *is the greater segment of that right* BH, *or* KL, *which is in power double to the side* AL, *of the Octohedron* AKDL, *inscribed in the same Icosahedron.*

Demonstration FOrasmuch as figures inscribed and circumscribed have one and the same center [a], let the same be the point I. Now right lines drawn by that center to the middle sections of the opposite sides, to wit, the lines AID and KIL, do in the point I cut one another into two equal parts, and perpendicularly [b]; and forasmuch as they couple the middle sections of the opposite lines BG and HF; therefore they cut them perpendicularly: Wherefore also the lines BG and HF are parallels [c]. Now then draw a line from B to H, and the said line BH shall be equal and parallel to the line KL [d]. But the line BH subtendeth two sides of the Pentagon which is composed of the sides of the Icosahedron, to wit, the sides BA and AH: Wherefore the line BH being cut by an extream and mean proportion, maketh the greater segment the side of the Pentagon [e]; which side is also the side of the Icosahedron, to wit, EC. And unto the line BH, the line KL is equal; and the line KL is in power double to AL, the side of the Octohedron [f]; for in the square AKDL, the angle KAL is a right angle. Wherefore EC the side of the Icosahedron, is the greater segment of the line BH or KL, &c. Which was required to be proved.

a) C. 21. 15.

b) C. 14. 15.

c) 4 C. 14. 15

d) 33. 1.

e) 8. 13.

f) 47. 1.

PROP. 17. THEOR. 17.

The side of a Cube is to the side of a Dodecahedron inscribed in it, in double proportion of an extream and mean proportion.

Demonstration FOr it was manifest [a] that the side of a Cube divided by an extream and mean proportion, maketh the lesser segment the side of the Dodecahedron inscribed in it; but the whole is to the lesser segment in double proportion of that in which it is to the greater [b]; for the whole, the greater segment, and the lesser, are lines in continual proportion [c]; Wherefore the whole, to wit, the side of the Cube, is to the side of the Dodecahedron inscribed in it, to wit, to his lesser segment, in double proportion of an extream and mean proportion, to wit, of that which the whole hath to the greater segment [d].

a) 2 C. 13. 15.
b) 10. def. 5.
c) 3. def. 6.
d) 2. 14.

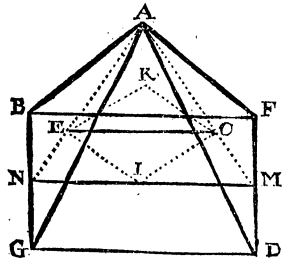
PROP. 18. THEOR. 18.

The side of a Dodecahedron, is to the side of the Cube inscribed in it, in converse proportion of an extream and mean proportion.

Demonstration IT was proved [a] that the side of a Dodecahedron circumscribed about a Cube, is the greater segment of the side of the same Cube. Wherefore the whole side of the Cube inscribed is to the greater segment, to wit, to the side of the Dodecahedron circumscribed, in an extream and mean proportion: Wherefore by conversion, the greater segment, that is, the side of the Dodecahedron, is to the whole, to wit, to the side of the Cube inscribed [b], &c. Which was required to be proved.

a) 3 C. 13. 15.
b) 13. def. 5.

PROP. 19. THEOR. 19.



The side GD, *of an Octohedron* ABGDF, *is sesquialter to the side* EC, *of a Pyramis inscribed in it.*

Construction FOr [a] the Octohedron is cut into two quadrilateral Pyramids, one of which let be ABGDF; and let the centers of the circles which contein the four bases of the Octohedron be K, E, I, and C, and draw these right lines KE, EI, IC, CK, and EC. Wherefore KEIC is a square, and one of the bases of the Cube inscribed in the Octohedron [b]. And forasmuch as the

a) C. 14. 13.
b) 4. 15.

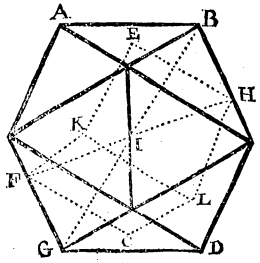
the angles of a Cube, and of the Pyramis inscribed in it, are set in the centers of the bases of the Octohedron circumscribed about the Cube [c]; and the side of the Pyramis coupleth the opposite angles of the base of the Cube [d]: It is manifest that the line EC is the side of the Pyramis inscribed in the Octohedron ABGDF. I say then that GD the side of the Octohedron, is sesquialter to EC, the side of the Pyramis inscribed in it. From the point A, draw to the bases BG and FD, perpendiculars AN and AM, which [e] shall passe by the centers E and C, and draw the line NM.

c) 6. 15.
d) 2. 15.
e) C. 13. 13.

Demonstration FOrasmuch as BGDF is a square [f], the lines NG and MD, shall be parallels and equal. For the lines BG and FD, are by the perpendiculars cut into two equal parts, in the points N and M [g]: Wherefore the lines NM and GD shall be parallels and equal [h]. And forasmuch as the lines AN and AM, which are the perpendiculars of equal and like triangles, are cut alike in the points E and C, the lines EC and NM shall be parallels [i]. And therefore [k] the triangles AEC and ANM shall be like. Wherefore as the line AN is to the line AE, so is the line NM to the line EC [l]. But the line AN is sesquialter to the line AE, for the line AE is double to the line EN [m]. Wherefore the line NM or the line GD, which is equal unto it, is sesquialter to the line EC. Wherefore, &c. Which was required to be proved.

f) 14. 13.
g) 3. 3.
h) 33. 1.
i) 2. 6.
k) Cor. 2. 6.
l) 4. 6.
m) C. 12. 13.

PROP. 20. THEOR. 20.



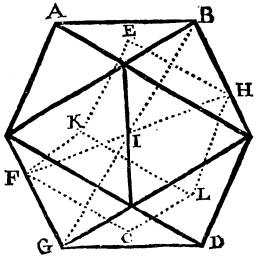
If from the power of the diameter BG, *of an Icosahedron* ABGD, *be taken away the power tripled of the side* EH, *of the Cube inscribed in the Icosahedron, the power remaining shall be sesquitertia to the power of the side* AB, *of the Icosahedron.*

Construction LEt the two bases of the Cube be EHKL, and LIKFC joyned; and let the diameter of the Cube be FH.

Demonstration FOrasmuch as the centers of inscribed and circumscribed figures are in one and the same point [a], the diameters BG and FH, shall in one and the same point cut one another into two equal parts; for by the same Corollary it hath been taught that the tops of equal and like Pyramids, do in that point concurre; let the point be the center I. Now the angles of the Cube which are at the points F and H, are set at the centers of the bases of the Icosahedron [b]: Wherefore the line FH shall be perpendicular to both the bases of the Icosahedron.

a) C. 21. 15.
b) 11. 15.

Wherefore the line I B conteineth in power the two lines I H and H B c. But the line H B is drawn from the center of the circle which conteineth the baſe of the Icoſahedron, to wit, the angle B, is placed in the circumference, and the point H is the center. Wherefore the whole line B G conteineth in power the whole lines F H, and the diameter of the circle (to wit, the double of the line B H) d. But the diameter which is double to the line H B, is in power ſeſquitertia to the ſide of the equilateral triangle inſcribed in the ſame circle e; for it is in proportion to the ſide, as the ſide is to the perpendicular f. And F H the diameter of the Cube, is in power triple to E H, the ſide of the ſame Cube g: If therefore from the power of the diameter B G, be taken away the power tripled of E H, the ſide of the Cube inſcribed; that is, the power of the line F H; the reſidue (to wit, the power of the diameter of the circle, which is double to the line H B) ſhall be ſeſquitertia to the ſide of the triangle inſcribed in that circle, which ſide is A B the ſide of the Icoſahedron: If therefore, &c. Which was required to be proved.



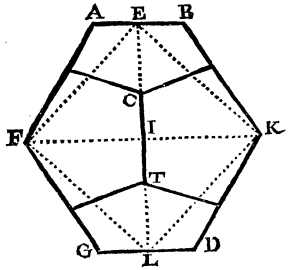
c) 47. 1.
d) 15. 5.
c) C. 12. 13.
f) Cor. 8. 6.
g) 15. 13.

COROLLARIE.

The Diameter of the Icoſahedron, conteineth in power two lines, to wit, the Diameter of the Cube inſcribed, which coupleth the centers of the oppoſite baſes, and the Diameter of the Circle which conteineth the baſe of the Icoſahedron.

For it was manifeſt, that B G the diameter, conteineth in power the line F H, which coupleth the centers, and the double of the line B H, that is, the diameter of the circle conteining the baſe wherein the center H is.

PROP. 21. THEOR. 21.



The ſide AB, *of a Dodecahedron* ABGDCT, *is the leſſer ſegment of that right line which is in power double to the ſide* EF, *of the Octohedron* ELFKI, *inſcribed in the ſame Dodecahedron.*

Conſtruction DRaw the diameters E L and F K, of

of the Octohedron, Now they couple the middle ſections of the oppoſite ſides of the Dodecahedron A B and G D a; and all thoſe diameters being divided by an extream and mean proportion, do make the leſſer ſegment the ſide of the Dodecahedron b: Wherefore the ſide A B is the leſſer ſegment of the line F K. But the line F K conteineth in power the two equal lines E F and E K c; for the angle F E K is a right angle of the ſquare F E K L, of the Octohedron. Wherefore the line F K is in power double to the line E F. Wherefore the line A B (the ſide of the Dodecahedron) is the leſſer ſegment of the line F K, which is in power double to E F, the ſide of the Octohedron: Therefore, The ſide of a Dodecahedron, &c. Which was required to be proved.

a) 9. 15. C. 3. 17. 13.
b) 4 Cor. of 17. 13.
c) 47. 1.

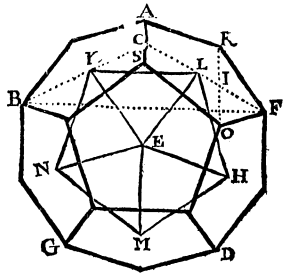
PROP. 22. THEOR. 22.

The Diameter of an Icoſahedron is in power ſeſquitertia to the ſide of the ſame Icoſahedron, and alſo is in power ſeſquialter to the ſide of the Pyramis inſcribed in the Icoſahedron.

Demonſtration FOraſmuch as it hath been proved a that if from the power of the diameter of the Icoſahedron, be taken away the triple of the power of the ſide of the Cube inſcribed in it, there ſhall be left a ſquare, ſeſquitertia to the ſquare of the ſide of the Icoſahedron: But the power of the ſide of the Cube tripled, is the diameter of the ſame Cube b. And the Cube and the Pyramis inſcribed in it, are conteined in one and the ſame ſphere c, and in one and the ſame Icoſahedron d: Wherefore one and the ſame Diameter of the Cube, or of the Sphere which conteineth the Cube and the Pyramis, is in power ſeſquialter to the ſide of the Pyramis e. Wherefore it followeth that if from the Diameter of the Icoſahedron be taken away the triple power of the ſide of the Cube, or the ſeſquialter power of the ſide of the Icoſahedron: The diameter therefore, &c. Which was required to be proved.

a) 20. 16.
b) 15. 13.
c) 1. 16.
d) Co. 1. 16.
e) 13. 13.

PROP. 23. THEOR. 23.



The ſide AS, *or* SO, *of a Dodecahedron* ABG DFSO, *is to the ſide* KL, *of an Icoſahedron* KL HMNE, *inſcribed in it, as the leſſer ſegment* IF, *of the perpendicular* CF, *of the Pentagon, is to that line* LC, *which is drawn from the center to the ſide* AS, *of the ſame Pentagon.*

Demon-

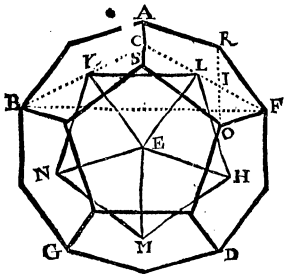
Conſtruction FRom the two angles of the Pentagons B A S and F A S, of the Dodecahedron, to wit, from the angles B and F, let there be drawn to the common baſe A S, perpendicular lines B C and F C, which ſhall paſſe by the centers K and L of the ſaid Pentagons [a]: Draw the lines B F and R O. Now foraſmuch as the line R O ſubtendeth the angle O F R, of the Pentagon of the Dodecahedron, it ſhall cut the line F C by an extream and mean proportion [b], let it cut it in the point I. And foraſmuch as the line K L is the ſide of the Icoſahedron inſcribed in the Dodecahedron, it coupleth the centers of the baſes of the Dodecahedron; for the angles of the Icoſahedron are ſet in the centers of the baſes of the Dodecahedron [c].

a) C. 10. 13.

b) 3. 16.

c) 7. 15.

Demonſtration FOraſmuch as in the triangle B C F, the two ſides C B and C F, are in the centers L and K, cut like proportionally, the lines B F and K L ſhall be parallels [d]: Wherefore the triangles B C F and K C L, ſhall be equiangled [e]. Wherefore as the line C L is to the line K L, ſo is the line C F to the line B F [f]. But C F maketh the leſſer ſegment the line I F [g]; and the line B F maketh the leſſer ſegment the line S O, to wit, the ſide of the Dodecahedron [h]; for the line B F, which coupleth the angles B and F, of the baſes of the Dodecahedron, is equal to the ſide of the Cube which conteineth the Dodecahedron [i]: Wherefore as the whole line C F, is to the whole line B F, ſo is the leſſer ſegment I F, to the leſſer ſegment S O [k]. But as the line C F is to the line B F, ſo is the line C L to the line K L. Wherefore alternately, [l] as the line I F the leſſer ſegment of the perpendicular of the Pentagon F A S, is to the line L C, which is drawn from the center of the Pentagon, to the baſe; ſo is the line S O the ſide of the Dodecahedron; to the line K L the ſide of the Icoſahedron inſcribed in it. Therefore, The ſide, &c. Which was required to be proved.



d) 2. 6.

e) Cor. 2. 6.

f) 4. 6.

g) 3. 16.

h) 2 C. 13. 15

i) 13. 15.

k) 2. 14.

l) 16. 5.

PROP. 24. THEOR. 24.

If half of the ſide of an Icoſahedron be divided by an extream and mean proportion, and if the leſſer ſegment thereof be taken from the whole ſide; and again, from the reſidue be taken away the third part, that which remaineth ſhall be equal to the ſide of the Dodecahedron inſcribed in the ſame Icoſahedron.

Conſtruction SUppoſe A B G D F be a Pentagon, conteining five ſides of the Icoſahedron [a], and let it be inſcribed in a circle, whoſe center

a) 16. 13.

center let be E. And upon the ſides of the Pentagon let there be raiſed triangles, making a ſolid angle of the Icoſahedron at the point I [b]. And in the circle A B D, inſcribe an equilateral triangle A H K. From the center E, draw to H K the ſide of the triangle, and G D the ſide of the Pentagon, a perpendicular line, which let be E C N M; and draw the lines E G, E D, I G, and I D. And divide the line B G into two equal parts in the point T. And draw the lines I N, I T, T N, and E T. And foraſmuch as in the perpendiculars I T and I N, are the centers of the circles which contein the equilateral triangles I B G and I G D [c]; Let thoſe centers be the points S and O, and draw the line S O. Divide the line T B the half of B G, the ſide of the Icoſahedron, by an extream and mean proportion in the point R [d]; and let the leſſer ſegment thereof be R B. And foraſmuch as the line S O coupleth the centers of the triangles I B G and I G D, it is [e] the ſide of the Dodecahedron inſcribed in the Icoſahedron, whoſe ſide is the line B G, from the ſide B G take away B R, the leſſer ſegment of the half ſide. And from the reſidue G R, take away the third part G V [f]; then I ſay that the reſidue R V is equal to S O, the ſide of the Dodecahedron inſcribed.

b) 16. 13.

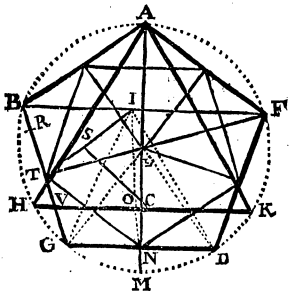
c) Cor. 1. 3

d) 30. 6.

e) 5. 13.

f) 9. 6.

Demonſtration FOraſmuch as the perpendicular E N is in the point C divided by an extream and mean proportion [g], and the greater ſegment thereof is the line E C, and unto the line E C the line C M is equal [h]. Wherefore the line E C is to the line C N, as the line C N is to the ſame C N [i]. But as the line E C is to the line C N, ſo is the whole line E N, to the greater ſegment E C [k]. Wherefore [l] as the whole line E N is to the greater ſegment E C, ſo is the line C M to the line C N. Wherefore the line C M is divided by an extream and mean proportion in the point N, to wit, is divided like unto the line E N [m]. Wherefore the line E M exceedeth the line E N by the leſſer ſegment of his half, to wit, by M N. And foraſmuch as E G D is the triangle of an equilateral and equiangled Pentagon A B G D F. Therefore [n] the triangle E T N is like to the triangle E G D. Wherefore as the line E G is to the line E N, ſo [o] is the line G D to the line N T. Wherefore the line G D (or B G which is equal unto it) exceedeth the line N T by the leſſer ſegment of the halfe of B G; for the line E G did in like ſort exceed the line E N. But that leſſe ſegment is the line B R. Wherefore the reſidue R G is equal to the line T N. And foraſmuch as I B G is an equilateral triangle, the perpendicular S T ſhall be the half of the line S I, which is drawn from the center P. Wherefore the line I T exceedeth the line I S by his third part. And foraſmuch as the line S O, which coupleth the ſections, is a parallel to the line T N [q]; for the equal perpendiculars I T and I N, are cut like in the points S and O. Therefore the



g) Co. 1. 14.

h) C. 12. 13.

i) 7. 5.

k) 2. def. 6

l) 11. 5.

m) 2. 14.

n) 20. 6.

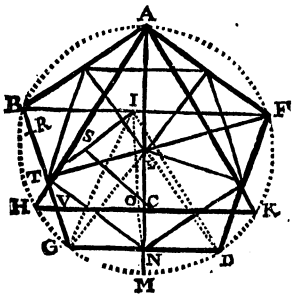
o) 4. 6.

p) C. 12. 13.

q) 2. 6.

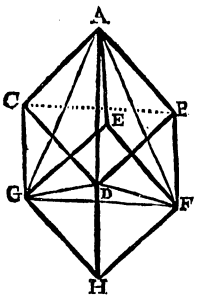
r) Cor. 2. 6. s) 4. 6.

the triangles ITN and ISO are like [r]. Wherefore as the line IT is to the line IS; ſo [s] is the line TN to the line SO. But the line IT exceedeth the line IS by a third part. Wherefore the line TN exceedeth the line SO by a third part. But the line TN is proved equal to the line RG. Wherefore the line RG exceedeth the line SO by a third part of it ſelfe which is GV. Wherefore the reſidue RV is equal to the line SO, which is the ſide of the Dodecahedron inſcribed in the Icoſahedron, whoſe ſide is the line BG. If therefore half of the ſide of an Icoſahedron be divided by an extream and mean proportion, &c. Which was required to be proved.



PROP. 25. THEOR. 25.

To prove that a given Cube AB CH *is triple to a trilateral equilateral Pyramis* AGDF, *inſcribed in it.*



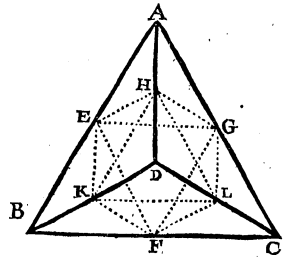
Demonſtration For ſmuch as the baſe AFD is common to the Pyramis AFDB and AFDG, the Pyramis AFDB ſhall be ſet without the Pyramis AFDC. Likewiſe the reſt of the baſes of the inſcribed Pyramis are common to the reſt of the Pyramids ſet without, which are the Pyramis AGDC upon the baſe AGD, the Pyramis AGFE upon the baſe AGF, and GDFH upon the baſe GDF, which Pyramids taken without, are four in number, equal and alike [a]; for every one of them is conteined under three half ſquares of the Cube, and one of the baſes of the Pyramis inſcribed. Wherefore each of them is conteined under the half baſe of the Cube, and the altitude of the Cube. As the Pyramis AEGF hath to his baſe half of the ſquare EH, to wit, the triangle EGF, and hath to his altitude the altitude of the Cube, to wit, the line AE. Wherefore the ſaid Pyramis is the ſixth part of the Cube. For if the Cube be divided into two Priſmes by the Plain CBFG, the priſme ACBGEF ſhall be triple to the Pyramis AEGF, having one and the ſame baſe with it EGF, and one and the ſame altitude EA [b]. Wherefore the ſaid outward Pyramis AEGF is the ſixth part of the whole Cube. Wherefore alſo the ſame Pyramis, together with the other three outward Pyramids AFDB, AGDC, and GDFH, ſhall contein two third parts of the Cube. Wherefore the reſidue, to wit, the Pyramis inſcribed AGDF, ſhall contein one third part of the Cube. And therefore converſely, the Cube ſhall be triple to it. Wherefore we have proved that a Cube, &c.

a) 8. de. 11.

b) 1 C. 7. 12.

PROP. 26. THEOR. 26.

To prove that a trilateral Pyramis ABCD, *is double to an Octohedron inſcribed in it* EGLKHF.



Conſtruction LEt the ſides of the Pyramis be cut into two equal parts in the points E, K, F, G, L, and H, inſcribing thereby an Octohedron in the Pyramis [a]. Wherefore the Pyramids AEGH, BEFK, CFGL, and DKHL, fall without the Octohedron inſcribed [b]. But the outward Pyramis, to wit, AEGH, and the three other, are like unto the whole Pyramis [c]. For the baſes of the whole Pyramis are by parallel lines drawn in them, cut into like triangles [d], of which the foreſaid Pyramids are made. Wherefore the whole Pyramis is to every one of them in treble proportion of that in which the ſides of like proportion are [e]. But by Conſtruction, the proportion of the ſide AB to the ſide AE is double. Wherefore the whole Pyramis ABCD is octuple to the Pyramis AEGH, and ſo is it to each of the Pyramids which are equal to AEGH. For double proportion multiplied into it ſelfe twice, maketh octuple. Wherefore it followeth that AEGH, BEFK, CFGL, and DKHL; taken together, make the halfe of the whole Pyramis ABCD. Wherefore the reſidue, to wit, the Octohedron EGLKHF, is the other halfe of the Pyramis. Wherefore the Pyramis is double to the Octohedron. Wherefore we have proved that a trilateral equilateral Pyramis, &c.

a) 2. 15.

b) 2. 15.

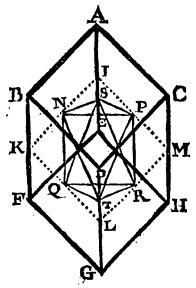
c) 7. def. 11.

d) 2. 6.

e) 8. 12.

PROP. 27. THEOR. 27.

To prove that a Cube is ſextuple to an Octohedron inſcribed in it.



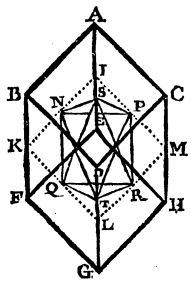
Conſtruction LEt the Cube ABCD, EFGH, whoſe four ſtanding lines AE, BF, CH, and DG, be cut into two equal parts in the points I, K, M, and L, and by thoſe points let there be extended a plain KLMI, which ſhall be a ſquare, and parallel to the ſquares BC and FH [a]. Wherefore in it ſhall be the baſe which is commen to the two Pyramids of the Octohedron inſcribed in the Cube [b]. Let that baſe be NPRQ, coupling the centers of the baſes of the Cube, & upon that baſe let be ſet the two

a) 15. 11.

b) 3. 15.

two Pyramids of the Octohedron, which let be N P Q R S and N P Q R T.

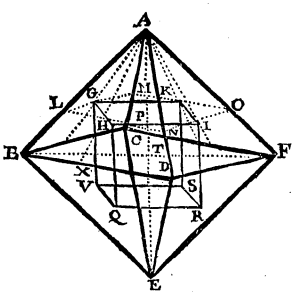
Demonſtration FOraſmuch as theſe two Pyramids taken together have their altitude equal with the altitude of the whole Cube, each of them apart hath to his altitude half the altitude of the Cube, to wit, half the ſide of the Cube, as the line K B. And foraſmuch as the ſquare K L M I is double to the ſquare N R Q P [c], the other ſquares of the Cube ſhall alſo be double to the ſquare N R Q P. And foraſmuch as the Cube [d] is reſolved into ſix Pyramids, whoſe baſes are the baſes of the Cube, and the altitudes the lines drawn from the center to the baſes, which are equal to half the ſide of the Cube, it followeth that each of the ſix Pyramids of the Cube, having his baſe double to the baſe of each of the Pyramids of the Octohedron, and the ſame altitude that the ſaid Pyramids of the Octohedron have, is double to either of the Pyramids of the Octohedron [e]. And foraſmuch as each of the Pyramids of the Cube is equal to the two Pyramids of the Octohedron, the ſix Pyramids of the Cube ſhall be ſextuple to the whole Octohedron. Wherefore we have proved that a Cube is ſextuple to an Octohedron inſcribed in it.



c) 47. 1.
d) ult. 15.
e) 6. 12.

PROP. 28. THEOR. 28.

To prove that an Octohedron ABCDEF, *is quadruple ſeſquialter to a Cube* GHIK, VQRS, *inſcribed in it.*



Demonſtration FOraſmuch as the lines drawn from the center of the Octohedron, or of the Sphere which conteineth it, unto the centers of the baſes of the Octohedron are proved equal [a]; and the angles of the Cube are ſet in the centers of theſe baſes [b]: It followeth that the ſame right lines are drawn from one and the ſame center of the Cube, and of the Octohedron; for they have each one and the ſame center [c]. Let that center be the point T. Wherefore the baſe BDFC, which cutteth the Octohedron into two equal and quadrilateral Pyramids [d], ſhall alſo cut the Cube into two equal parts [e]. For it paſſeth by the center T [f]. And foraſmuch as the baſe of the Cube is in the four centers G, H, I, and K, of the baſes of the Pyramis ABDFC, a plain LNOM extended by thoſe points, ſhall be parallel to the plain BDFC [g], and ſhall cut the Pyramis in

a) 21. 15.
b) 4. 15.
c) C. 21. 15.
d) C. 14. 13.
e) C. 39. 11.
f) 14. 13.
g) 4. 15.

in the points L, N, O, and M, and the lines LN, BD, and NO, DF, ſhall be parallels, ſo alſo ſhall the lines OM, FC, and LM, BC, and the ſquare GHIK, of the Cube, ſhall be inſcribed in the ſquare LNOM. Wherefore the ſquare LNOM is double to the ſquare GHIK [h]. From the ſolid angle A, let there be drawn to the plain Superficies BDCF, a perpendicular, which let fall upon it, in the point T; and let the ſame pependicular be AT, cutting the plain LNOM in the point P. And it ſhall alſo be a perpendicular to the plain LNOM [i]. Again, from the angle BAD of the triangle ADB, let there be drawn by the center H, of the triangle, to the baſe, a line AHX. Wherefore the line AX is ſeſquialter to the line AH [k]. Wherefore the line AH is double to the line HX. But the other lines AB, AD, AF, and AC, and the perpendicular APT, are cut like unto the line AHX [l]. Wherefore the line AP is double to the line PT. Wherefore the line AP is the altitude of the Cube; for the line PT is the half thereof. And foraſmuch as upon the baſe GHIK of the Cube, and under the altitude AP, of the ſame Cube, is ſet the Pyramis AGHIK, the ſaid Pyramis is the third part of the Cube [m]. But unto the Pyramis AGHIK the Pyramis ALNOM is double [n]; for the baſe of the one is double to the baſe of the other. Wherefore the Pyramis ALNOM is two third parts of the Cube. And foraſmuch as the Pyramids ALNOM and ABDFC, are like [o], therefore they are in triple proportion of that in which the ſides of like proportion AH to AX, or AL to AB are [p]. But the ſide AB is proved to be ſeſquialter to the ſide AL. Wherefore the Pyramis ABCDF is to the Pyramis ALNOM, as 27 is to 8 (which is ſeſquialter proportion tripled; for the quantity or denomination of ſeſquialter proportion, to wit, $1\frac{1}{2}$, multiplied into it ſelfe once, it maketh $2\frac{1}{4}$, which again multiplied by $1\frac{1}{2}$, maketh $3\frac{3}{8}$, that is 27 to 8. But of what parts the Pyramis ALNOM conteineth 8, of the ſame the Cube conteineth 12, of the ſame the whole Octohedron (which is double to the Pyramis ABDFC) conteineth 54, which 54 hath to 12 quadruple ſeſquialter proportion: Wherefore the whole Octohedron is to the Cube inſcribed in it in quadruple ſeſquialter proportion. Wherefore we have proved, &c.

h) 47. 1.
i) Co. 14. 11.
k) C. 12. 13.
l) 17. 11.
m) C. 7. 12.
n) 6. 12.
o) 7. def. 11.
p) Co. 8. 12.

COROLLARIE.

An Octohedron is to a Cube inſcribed in it, in that proportion that the ſquares of their ſides are.

For [q] the ſide of the Octohedron is in power quadruple ſeſquialter to the ſide of the Cube inſcribed in it.

q) 14. 16.

PROP. 29. THEOR. 29.

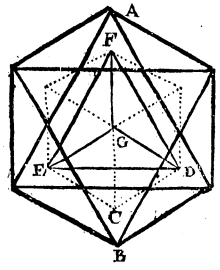
To prove that a given Octohedron AB, *is tredecuple ſeſquialter (that is, as* $13\frac{1}{2}$ *to* 1*) to a trilateral equilateral Pyramis* FEGD, *inſcribed in it.*

Conſtruction LEt there be inſcribed in the given Octohedron a Cube FCED [a], and in the Cube a Pyramis [b].

Demonſtration FOraſmuch as the angles of the Pyramis are [c] ſet in the angles of the Cube, and the angles of the Cube are ſet in

a) 4. 15.
b) 1. 15.
c) 1. 15.

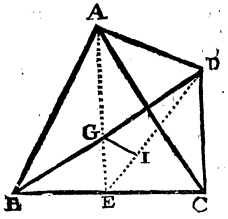
the centers of the base of the Octohedron, to wit, in the points F, E, C, D, and G [d]. Wherefore the angles of the Pyramis are set in the centers FC and ED of the Octohedron. Wherefore the Pyramis FEDG is inscribed in the Octohedron [e]. And forasmuch as the Octohedron AB is to the Cube FCED inscribed in it, quadruple sesquialter (by the former Prop.) and the Cube CDEF is to the Pyramis FEDG inscribed in it triple [f]. Wherefore three Magnitudes being given, to wit, the Octohedron, the Cube, and the Pyramis, the proportion of the extreames (to wit, of the Octohedron to the Pyramis) is made of the proportion of the meanes (to wit, of the Octohedron to the Cube, and of the Cube to the Pyramis [g].) Now then multiply the quantities or denominations of the proportions (to wit, of the Octohedron to the Cube, which is $4\frac{1}{2}$, and of the Cube to the Pyramis, which is 3 [h],) there shall be produced $13\frac{1}{2}$, to wit, the proportion of the Octohedron to the Pyramis inscribed in it. For $4\frac{1}{2}$ multiplyed by 3, produce $13\frac{1}{2}$. Wherefore the Octohedron is to the Pyramis inscribed in it in tredecuple sesquialter proportion. Wherefore we have proved, &c.



d) 4. 15.
e) 6. 15.
f) 25. 16.
g) 10. def. 5.
h) def. 6.

PROP. 30. THEOR. 30.

To prove that a trilateral equilateral Pyramis is noncuple to a Cube inscribed in it.



Construction. LEt the given pyramis be ABCD, whose two bases let be ABC and DBC, and let their centers be G and I, and from the angle A, draw unto the base BC, a perpendicular AE, also from the angle D, draw unto the same base BC, a perpendicular DE, and they shall meet in the section E [a], and in them shall be the centers G and I [b].

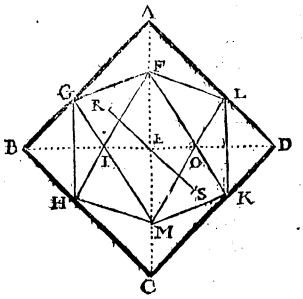
Demonstration. Forasmuch as the line GI coupleth the centers of the bases of the pyramis, the said line GI shall be the diameter of the base of the Cube inscribed in the pyramis [c]. And forasmuch as the line AG is double to the line GE [d], the whole line AE shall be triple to the line GE, and so is also the line DE to the line IE. Wherefore the lines AD and GI are parallels [e]. And therefore the triangles AED and GEI are like [f]. And forasmuch as the triangles AED and GEI are like, the line AD shall be triple to the line GI [g]. But the line AD is the diameter of the base of the Cube circumscribed about the pyramis ABCD, and the line GI is the diameter of the base of the Cube inscribed in the pyramis ABCD; but the diameters of the bases are

a) 3. 3.
b) Co. 1. 3.
c) 18. 15.
d) C. 12. 13.
e) 2. 6.
f) Cor. 2. 6.
g) 4. 6.

are equimultiplices to the sides (to wit, are in power double.) Wherefore the side of the Cube circumscribed about the pyramis ABCD, is triple to the side of the Cube inscribed in the same pyramis [h]. But like Cubes are in triple proportion to one another of that in which their sides are [i], and the sides are in triple proportion to one another. Wherefore triple taken three times, bring forth twenty seven cuple, which is 27 to 1; for the four terms 27, 9, 3, and 1, being set in triple proportion, the proportion of the first to the fourth (to wit, of 27 to 1,) shall be triple to the proportion of the first to the second, to wit, of 27 to 9 [k]. Which proportion of 27 to 1, is the proportion of the sides tripled, which proportion also is found in like solids. Wherefore of what parts the Cube circumscribed conteineth 27, of the same the Cube inscribed conteines 1. But of what parts the Cube circumscribed conteineth 27, of the same the pyramis inscribed in it conteineth 9 [l]. Wherefore of what parts the pyramis ABCD conteineth 9, of the same the Cube inscribed in the Pyramis conteineth 1. Wherefore we have proved, &c.

h) 15. 5.
i) 33. 11.
k) 10. def. 5.
l) 25. 6.

PROP. 31. THEOR. 31.

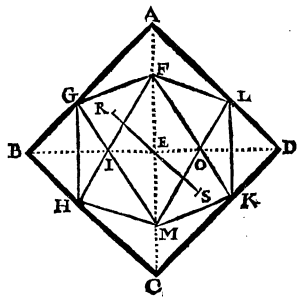


An Octohedron ABCD, *hath to an Icosahedron* FGHMKLIO, *inscribed in it, that proportion which two bases of the Octohedron have to five bases of the Icosahedron.*

Demonstration. Forasmuch as the solid of the Octohedron consisteth of eight pyramids, set upon the bases of the Octohedron, and having to their altitude a perpendicular line drawn from the center to the base. Let that perpendicular be ER or ES, being drawn from the center E (which center is common to either of the solids [a]) to the centers of the bases, to wit, to the points R and S. Wherefore seeing that three pyramids are equal and like, they shall be equal to a prisme set upon the same base, and under the same altitude [b]. But to this prisme is double that prisme which is set upon the same base, and hath his altitude double, to wit, the whole line RS [c]; for it is equal to the two equal and like prismes whereof it is composed. Wherefore the prisme set upon the base of the Octohedron, and having to his altitude the line RS, is equal to six piramids, set upon six bases of the Octohedron, and having to their altitude the line ER, so there remain two pyramids (for in the Octohedron are eight bases) which shall be equal to the prisme which is set upon the third part of the base of the Octohedron, and under the altitude RS, for prismes under one and the same altitude, are in proportion to one another as are their bases [d]. Wherefore the two prismes which are set upon the base of the

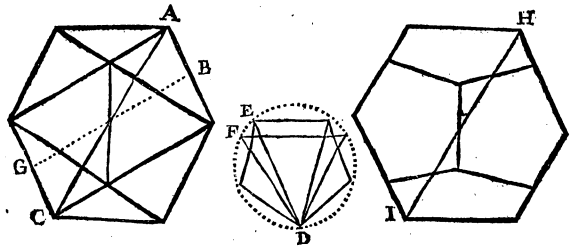
a) C. 21. 15.
b) Co. 7. 12.
c) C. 25. 11.
d) Co. 7. 12.

the Octohedron, and upon a third part thereof, and under the altitude R S, are equal to the eight pyramids of the Octohedron, or to the whole Solid of the Octohedron. And forasmuch as the Icosahedron inscribed in
c) 17. 15. the Octohedron, hath his bases set in the bases of the Octohedron [c]; it followeth that the pyramids set upon the bases of the Icosahedron, and having to their tops one and the same center E, are conteined under the same altitude that the pyramids of the Octohedron are conteined under, to wit, under the line E R or E S; and therefore a prisme set upon the base of the Icosahedron, and having his altitude double to the altitude of the Pyramis, to wit, the whole line R S, is equal to the six pyramids set upon the base of the Icosahedron, and under the altitude E R or E S, as we have proved in the Octohedron. Wherefore the twenty pyramids set upon the twenty bases of the Icosahedron, are equal to three prismes set upon the base of the Icosahedron, and under the altitude R S; and morover, to another prisme set upon a third part of the base of the Icosahedron, and under the same altitude R S, which prisme is
f) Co. 7. 12. a third part of the former prisme [f]; for their proportion is as the proportion of the bases. Wherefore two prismes set upon the base of the Octohedron, and a third part thereof, and under the altitude R S, is to four prismes set upon the three bases of the Icosahedron, and a third part thereof, and under the same altitude R S, in the same proportion that the bases are, that is, as four third parts of the base of the Octohedron (which are equal to one base and $\frac{1}{3}$) to 10 third parts of the base of the Icosahedron (which are equal to 3 bases and $\frac{1}{3}$) or as 2 third parts of the base of the Octohedron are to 5 third parts of the base of the Icosahedron. But 2 third parts of the base of the Octohedron, are to 5 third parts of the base of the Icosahedron, as two bases are to 5 bases
g) 15. 5. [g] (for they are parts of equimultiplices.) And two prismes of the Octohedron are to four prismes of the Icosahedron, as the solid of the Octohedron is to the solid of the Icosahedron, when as each are equal to each of the solids. Wherefore
h) 11. 5. [h] the solid of the Octohedron is to the solid of the Icosahedron inscribed in it, as two bases of the Octohedron are to five bases of the Icosahedron. Therefore, &c. Which was required to be proved.



PROP. 32. THEOR. 32.

The proportion of the solid of an Icosahedron ABGC, *to the solid of a Dodecahedron inscribed in it, consisteth of the proportion of the side* DF *of the Icosahedron, to the side* DE *of the Cube, conteined in the same Sphere, and of the proportion tripled of the Diameter* AC, *to the line* BG, *which coupleth the centers of the opposite bases of the Icosahedron.*



Demonstration FOrasmuch as the solid of the Icosahedron A B G C is to the solid of the Dodecahedron H I, being conteined in one and the same sphere, as D F is to D E [a]. But the Dodecahedron whose diameter is H I, is to the Dodecahedron, whose diameter is B G, in triple proportion of that in which the diameter H I is to the diameter B G [b]; and the lines H I and A C are equal by supposition (to wit, the diameters of one and the same sphere.) Wherefore as H I is to B G, so is A C to B G. Wherefore the proportion of the extreames (to wit, of the Icosahedron A B G C, to the Dodecahedron set upon the diameter B G, which coupleth the centers, consisteth [c] of the proportions of the means, to wit, of the proportion of A B G C to the Dodecahedron H I, (which is one and the same with the proportion of D F to D E) and of the proportion of the same H I to the other Dodecahedron set upon the diameter B G, inscribed in the same Icosahedron A B G C [d]; which proportion is triple to the proportion of the line H I (or the line A C) to G B, which coupleth the centers of the opposite bases of the Icosahedron. The proportion therefore, &c. Which was required to be proved.

a) 8. 14.
b) C. 17. 12.
c) 5. def. 6.
d) 5. 15.

PROP. 33. THEOR. 33.

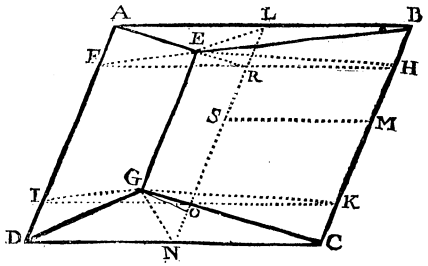
The solid of a Dodecahedron exceedeth the solid of a Cube inscribed in it, by a Parallelepipedon, whose base wanteth of the base of the Cube by a third part of the lesser segment, and whose altitude wanteth of the altitude of the Cube, by the lesser segment of the lesser segment of half the side of the Cube.

Construction FOrasmuch as [a] it was manifest that the base of a Cube inscribed in a Dodecahedron, doth with his sides subtend the angles of four Pentagons, meeting at one and the same side of the Dodecahedron: Let that base of the Cube be A B C D; and let the side whereat four bases of the Dodecahedron circumscribed meet, be E G, which shall contein a solid A E B D G C, set upon the base A B C D. Divide the

a) 17. 13. 18. 15.

the sides A B and D C into two equal parts in the points L and N, and
b) 17. 13. draw the line L N, which is a parallel to the side E G [b]. The perpendiculars also E R and G O, which couple those parallels, are each equal to halfe of the side E G, and each is the greater segment of half the side of the Cube. And therefore the whole line E G is the greater segment of the whole line L N, the side of the Cube
c) 17. 13. [c]; by the points R and O, draw unto the sides A B and C D, parallel lines F H and I K, and draw these right lines EF, E H, G I, and G K.

Demonstration Forasmuch as the two lines F H and E R, touching one another, are parallels to the two lines I K and G O, touching also one another, and not being in the same plain with the two first lines. Therefore the plain Superficies E F H and G I K, passing by those lines,
d) 15. 11. are parallels [d]; which plains do cut the solid A E B D G C. Wherefore there are made four quadrangled pyramids set upon the rectangle parallegrams L H, L F, N K, and N I, and having their tops the points E and
e) 4. 1. G. And forasmuch as the triangles G O K and E R H, are equal and like [e]; to wit, they contein equal angles comprehended under equal sides,



and they are parallels by construction, being set in the plains G I K and E F H, the figures G K H E, O K H R, and G O R E, shall be Parallelograms, by the definition of Parallelograms; and therefore the solid G O K E R H is a prisme
f) 11. de. 11. [f]. And by the same reason may the solid G O I E R F be proved to be a prisme. And forasmuch as upon equal bases N O K C and R L B H, and under equal altitudes O G and R E are set pyramids, these pyramids shall be equal to that pyramis which is set upon the base C K I D (which is double to either of the bases N O K C and R L B H) and under the same altitude
g) 6. 12. O G [g]. And forasmuch as the side G E is the greater segment of the line
h) 33. 1. C B, the line K H which [h] is equal to the line G E, shall be the greater
i) 2. 14. segment of the same line C B [i]. Wherefore the residues C K and H B, shall make the lesser segment of the whole line C B. But as the greater segment K H is to the two lines C K and H B, the lesser segment, so is the
k) 1. 6. rectangle Parallelogram O H, to the two rectangle Parallelograms O C
l) Cor. 7. 12. H L [k]. Wherefore the pyramis set upon the base O K M S, conteineth two third parts of the prisme set upon the same base [l]. Wherefore the prisme which is set upon two third parts of the base O K M S, is equal to the two pyramids N O K C G and R L B H E. For the sections of a
m) Cor. of 25. 11. prisme are to one another, as the sections of the base are [m]. But the sections of the base are as the sections of the line C B or K M
n) 1. 6. [n]. Wherefore adde the two pyramids N Q K C G and R L B H E, unto the prisme G O K E F H,

G O K E R H, two third parts of the prisme set upon the base O K M S. And forasmuch as the line K M is the lesser segment of the whole line B C (for it is equal to the two lines C K and H B) and the prisme set upon the base O K H R is cut like unto the line K M, to wit, in each are taken two thirds, as hath been proved, the prisme equal to the two pyramids, shall adde unto the prisme G O K E R H, which is set upon the greater segment K H, two thirds of the lesser segment. Wherefore in the line B C there shall remain one third part of the lesser segment; and therefore in the rectangle Parallelogram N B, which is half the base of the Cube, there shall remain the same third part of the lesser segment. And by the same reason may we prove that in the other pyramids O N D I G and R L A F E, and in the prisme G O I E R F is left the same exceese of the base L A N D, to wit, the third part of the lesser segment. Wherefore the whole prisme conteined between the triangles I G K and F E H, and under the length of the greater segment, and two third parts of the lesser segment of B C the side of the Cube, is equal to the solid composed of four bases of the Dodecahedron, and set upon the base of the Cube. Wherefore the base of that prisme wanteth of the whole base of the Cube, only a third part of the lesser segment, and the altitude of that prisme was the line G O, which is the greater segment of half the side of the Cube. And forasmuch as unto the triangle I G K, is double the rectangle Parallelogram set upon the same base I K (the side of the Cube) and under the altitude G O [o]. It followeth that three rectangle parallelograms set upon the same I K, the side of the Cube, and under the altitude O G, the greater segment of half the side of the Cube, are sextuple to the triangle I G K. Wherefore these three rectangle parallelograms do make one rectangle Parallelogram set upon the base I K, and under the altitude of the line G O tripled. But [p] there are six prismes equal and like unto the foresaid prisme, being set upon each of the six bases of the Cube; which prismes are in proportion to one another as their bases are [q]. Wherefore the solid composed of these six Prismes, shall want of the base A B C D the third part of the lesser segment, and taking his altitude of the foresaid rectangle Parallelogram, the said altitude shall be equal to three greater segments (one of which is G O) of halfe the side of the Cube.

o) 41. 1.

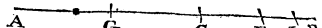
p) 7. def. 11.

q) 3 C. 7. 12.

Now resteth to prove that these three segments want of the side of the Cube by the lesser segment of the lesser segment of half the side of the Cube.

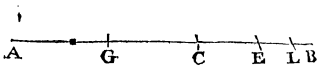
Suppose that A B the side of the Cube, be divided into the greater segment A C, and into the lesser segment C B [r]; divide into two equal parts the line A C in the point G, and the line C B in the point E. And

r) 30. 6.



unto the line C G, put the line C L equal. Now forasmuch as the lines A G and G C, are the greater segments of half the line A B, for each of them is the half of the greater segment of the whole line A B; the lines E B and E C shall be the lesser segments of half the line A B. Wherefore the whole line C L is the greater segment; and the line C E is the lesser

segment. But as the line C L is to the line C E, ſo is the line C E to the reſidue E L. Wherefore the line E L is the greater ſegment of the line C E, or of the line E B, equal thereto. Wherefore the reſidue L B is the leſſer ſegment of the ſame E B (which is the leſſer ſegment of half the ſide of the Cube.) But the lines A G, G C, and C L, are three greater ſegments of the half of the whole line A B, which three greater ſegments



make the altitude of the foreſaid ſolid. Wherefore the altitude of the ſaid ſolid wanteth of A B the ſide of the Cube by the line L B, which is the leſſer ſegment of the line B E, which line B E again is the leſſer ſegment of half the ſide A B of the Cube. Wherefore the foreſaid ſolid conſiſting of ſix ſolids, whereby the Dodecahedron exceedeth the Cube inſcribed in it, is ſet upon a baſe which wanteth of the baſe of the Cube, by a third part of the leſſer ſegment, and is under an altitude wanting of the ſide of the Cube by the leſſer ſegment of the leſſer ſegment of half the ſide of the Cube. Therefore, &c. Which was required to be proved.

COROLLARIE.

A Dodecahedron is double to a Cube inſcribed in it, taking away the third part of the leſſer ſegment of the Cube, and moreover, the leſſer ſegment of the leſſer ſegment of half of that exceſſe.

For if there be given a Cube, from which is cut off a ſolid ſet upon a third part of the leſſer ſegment of the baſe, and under one and the ſame altitude with the Cube, that ſolid taken away, hath to the whole ſolid the proportion of the ſection of the baſe to the baſe s. Wherefore from the Cube is taken away a third part of the leſſer ſegment. Again, foraſmuch as the reſidue wanteth of the altitude of the Cube, by the leſſer ſegment of the leſſer ſegment of half the altitude or ſide, and that reſidue is a Parallelepipedon, if it be cut by a plain ſuperficies parallel to the oppoſite plain ſuperficies, cutting the altitude of the Cube by a point, it ſhall take away from that Parallelepipedon a ſolid, having to the whole the proportion of the ſection to the altitude t. Wherefore the exceſſe wanteth of the ſame Cube, by the third part of the leſſer ſegment, and moreover, by the leſſer ſegment of the leſſer ſegment of half of that exceſſe.

s) 32. 11.

t) 3 Cor. of 25. 11.

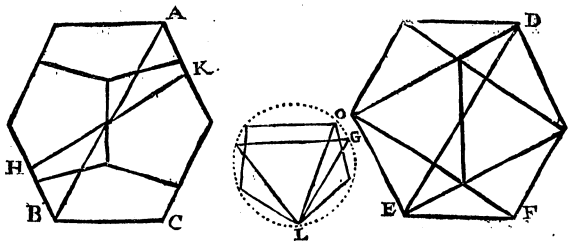
PROP. 34. THEOR. 34.

The proportion of the ſolid of a Dodecahedron AHBCK, *to the ſolid of an Icoſahedron* DEF, *inſcribed in it, conſiſteth of the proportion tripled of the diameter* AB, *to that line* KH, *which coupleth the oppoſite baſes of the Dodecahedron, and of the proportion of the ſide* IO *of the Cube, to the ſide* IG, *of the Icoſahedron inſcribed in one and the ſame Sphere.*

Con-

Conſtruction LEt A B be the diameter of the Dodecahedron, and let the line which coupleth the centers of the oppoſite baſes be K H, and let the diameter of the Icoſahedron be D E. Now foraſmuch as one and the ſame circle conteineth the Pentagon of a Dodecahedron, and the triangle of an Icoſahedron deſcribed in one and the ſame Sphere a: Let that circle be I G O. Wherefore I O is the ſide of the Cube, and I G the ſide of the Icoſahedron. I ſay then that the proportion of the Dodecahedron A H B C K, to the Icoſahedron D E F inſcribed in it, conſiſteth of the proportion tripled of the line A B to the line K H, and of the proportion of the line I O to the line I G.

a) 4. 14.



Demonſtration FOraſmuch as the Icoſahedron D E F is inſcribed in the Dodecahedron A B C, by ſuppoſition, the diameter D E ſhall be equal to the line K H b. Wherefore the Dodecahedron ſet upon the diameter K H ſhall be inſcribed in the ſame ſphere, wherein the Icoſahedron D E F is inſcribed: But the Dodecahedron A H B C K is to the Dodecahedron upon the diameter K H, in triple proportion of that in which the diameter A B is to the diameter K H c, and the ſame Dodecahedron which is ſet upon the diameter K H, hath to the Icoſahedron D E F (which is ſet upon the ſame diameter, or upon a diameter equal to it, to wit, D E,) that proportion which I O the ſide of the Cube, hath to I G, the ſide of the Icoſahedron, inſcribed in one and the ſame ſphere d. Wherefore the proportion of the Dodecahedron A H B C K, to the Icoſahedron D E F inſcribed in it, conſiſteth of the proportion tripled of the diameter A B, to the line K H, which coupleth the centers of the oppoſite baſes of the Dodecahedron (which proportion is that which the Dodecahedron A H B C K hath to the Dodecahedron ſet upon the diameter K H) and of the proportion of I O the ſide of the Cube, to I G the ſide of the Icoſahedron (which is the proportion of the Dodecahedron ſet upon the diameter K H, to the Icoſahedron D E F, deſcribed in one and the ſame ſphere e:) Therefore the proportion of the ſolid of a Dodecahedron, &c. Which was required to be proved.

b) 7. 15.

c) G. 17. 12.

d) 8. 14.

e) 5. def. 6.

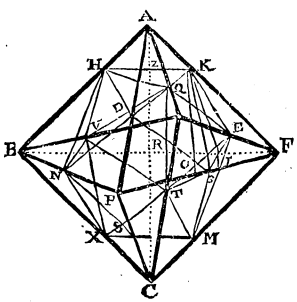
PROP. 35. THEOR. 35.

The ſolid of a Dodecahedron conteineth of a Pyramis circumſcribed about it, two ninth parts, taking away a third

part of one ninth part of the leſſer ſegment (of a line divided by an extream and mean proportion) and moreover the leſſer ſegment of the leſſer ſegment of half the reſidue.

Demonſtration IT hath been proved that the Dodecahedron, together with the Cube inſcribed in it, is conteined in one and the ſame
a) Co. 1. 16 Pyramis [a]. And it is manifeſt [b] that the Dodecahedron is double to the
b) C. 33. 16. ſame Cube, taking away the third part of the leſſer ſegment; and more-
over, the leſſer ſegment of the leſſer ſegment of half the reſidue, or of this
c) 30. 16. exceſſe. But a Pyramis is to the ſame Cube inſcribed in it noncuple [c]. Wherefore the Dodecahedron inſcribed in the Pyramis, and conteining the ſame Cube twice, taking away the ſame third of the leſſer ſegment; and moreover, the leſſer ſegment of the leſſer ſegment of half the reſidue, ſhall contein two ninth parts of the ſolid of the Pyramis (of which ninth parts each is equal to the Cube) taking away the ſame exceſſe. Therefore, &c. Which was required to be proved.

PROP. 36. THEOR. 36.



An Octohedron ABCF PL, *exceedeth an Icoſahedron* HKEGMXNVDSQT, *by a Parallelepipedon ſet upon the ſquare of the ſide of the Icoſahedron, and having to his altitude the line which is the greater ſegment of half the ſemidiameter of the Octohedron.*

Conſtruction DRaw the diameters AZRC and BROIF, and the perpendicular KO parallel to the line AZR.

Demonſtration FOraſmuch as [a] the triangles KDG and KEQ are deſcri-
a) 16. 15. bed in the baſes APF and ALF, of the Octohedron. Therefore about the ſolid angle there remain upon the baſe FEG three triangles KEG, KFE, and KFG, which contein a Pyramis KEFG, unto which Pyramis ſhall be equal and like the oppoſite Pyramis
b) 8. def. 11. MEFG, ſet upon the ſame baſe FEG [b]. And by the ſame reaſon ſhall there of every ſolid angle of the Octohedron remain two Pyramids, equal and like; to wit, two upon the baſe AHK, two upon the baſe BNV, two upon the baſe DPS, and two upon the baſe QLT. Now then there ſhall be made twelve Pyramids, ſet upon a baſe conteined of the ſide of the

Icoſahedron, and under two leſſer ſegments of the ſide of the Octohedron conteining a right angle, as for example, the baſe GEF. And foraſmuch as the ſide GE ſubtending a right angle is [c] in power double to either of the lines EF and FG, and ſo the ſide KH is in power double to either of the ſides AH and AK, and either of the lines AH, AK, or EF, FG, is in power double to either of the lines AZ or ZK, which contein a right angle, made in the triangle or baſe AHK, by the perpendicular AZ. Wherefore it follows that the ſide GE or HK, is in power quadruple to the triangle EFG or AHK. But the Pyramis KEFG, having his baſe EFG in the plain FLBP, of the Octohedron, ſhall have to his altitude the perpendicular KO [d], which is the greater ſegment of the ſemidiameter of the Octohedron [e]. Wherefore three Pyramids ſet under the ſame altitude, and upon equal baſes ſhall be equal to one Priſme ſet upon the ſame baſe, and under the ſame altitude [f]. Wherefore four priſmes ſet upon the baſe GEF quadrupled (which is equal to the ſquare of the ſide GE) and under the altitude KO (or RZ the greater ſegment (which is equal to KO) ſhall contein a ſolid equal to the twelve Pyramids, which twelve Pyramids make the exceſſe of the Octohedron above the Icoſahedron inſcribed in it. Therefore an Octohedron, &c. Which was required to be proved.

c) 47. 1.
d) 4. def. 6.
e) 16. 15.
f) 1 C. 7. 12.

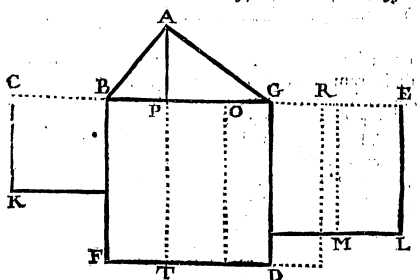
COROLLARIE.

A Pyramis exceedeth the double of an Icoſahedron inſcribed in it, by a ſolid ſet upon the ſquare of the ſide of the Icoſahedron inſcribed in it, and having to his altitude that whole line of which the ſide of the Icoſahedron is the greater ſegment.

For [g] it is manifeſt that an Octohedron and an Icoſahedron inſcribed in it, are inſcribed in one and the ſame Pyramis. It hath been proved likewiſe [h] that a Pyramis is double to an Octohedron inſcribed in it. Wherefore the two exceſſes of the two Octohedrons (unto which the Pyramis is equal) above the two Icoſahedrons (inſcribed in the ſaid two Octohedrons) being brought into a ſolid, the ſaid ſolid ſhall be ſet upon the ſame ſquare of the ſide of the Icoſahedron, and ſhall have to his altitude the perpendicular KO doubled, whoſe double coupling the oppoſite ſides HK and XM, maketh the greater ſegment the ſame ſide of the Icoſahedron [i].

g) 19. 15.
h) 26. 16.
i) 1, 2. Cor. 14. 15.

PROP. 37. THEOR. 37.



If in a triangle ABG, *having to his baſe a rational line ſet* BG, *the ſides* AB *and* AG *be commenſurable in power to the baſe, and from the top* A *be drawn to the baſe a perpendicular*

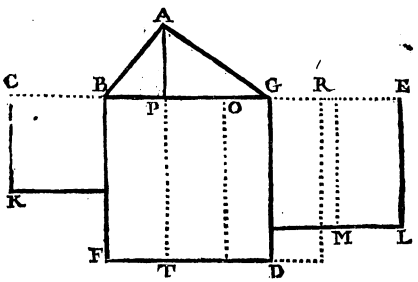
dicular line AP, *cutting the base, the sections of the base shall be commensurable in length to the whole base, and the perpendicular shall be commensurable in power to the said whole base.*

Construction PRoduce on either side the line BG, to the points C and E, and unto the line AG put the line GE equal, and unto the line AB put the line BC equal. And upon the lines CB, BG, and GE, describe the squares BK, BD, and GL, and from the greater of the squares of the lines AB or AG, which let be GL, cut off a Parallelogram EM, equal to the lesser square BK [a], and [b] unto the residue GM, let there be applyed upon the line GD, an equal rectangle Parallelogram OD.

a) 45. 1.
b) 45. 1.

Demonstration FOrasmuch as the angles APB and APG are right angles; therefore [c] the line AG conteineth in power the two lines AP and PG, and the line AB the two lines AP and PB. Wherefore how much the line AG conteineth in power more than the line AB, so much also doth the line PG contein in power more than the line BP, to wit, taking away the common square of AP, there is left the excesse of the square PG, above the square of BP. But the square of

c) 47. 1.

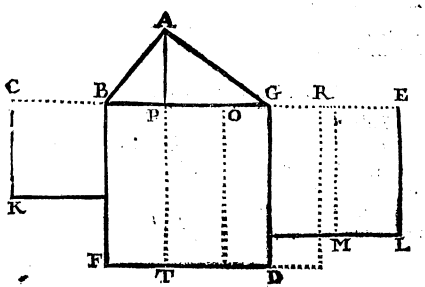


AG (which is GL) exceedeth the square of AB (to wit, the square BK) by the rectangle Parallelogram GM or OD, by Construction. Wherefore the square of PG exceedeth the square of BP, by the rectangle Parallelogram OD. And forasmuch as unto the squares of AB and AG, are equal the squares of AP and PB, and of AP and PG, and their excesse is taken away, to wit, the rectangle Parallelogram OD, there shall be left the squares of AP and PO, equal to the squares of AP and PB, and taking away the square of AP, which is common, the residues, to wit, the squares of BP and PO, shall be equal; and therefore their sides (to wit, the lines BP and PO) are equal. And forasmuch as the squares GL and BK are (by supposition) rational, and therefore commensurable, their excesse OD shall be commensurable unto them [d]; and therefore it is rational [e]. Wherefore the rational Parallelogram OD being applyed upon the rational line GD (or BG) maketh the breadth OG rati-

d) 15. 10.
e) 9. def. 10.

rational and commensurable in length to the whole line BG [f]. But if the whole line BG be commensurable to one of the parts OG, the lines BO, OG, and BG, shall be commensurable [g]. Wherefore also the line OG shall be commensurable to the half of the line BO, to wit, to the line PO, or PB [h]. And forasmuch as the two lines PO and OG are commensurable, the whole line PG shall be commensurable to the line PO, or to the line PB [i]. Wherefore either of the lines PG and PB shall be commensurable unto the whole line GB [k]. Wherefore the lines BG, PB, and PG, have to one another that proportion which numbers have [l]. Wherefore the sections PB and PG, of the base BG, are commensurable in length to the same base [m].

f) 20. 10.
g) 15. 1.0
h) 12. 10.
i) 15. 10.
k) 15. 10.
l) 5. 13.
m) 6. 10.



And now that the perpendicular AP is commensurable in power to the base BG, is thus proved. Forasmuch as the square of AB is by supposition commensurable to the square of BG, and unto the rational square of AB, is commensurable the rational square of PB [n]. Wherefore the residue, to wit, the square of PA, is commensurable to the same square of BP [o]. Wherefore [p] the square of PA is commensurable to the whole square of BG. Wherefore the perpendicular AP is commensurable in power to the base BG [q]. Which was required to be proved.

n) 12. 11.
o) 15. 11.
p) 12. 10.
q) 3. de. 10.

In demonstrating of this we made no mention at all of the length of the sides AB and AG, but only of the length of the base BG, for that the line BG is the rational line first set, and the other lines AB and AG, are supposed to be commensurable in power only to the line BG. Wherefore if that be plainly demonstrated, when the sides are commensurable in power only to the base, much more easily will it follow, if the same sides be supposed to be commensurable both in length and in power to the base, that is, if their lengths be expressed by the Roots of square numbers.

COROLLARIE I.

By the former things demonstrated, it is manifest that if from the powers of the base, and of one of the sides be taken away the power of the other side, and if the half of the power remaining be applyed upon the whole base, it shall make the breadth that section of the base which is coupled to the first side.

For from the powers of the base BG, and of one of the sides AG, that

that is, from the squares B D and G L, the power of the other side A B, to wit, the square B K (or the Parallelogram E M is taken away.) And of the residue (to wit, of the square B D, and of the Parallelogram O D or D R, which by supposition is equal to O D) the half (to wit, of the whole F R, which is P D; for the lines G R and P B are equal to the lines G O and P O) is applyed to the whole line B G or G D, and maketh the breadth the line P G, the section of the base B G, which section is coupled to the first side A G. And by the same reason in the other side, if from the squares B D and B K, be taken away the square G L, there shall remain the rectangle Parallelogram F O; for the Parallelogram E M is equal to the square B K, and the Parallelogram G M to the Parallelogram G M to the Parallelogram O D. Wherefore E P the half of the residue F O, maketh the breadth B P, which is coupled to the first side taken A B.

COROLLARIE II.

If a Perpendicular drawn from an angle of a triangle do cut the base, the sections are to the other sides in power proportional by an Arithmetical proportion.

For it was proved that the excesse of the powers of the lines A G and A B, is one and the same with the excesse of the powers of the lines P G and P B: If therefore the powers do equally exceed one another, they shall by an Arithmetical proportion be proportional.

The End of the Sixteenth Element of EUCLIDE, *added by* FLUSSAS.

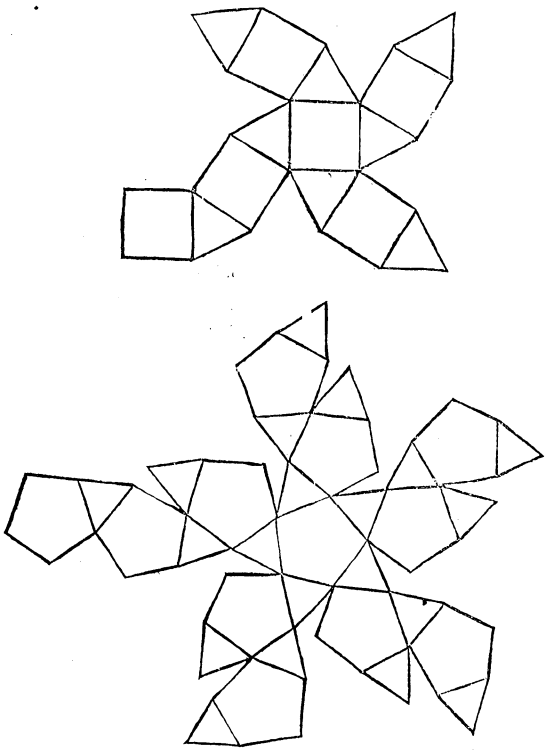
A BRIEFE TREATISE ADDED BY *FLUSSAS*, OF Regular Solids.

REgular Solids are said to be composed and mixt when each of them is transformed into other Solids, keeping still the forme, number, and inclination of the bases, which they before had to one another; some of which yet are transformed into mixt Solids, and other some into simple. Into mixt, as a Dodecahedron and an Icosahedron, which are transformed or altered, if you divide their sides into two equal parts, and take away the solid angles subtended of plain superficial figures, made by the lines coupling those middle sections; for the Solid remaining after the taking away of those solid angles, is called an Icosidodecahedron. If you divide the sides of a Cube and of an Octohedron into two equal parts, and couple the sections, the solid angles subtended of the plain superficies made by the coupling lines, being taken away, there shall be left a solid, which is called an Exoctohedron. So that both of a Dodecahedron and also of an Icosahedron, the Solid which is made shall be called an Icosidodecahedron; and likewise the Solid made of a Cube, and also of an Octohedron, shall be called an Exoctohedron. But the other Solid, to wit, a Pyramis or Tetrahedron, is transformed into a simple Solid; for if you divide into two equal parts each of the sides of the Pyramis, triangles described of the lines which couple the sections, and subtending and taking away the solid angles of the Pyramis, are equal and like unto the equilateral triangles left in each of the bases, of all which triangles is produced an Octohedron, to wit, a simple, and not a composed Solid. For the Octohedron hath four bases,

like in number, form, and mutual inclination with the bases of the Pyramis, and hath the other four bases with like situation opposite and parallel to the former. Wherefore the application of the Pyramis taken twice, maketh a simple Octohedron, as the other Solids make a mixt compound Solid.

DEFINITIONS.

1 *An Exoctohedron is a solid figure conteined of six equal squares, and eight equilateral and equal triangles.*



2 *An*

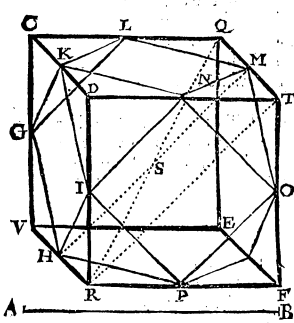
2 *An Icosidodecahedron is a solid figure conteined under twelve equilateral, equal, and equiangled Pentagons, and twenty equal and equilateral triangles.*

For the better understanding of the two former Definitions, and also of the two Propositions following, I have here set two figures, whose figures if you first describe upon pasted paper or such like matter, and then cut them and fold them accordingly, they will represent unto you the perfect formes of an Exoctohedron, and of an Icosidodecahedron.

PROBLEME 1.

To describe an equilateral and equiangled Exoctohedron, and to contein it in a given Sphere, and to prove that the Diameter of the Sphere is double to the side of the said Exoctohedron.

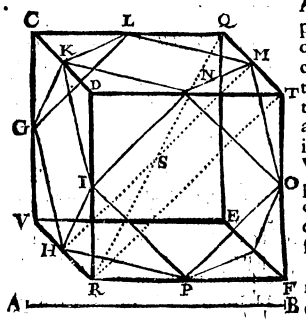
Construction SUppose a Sphere, whose diameter let be AB, and about the diameter AB let there be described a square [a], and upon the square let there be described a Cube [b], which let be CDEF QTVR, and let the diameter thereof be QR, and the center S. Divide the sides of the Cube into two equal parts in the points G, H, I, K, L, M, N, O, P, &c. and couple the middle sections by the right lines IN, NO, OP, PI, and such like, which subtend the angles of the squares or bases of the Cube; and they are equal [c], and contein right angles, as the angle NIP. For the angle NID, which is at the base of the Isosceles triangle NDI, is the half of a right angle, and so likewise is the opposite angle RIP. Wherefore the residue NIP is a right angle, and so the rest. Wherefore NIPO is a square. And by the same reason shall the rest NML, KGHI, &c. inscribed in the bases of the Cube, be squares, and they shall be six in number, according to the number of the bases of the Cube. Again, forasmuch as the triangle KIN subtendeth the solid angle D of the Cube, and likewise the triangle KGL the solid angle C, and so the rest which subtend the right solid angles of the Cube, and these triangles are equal and equilateral (to wit) being made of equal sides, and they are the limits or borders of the squares, and the squares the limits or borders of them; as hath been before proved. Wherefore



a) 6. 4.
b) 15. 13.
c) 4. 1.

LMNOPHGK is an Exoctohedron by the definition, and is equilateral; for it is conteined of equal ſubtendant lines, it is alſo equiangled; for every ſolid angle thereof is conteined under two ſuperficial angles of two ſquares, and two ſuperficial angles of two equilateral triangles.

Demonſtration FOraſmuch as the oppoſite ſides and diameters of the baſes of the Cube are parallels, the plain extended by the right lines Q T and V R, ſhall be a Parallelogram. And for that alſo in that plain lyeth Q R, the diameter of the Cube, and in the ſame plain alſo is the line M H, which divideth the ſaid plain into two equal parts, and alſo coupleth the oppoſite angles of the Exoctohedron; this line M H therefore divideth the diameter into two equal parts [d]; and alſo divideth it ſelfe in the ſame point, which let be S, into two equal parts [e].



And by the ſame reaſon may we prove that the reſt of the lines which couple the oppoſite angles of the Exoctohedron, do in S the center of the Cube, divide one another into two equal parts, for each of the anangles of the Exoctohedron are ſet in each of the baſes of the Cube. Wherefore making the center the point S, with the diſtance S H or S M deſcribe a Sphere, and it ſhall touch every one of the angles equidiſtant from the point S.

And foraſmuch as A B the diameter of the ſphere given, is put equal to the diameter of the baſe of the Cube, to wit, to the line R T, and the ſame line R T is equal to the line M H [f], which line M H coupling the oppoſite angles of the Exoctohedron, is drawn by the center. Wherefore it is the diameter of the Sphere given which conteineth the Exoctohedron.

Laſtly, foraſmuch as in the triangle R F T, the line P O doth cut the ſides into two equal parts, it ſhall cut them proportionally with the baſes, to wit, as F R is to F P, ſo ſhall R T be to P O [g]. But F R is double to F P by ſuppoſition: Wherefore R T, or the diameter H M, is alſo double to the line P O, the ſide of the Exoctohedron. Wherefore we have deſcribed, &c. Which was required to be done.

d) Co. 34. 1.
e) 4. 1.
f) 33. 1.
g) 2. 6.

PROBLEME II.

To deſcribe an equilateral and equiangled Icoſidodecahedron, and to comprehend it in a ſphere given, and to prove that the Diameter being divided by an extream and mean proportion, maketh the greater ſegment double to the ſide of the Icoſidodecahedron.

Con-

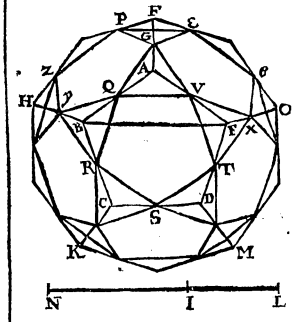
Conſtruction SUppoſe that the diameter of the ſphere given be NL, and [a] divide the line N L, by an extream and mean proportion in the point I, and the greater ſegment thereof let be N I; and upon the line NI deſcribe a Cube [b]; and about this Cube let there be circumſcribed a Dodecahedron [c]; and let the ſame be A B C D E F H K M O, and divide each of the ſides into two equal parts in the points Q, R, S, T, V, X, Y, Z, P, ϱ, ϑ, G, &c. and couple the ſections with right lines, which ſhall ſubtend the angles of the Pentagons, as the lines P G, G V, V Q, QY, Y R, R Q, V T, T X, X V, and ſo the reſt.

a) 30. 6.
b) 15. 13.
c) 17. 13.

Demonſtration FOraſmuch as theſe lines ſubtend equal angles of the Pentagons, and thoſe equal angles are conteined of equal ſides to wit, of the halves of the ſides of the Pentagons; therefore thoſe ſubtending lines are equal [d]. Wherefore the triangles G Q V, Y Q R, and V X T, and the reſt, which take away ſolid angles of the Dodecahedron, are equilateral.

d) 4. 1.

Again, foraſmuch as in every Pentagon are deſcribed five equal right lines, coupling the middle ſections of the ſides, as are the lines QV, V T, T S, S R, and R Q, they deſcribe a Pentagon in the plain of the Pentagon of the Dodecahedron. And the ſaid Pentagon is conteined in a circle, to wit, whoſe center is the center of a Pentagon of the Dodecahedron. For the lines drawn from that center to the angles of this Pentagon are equal, for that they are perpendiculars upon the baſes cut [e]. Wherefore the Pentagon QRSTV, is equiangled [f]. And by the ſame reaſon may the reſt of the Pentagons deſcribed in the baſes of the Dodecahedron, be proved equal and like.



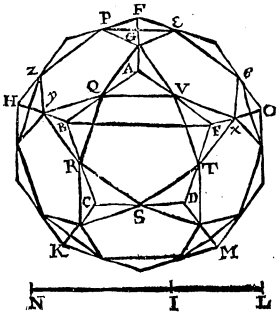
e) 12. 4.
f) 11. 4.

Wherefore thoſe Pentagons are twelve in number: And foraſmuch as the equal and like triangles do ſubtend and take away twenty ſolid angles of the Dodecahedron; therefore the ſaid triangles ſhall be twenty in number. Wherefore we have deſcribed an Icoſidodecahedron by the definition, which Icoſidodecahedron is equilateral; for that all the ſides of the triangles are equal and common with the Pentagons; and it is alſo equiangled. For each of the ſolid angles is made of two ſuperficial angles of an equilateral Pentagon, and of two ſuperficial angles of an equilateral triangle.

Now let us prove that it is conteined in the given ſphere whoſe Diameter is N L. Foraſmuch as perpendiculars drawn from the centers of the Dodecahedron, to the middle ſections of his ſides, are the halves of the lines which couple the oppoſite middle ſections of the ſides of the Dodecahedron [g]; which lines alſo [h] do in the center divide one another into two equal parts. Therefore right lines drawn from that point to the angles

g) 3 Cor. of 17. 13.
h) idem.

angles of the Icosidodecahedron (which are set in those middle sections) are equal; which lines are 30 in number, according to the number of the sides of the Dodecahedron; for each of the angles of the Icosidodecahedron are set in the middle sections of each of the sides of the Dodecahedron. Wherefore making the center the center of the Dodecahedron, and the space any one of the lines drawn from the center to the middle sections, describe a sphere, and it shall passe by all the angles of the Icosidodecahedron, and shall contein it.



i) 4C. 17. 13

And forasmuch as the diameter of this solid, is that right line whose greater segment is the side of the Cube inscribed in the Dodecahedron [i], which side is N I by supposition. Wherefore that solid is conteined in the sphere given, whose diameter is put to be the line N L.

Now let us prove that the greater segment of the diameter is double to Q V the side of the solid. Forasmuch as the sides of the triangle A E B, are in the points Q and V divided into two equal parts, the lines Q V and B E are parallels [k]. Wherefore as A E is to A V, so is E B to V Q [l]. But the line A E is double to the line A V. Wherefore the line B E is double to the line Q V [m]. Now the line B E is equal to N I, or to the side of the Cube [n]; which line N I is the greater segment of the diameter N L. Wherefore the greater segment of the diameter given is double to the side of the Icosidodecahedron inscribed in the given sphere. Wherefore, We have described, &c. Which was required to be done.

k) Co. 39. 1.
l) 2. 6.
m) 4. 6.
n) 2 Cor. of 17. 13.

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To the understanding of the nature of this Icosidodecahedron, you must well conceive the passions and proprieties of both these solids, of whose bases it consisteth, to wit, of the Icosahedron and of the Dodecahedron. And although in it the bases are placed oppositely, yet have they to one another one and the same inclination. By reason whereof there lie hidden in it the actions and passions of the other Regular Solids. And I would have thought it not impertinent to the purpose to have set forth the inscriptions and circumscriptions of this Solid, if want of time had not hindred. But to the end the Reader may the better atteine to the understanding thereof, I have here following briefly set forth, how it may in or about every one of the five Regular Solids be inscribed or circumscribed; by the help whereof he may, with small travail or rather none at all, having well poised and considered the Demonstrations apperteining to the foresaid five Regular Solids, demonstrate both the inscription of the said Solids in it, and the inscription of it in the said Solids.

Of

Of the inscriptions and circumscriptions of an Icosidodecahedron.

AN Icosidodecahedron may contein the other five regular bodies. For it will receive the angles of a Dodecahedron in the centers of the triangles which subtend the solid angles of the Dodecahedron, which solid angles are twenty in number, and are placed in the same order in which the solid angles of the Dodecahedron taken away, or subtended by them, are. And by that reason it shall receive a Cube and a Pyramis conteined in the Dodecahedron, when as the angles of the one, are set in the angles of the other.

An Icosidodecahedron receiveth an Octohedron, in the angles cutting the six opposite sections of the Dodecahedron, even as if it were a simple Dodecahedron.

And it conteineth an Icosahedron, placing the twelve angles of the Icosahedron in the same centers of the twelve Pentagons.

It may also by the same reason be inscribed in each of the five regular bodies; to wit, In a Pyramis, if you place four triangular bases concentrical with four bases of the Pyramis, after the same manner that you inscribed an Icosahedron in a Pyramis; so likewise may it be inscribed in an Octohedron, if you make eight bases thereof concentrical with the eight bases of the Octohedron. It shall also be inscribed in a Cube, if you place the angles which receive the Octohedron in it, in the centers of the bases of the Cube. Again, you shall inscribe it in an Icosahedron, when the triangles compassed in of the Pentagon bases, are concentrical with the triangles which make a solid angle of the Icosahedron.

Lastly, it shall be inscribed in a Dodecahedron, if you place each of the angles thereof in the middle sections of the sides of the Dodecahedron, according to the order of the Construction thereof.

The opposite plain superficieces also of this solid are parallels. For the opposite solid angles are subtended of parallel plain superficieces, as well in the angles of the Dodecahedron subtended by triangles, as in the angles of the Icosahedron subtended of Pentagons, which thing may easily be demonstrated. Moreover in this solid are infinite properties and passions, springing of the solids whereof it is composed.

Wherefore it is manifest that a Dodecahedron and an Icosahedron, mixed, are transformed into one and the self same solid of an Icosidodecahedron. A Cube also and an Octohedron are mixed and altered into another solid, to wit, into one and the same Exoctohedron. But a Pyramis is transformed into a simple and perfect solid, to wit, into an Octohedron.

If we will frame these two solids joyned together into one solid, this onely must we observe.

In the Pentagon of a Dodecahedron inscribe a like Pentagon, and let its angles be set in the middle sections of the Pentagon circumscribed, and then upon the said Pentagon inscribed let there be set a solid angle of an Icosahedron, and so observe the same order in each of the bases of the Dodecahedron, and the solid angles of the Icosahedron set upon these Pentagons shall produce a solid consisting of the whole Dodecahedron, and whole

whole Icosahedron. In like sort, if in every base of the Icosahedron, the sides being divided into two equal parts, be inscribed an equilateral triangle, and upon each of those equilateral triangles be set a solid angle of a Dodecahedron, there shall be produced the same solid consisting of the whole Icosahedron, and of the whole Dodecahedron.

And after the same order, if in the bases of a Cube be inscribed squares subtending the solid angles of an Octohedron, or in the bases of an Octohedron be inscribed equilateral triangles subtending the solid angles of a Cube, there shall be produced a solid consisting of either of the whole solids, to wit, of the whole Cube, and of the whole Octohedron.

But equilateral triangles inscribed in the bases of a Pyramis, having their angles set in the middle sections of the sides of the Pyramis, and the solid angles of a Pyramis, set upon the said equilateral triangles, there shall be produced a solid consisting of two equal and like Pyramids.

And now if in these solids thus composed, you take away the solid angles, there shall be restored again the first composed solids, to wit, the solid angles taken away from a Dodecahedron and an Icosahedron composed into one, there shall be left an Icosidodecahedron, the solid angles taken away from a Cube and an Octohedron composed into one solid, there shall be left an Exoctohedron. Moreover, the solid angles taken away from two Pyramids composed into one solid, there shall be left an Octohedron.

Of the nature of a trilateral and equilateral Pyramis.

1 A trilateral equilateral Pyramis is divided into two equal parts, by three equal squares, which in the center of the Pyramis cut one another into two equal parts, and perpendicularly, and whose angles are set in the middle sections of the sides of the Pyramis.

2 From a Pyramis are taken away 4 Pyramids like unto the whole, which utterly take away the sides of the Pyramis, and that which is left is an Octohedron inscribed in the Pyramis in which all the solids inscribed in the Pyramis are conteined.

3 A perpendicular drawn from the angle of the Pyramis to the base, is double to the diameter of the Cube inscribed in it.

4 And a right line coupling the middle sections of the opposite sides of the Pyramis is triple to the side of the same Cube.

5 The side also of a Pyramis is triple to the diameter of the base of the Cube.

6 Wherefore the same side of the Pyramis is in power double to the right line which coupleth the middle sections of the opposite sides.

7 And it is in power sesquialter to the perpendicular which is drawn from the angle to the base.

8 Wherefore the perpendicular is in power sesquitertia to the line which coupleth the middle sections of the opposite sides.

9 A Pyramis and an Octohedron inscribed in it, also an Icosahedron inscribed in the same Octohedron, do contein one and the same sphere.

Of the nature of an Octohedron.

1 Four perpendiculars of an Octohedron, drawn in four bases thereof from two opposite angles of the said Octohedron, and coupled together by

by those 4 bases, describe a Rhombus, or Diamond figure; one of whose diameters is in power double to the other diameter.

2 For it hath the same proportion that the diameter of the Octohedron hath to the side of the Octohedron.

3 An Octohedron and an Icosahedron inscribed in it, do contein one and the same sphere.

4 The diameter of the solid of the Octohedron is in power sesquialter to the diameter of the circle which conteineth the base, and is in power duple superbipartiens tertias (that is, as 8 to 3,) to the perpendicular or side of the foresaid Rhombus; and moreover is in length triple to the line which coupleth the centers of the next bases.

5 The angle of the inclination of the bases of the Octohedron, doth with the angle of the inclination of the bases of the Pyramis, make angles equal to two right angles.

Of the nature of a Cube.

1 The diameter of a Cube is in power sesquialter to the diameter of his base.

2 And is in power triple to his side.

3 And unto the line which coupleth the centers of the next bases, it is in power sextuple.

4 Again, the side of the Cube, is to the side of the Icosahedron inscribed in it, as the whole is to the greater segment.

5 Unto the side of the Dodecahedron, it is as the whole is to the lesser segment.

6 Unto the side of the Octohedron it is in power duple.

7 Unto the side of the Pyramis it is in power subduple.

8 Again, the Cube is triple to the Pyramis, but to the Cube the Dodecahedron is in a manner double. Wherefore the same Dodecahedron is in a manner sextuple to the said Pyramis.

Of the nature of the Icosahedron.

1 Five triangles of an Icosahedron, do make a solid angle, the bases of which triangles make a Pentagon. If therefore from the opposite bases of the Icosahedron be taken the other Pentagon by them described, these Pentagons shall in such sort cut the diameter of the Icosahedron which coupleth the foresaid opposite angles, that that part which is conteined between the plaines of these two Pentagons shall be the greater segment, and the residue which is drawn from the plain to the angle, shall be the lesser segment.

2 If the opposite angles of two bases joyned together, be coupled by a right line, the greater segment of that right line is the side of the Icosahedron.

3 A line drawn from the center of the Icosahedron to the angles, is in power quintuple to half that line, which is taken between the Pentagons, or of the half of that line, which is drawn from the center of the circle which conteineth the foresaid Pentagon, which two lines are therefore equal.

4 The side of the Icosahedron conteineth in power either of them, and also the lesser segment, to wit, the line which falleth from the solid angle to the Pentagon.

5 The diameter of the Icosahedron conteineth in power the whole line,

which coupleth the oppoſite angles of the baſes joyned together, and the greater ſegment thereof, to wit the ſide of the Icoſahedron.

6 The diameter alſo is in power quintuple to the line which was taken between the Pentagons, or to the line which is drawn from the center to the circumference of the circle which conteineth the Pentagon compoſed of the ſides of the Icoſahedron.

7 The dimetient conteineth in power the right line which coupleth the centers of the oppoſite baſes of the Icoſahedron, and the diameter of the circle which conteineth the baſe.

8 Again, the ſaid dimetient conteineth in power the diameter of the circle which conteineth the Pentagon, and alſo the line which is drawn from the center of the ſame circle to the circumference: That is, it is quintuple to the line drawn from the center to the circumference.

9 The line which coupleth the centers of the oppoſite baſes, conteineth in power the line which coupleth the centers of the next baſes, and alſo the reſt of that line of which the ſide of the Cube inſcribed in the Icoſahedron is the greater ſegment.

10 The line which coupleth the middle ſections of the oppoſite ſides, is triple to the ſide of the Dodecahedron inſcribed in it.

11 Wherefore if the ſide of the Icoſahedron, and the greater ſegment thereof be made one line, the third part of the whole is the ſide of the Dodecahedron inſcribed in the Icoſahedron.

Of the Dodecahedron.

1 The diameter of a Dodecahedron conteineth in power the ſide of the Dodecahedron, and alſo that right line to which the ſide of the Dodecahedron is the leſſer ſegment, and the ſide of the Cube inſcribed in it is the greater ſegment, which line is that which ſubtendeth the angle of the inclination of the baſes, conteined under two perpendiculars of the baſes of the Dodecahedron.

2 If there be taken two baſes of the Dodecahedron, diſtant from one another by the length of one of the ſides, a right line coupling their centers being divided by an extream and mean proportion, maketh the greater ſegment the right line which coupleth the centers of the next baſes.

3 If by the centers of five baſes ſet upon one baſe, be drawn a plain ſuperficies, and by the centers of the baſes which are ſet upon the oppoſite baſe, be drawn alſo a plain ſuperficies, and then be drawn a right line, coupling the centers of the oppoſite baſes, that right line is ſo cut, that each of his parts ſet without the plain ſuperficies, is the greater ſegment of that part which is conteined between the plaines.

4 The ſide of the Dodecahedron is the greater ſegment of the line which ſubtendeth the angle of the Pentagon.

5 A perpendicular line drawn from the center of the Dodecahedron to one of the baſes, is in power quintuple to half the line which is between the plains.

6 And therefore the whole line which coupleth the centers of the oppoſite baſes is in power quintuple to the whole line which is between the ſaid plains.

7 The line which ſubtendeth the angle of the baſe of the Dodecahedron, together with the ſide of the baſe are in power the quintuple to the line which is drawn from the center of the circle which conteineth the baſe, to the circumference.

8 A Section of a ſphere conteining 3 baſes of the Dodecahedron, taketh a third part of the diameter of the ſaid ſphere.

9 The ſide of the Dodecahedron and the line which ſubtendeth the angle of the Pentagon, are equal to the right line which coupleth the middle ſections of the oppoſite ſides of the Dodecahedron.

THE END.

Euclides Data.

A Commentary or Preface written by the Phyloſopher MARINUS, *on* EUCLIDES DATA.

IN the firſt place we ought to ſet down (as a Foundation) what that is, which we call *DATUM* or GIVEN; then to conſider the profit and utility thereof; and in the third place, to what Art or Science this Tract doth appertein.

The Word *DATUM* therefore is diverſly defined, for the Ancients have defined it after one manner, and later Writers after another, whence it follows that it ſeemeth a difficult thing to give a true explication thereof; for ſome of them have not delivered the Definition of the Word; but have with much labour and trouble ſought certain propricties thereof, and ſome others collecting and mingling what hath been delivered by others before, have endevoured to define the Word *DATUM*; but not ſo exquiſitely but that they have contradicted themſelves; although what hath been ſaid by all of them, ſeemes to be grounded on one and the ſame notion and ſuppoſition; for they all take the Word *DATUM* to be a thing comprifed; and therefore among ſuch as have endevoured to deſcribe it moſt ſimply, and with ſome ſimple difference, ſome of them have taken the Word *DATUM* to be the ſame with *ORDINATUM*, and ſo *Appollonius* underſtands it in his Tract of Inclinations, and in his univerſal Tract; and ſo others, as *Diodorus*, takes it to be *COGNITUM* KNOWN; for in this ſignification he takes the right line and the angles to be given, and all that may arrive to our Knowledge, although we may not be able well to expreſſe it. But others have believed that it hath the ſame ſignification as the Word [*Effabile*] that may be declared, and ſo *Ptolomie* would have it, who calls thoſe things GIVEN, whoſe meature is known whether preciſely, or neer the matter. Others alſo have thought the Word *DATUM* to be what is granted

granted us by the propoſer in the Hypotheſis; being that in the Firſt Elements, a point given, and a right line given, is diverſly taken (that is to ſay, that who ſo would give and determine the quantity of a right line) all which things ſignifie ſome COMPREHENSION; and therefore of all theſe Definitions, thoſe are moſt agreeable, which do moſt openly declare the COMPREHENSION, as we ſhall make evident by what follows.

Let us now unfold the diverſe opinions of thoſe, who writing the nature of *DATUM* GIVEN, have not taken one ſimple Marke, or only Character for its Definition; and let us reduce it as in a Summary or Epitomy, to the end we may with the more eaſe know or number all their differences. Some of them then have defined *DATUM* to be *Ordinatum* and *Porimon* together, and others *Ordinatum* and *Cognitum* together, and others *Porimon* and *Cognitum* together. Wherefore all ſeem to have ſo defined it as to have have had regard to the *Comprehenſion*, or *Aſſuming* and *Inventi-on*, of the thing given; and to the end that we may the better conceive their opinions, and that from the ſaying of many we may be able to draw a true Definition of what is propoſed, we will take notice in the firſt place of the ſignification of all the ſimple terms which they make uſe of, as alſo of the terms oppoſed to them, to wit, *Inordinatum*, and *Incognitum*, *Aporon*, and *Irrational*; for thoſe things appertein to this Geometrical buſineſſe, to natural things, and to Mathematical Diſcipline.

Now we may call that *Ordinatum* (or Regulated) which doth alwayes keep and obſerve that for which it is ſaid to be ordered, whether you regard its Magnitude or Species, or touching ſome other ſuch like thing: It is alſo thus defined, *Ordinatum* is that which cannot be done in divers manners, but in one only manner, and in ſome determined place: As for Example, A right line drawn by two given points, is ſaid to be ordered, by reaſon it cannot be otherwiſe done, nor in divers manners. But an angle paſſing by two points is ſaid to be *Inordinatum* (or diſordinate and irregular) for that it is made in infinite and divers manners by a great or ſmall circle deſcribed by two points *ad infinitum*. Contrariwiſe, an angle conſtituted by three points, is ſaid to be *Ordinatum*, as alſo thoſe things which follow, are ſaid to be *Ordinatum*, as to conſtitute an equilateral triangle on a right line, for it cannot be diverſly made, but unchangeably, on both the extremities of the line. Again, To divide a given right line according to a given proportion, for that cannot be done but in one certain point. The things *Inordinatum* are ſuch as are done contrary to thoſe laſt mentioned, as to conſtitute a Scalene triangle, and to divide a right line indefinitely. Wherefore the Problem is ordered, is propoſed in the determination, conſidering that a certain thing may be in one manner ſaid to be *Ordinatum*, and *Inordinatum* in another, as an equilateral triangle, conſidering the equality of the ſides, it is *Ordinatum*, but conſidering its Magnitude it is *Inordinatum*, being in no wiſe determined.

But we call that *Cognitum* which is notorious, as clear and comprehended of us, and *Incognitum* that which is not known or comprehended of us, as the length of a way is ſaid to be known, when we know how many Miles it conteins; alſo that the three lines of a Rectiline triangle are equal to two right angles; and in like manner, that the Binomial is Irrational, ſuch things are known, as alſo that it is only one right line that can touch a ſpiral line from a given point without it, from both parts; for if there were yet another line, two right lines would encloſe a ſpace, which is im-

impoſſible. Again, Irrational things are not ſaid to be unknown, but ſuch of them only which are neither known nor comprehended of us.

Porimon is that which we may make and conſtitute, that is to ſay, bring to our underſtanding. Again, it is defined thus, *Porimon* is that which may be exhibited by Demonſtration, or which is apparent without Demonſtration, as to deſcribe a circle from a center and with a ſpace, as alſo to conſtitute not only an equilateral triangle, but alſo a Scalene; or to find a *Binomium*, or to find two right lines rational, commenſurable in power only, and other things which are known infinitely, are *Porimon*; as to deſcribe a circle by two points.

Aporon is wholly oppoſite to *Porimon*, as for Example, the Quadrature of a circle, for that it hath not as yet been found; although it be certainly known that it may be: Nevertheleſſe the manner of finding it out hath not been to this preſent comprehended. But we ſpeak here of that which is already known, which is called *Porimon principale*; for what hath not been as yet made, and yet nevertheleſſe is poſſible, is called *Poriſton*, (or feaſible) although the Conſtruction be yet unknown. But *Aporon* as hath been afore ſaid, is oppoſite to *Porimon*, and is that whoſe Nature is not as yet decided, nor well determined.

Effabile, that is to ſay, rational (or ſpeakable and explicable) is that whoſe Magnitude, Species, and Poſition, we may be able to declare; but this Definition is a little too general, for properly, and according to it ſelfe, *Effabile* is that which is known by certain things, and according to a meaſure given by poſition, as of a ſpan, or a fingers breadth, &c.

Theſe things then being thus unfolded, we may eaſily perceive in what all thoſe things that we have afore ſpoken of do agree together, and wherein they do differ; and firſt of all how *Ordinatum* and *Cognitum* do agree together, and likewiſe their oppoſites the one to the other, for it cannot be ſaid that any one of thoſe things by counterchanging is the other, nor yet that the one hath not more extent than the other, although they agree in many things, as to deſcribe a right line by two points, and to conſtitute an equilateral triangle by three Circles. But to ſquare a Circle, that is indeed *Ordinatum*, yet nevertheleſſe, *Incognitum*. Alſo that at a point of a ſpiral line there is but one touch line, that is, of the kind we call *Ordinatum*, and cannot be otherwiſe done, yet nevertheleſſe the Demonſtration and Conſtruction thereof is not yet known. Again, the indefinite ſection, and the Conſtruction of the Scalenum is *Cognitum*; but is not *Ordinatum*; in ſo much as it is manifeſt that amongſt thoſe things which are *Ordinatum*, there are ſome that are *Cognitum*, and others that are *Incognitum*; and contrariwiſe, that amongſt the things that are *Cognitum*, there are ſome that are *Ordinatum*, and others that are *Inordinatum*; and therefore thoſe things anſwer one another, as among Living Creatures, that which hath reaſon, with that which hath Feet, for there is no equality amongſt them, neither doth the one extend more than the other.

In like manner, *Ordinatum* and *Inordinatum* agree together, reſpecting *Porimon* and *Aporon*; ſeeing that between them there is a very great reſemblance, and becauſe that they do differ only in the manner before expreſſed; for in truth the ſpiral line is *Ordinatum*; but it was not *Porimon* before *Archimedes*; and by the ſame reaſon thoſe things that are inordinate and known by an infinity of wayes and meanes are *Porimon*, if any one ſhall undertake to invent their Conſtitution and Conſtruction. Yet nevertheleſſe

theleſſe they are not ordinate, as to conſtitute a Scalenum triangle, it being no difficult thing to make known the conſtitution thereof by an equilateral triangle, yea it is moſt eaſie, although it be inordinate, and known by an infinity of wayes.

And in the ſame manner do agree *Ordinatum* and *Inordinatum*, together with *Effabile* and *Irrationale*; for they agree together in many things, differing nevertheleſſe by the fore-going reaſon, ſeeing thoſe things there mentioned bear no equality to each other, neither doth one thing contein the other; for all Binomiums and ſuch as are taken as Irrationals, are indeed ordinate, but yet they are not *Effabile*, or expreſſible, or to be unfolded, as the diameter of the ſquare is in reſpect of its ſide. Now touching *Effabile* there are divers inordinate; becauſe they are diverſly known, and indeterminately, for a Scalenum triangle may be meaſured by a defined and propoſed meaſure, as explicable, although it be inordinate.

Now it is eaſie to ſee the agreement that there is between *Cognitum* and *Porimon*, but it is a difficult thing to expreſſe or unfold their difference; foraſmuch as in their natures they come ſo neer to one another as that there ſeems to be an equality between them: Nevertheleſſe, there will ſome difference appear to him that ſhall conſider it more ſtrictly; for let it be conſented to that there can be only one line that can touch a ſpiral line in a certain point, that is *Cognitum*, yet notwithſtanding the Problem is not *Porimon*, it being not as yet comprehended; ſo as that all that which is *Cognitum* is not therefore *Porimon*. But all that which is *Porimon* is alſo *Cognitum*, and therefore *Cognitum* appeares to be of a greater extent than *Porimon*.

Now *Cognitum*, *Porimon*, and *Effabile*, do agree in ſome certain things, and do differ in other things by the ſame reaſon before alledged; for thoſe lines which are there called Irrationals are in truth known; and yet nevertheleſſe are not *Effabile* or explicable. Contrariwiſe, every number is indeed *Effabile*, and yet every number is not *Cognitum*. But *Effabile* is alwayes of its own nature expreſſable, although that ſome lengths may be now *Effabile*, and at another time not, if it be examined with ſome other according to one and the ſame meaſure. But alſo that ſame length is ſometimes known, and other times not, though they wholly agree with one another. Now it is a difficult thing to find ſome thing that ſhall be *Effabile* and *Incognitum*, for *Cognitum* ſeemes to be of a greater extent than *Effabile*, and by thoſe things it is manifeſt that *Porimon* and *Aporon* do differ from RATIONAL or *Effabile*, and from IRRATIONAL, for of IRRATIONAL ſome of them may be *Porimon*; but of RATIONAL none of them can be Irrationals; and therefore it is very eaſie to perceive in what the before expreſſed things agree. Notwithſtanding they ſeem to agree together, in ſuch ſort as that *Porimon* ſeemes to be of a greater extent than *Effabile*.

Now by theſe things we may come to know the difference of thoſe things that have been before ſpoken of, for in truth *Effabile* and *Irrationale* are ſo termed in reſpect of meaſure, which notwithſtanding is not as yet arrived to our underſtanding, ſeeing that ſome thing, that is rational, may be as yet unknown to us, and in like manner may be rational, and yet may never be comprehended ſo to be. But *Ordinatum* and *Inordinatum* is ſo termed according to it ſelf, and according to the proper nature of the thing on which we contemplate, although it be not comprehended by us. As *Archimedes* had perceived ſome things to be ordinate from

from the nature of the things, the which *Serenus* had before contemplated. But *Cognitum* and *Incognitum* is ſpoken in reſpect of us, ſo as the things before mentioned do differ among themſelves; for theſe have reſpect to us, the others, ſome of them to their proper nature, and the reſt to meaſure.

Having then explained the agreements and differences of the things that have been propoſed, it remaines now that we conſider what is meant by the Word *DATUM*, for of all thoſe that believe the Word *DATUM* to be that which is conſented to by the Propoſer in the Hypotheſis, are wide from what is ſought; becauſe that all the Elements of the things GIVEN are not compoſed of this ſort of GIVEN, which is according to the Hypotheſis, as may be ſeen in thoſe Tracts which have been made on this ſubject GIVEN. Wherefore waving this opinion, let us judg of the Definitions of others.

Then, that which is conſented to or granted in the Hypotheſis, is ſome thing which is conſequently known by the principles; but ſuch as make uſe of Definitions of one only Word, do define it and remarke it by ſome one of the before mentioned, as hath been ſpoken in the beginning, ſo as that almoſt all ſeem to have had this common notion of GIVEN, to wit, that it is comprehended even as the Word *DATUM*, doth alſo manifeſt it to be; and amongſt thoſe, theſe are the chief that do define it by the Hypotheſis or Suppoſition; and others have had regard to what is conſented to or granted. But we making uſe of the ſaid things as of a Rule and Direction to judge rightly, we may be able to find out a perfect Definition of *DATUM*; for it is certain that it ought to equal, and be convertible with the thing defined, which is one thing proper to good Definitions. Now ſuch ſeemes to be the Definition of the thing propoſed, which among the moſt ſimple and plain Expoſitors is defined *Porimon*, and amongſt the more acute, that which defineth it to be *Porimon* and *Cognitum* together; but all the reſt are imperfect; for that which defineth it *Ordinatum* is not ſufficient for the comprehenſion and knowledge of *DATUM*; becauſe that neither wholly ordinate, nor alone ordinate, is not compriſed, ſeeing that there are things inordinate that have the ſame condition, as hath been ſhewn. Again, that reaſon gives not ſatisfaction neither, which deſcribes it to be *Cognitum*; for all that is known is not comprehended, although that alone *Cognitum* be comprehended. Moreover, that alſo is not perfect which defineth it to be *Effabile*, for *Effabile* is not alone comprehended; ſeeing that ſome of the Irrationals are alſo comprehended. In like manner, all *Effabile* is not comprehended, as hath been before declared. Now amongſt the Definitions which expound it by one only Word, there remains that which defineth it to be *Porimon*, which ſeemeth greatly to manifeſt the comprehenſion; for whole *Porimon* and alone *Porimon* is comprehended. Wherefore *EUCLIDE* himſelf uſeth ſuch a Definition in a Deſcription of all the kinds of GIVEN by him conceived and regarded. But amongſt ſuch Definitions as are compounded, that is a perfect Definition which defineth *DATUM* to be *Cognitum* and *Porimon* together, having *Cognitum* for analogical kind, and *Porimon* for difference; but that is imperfect which hath *Ordinatum* and *Porimon* together; for thoſe things which are ſuch, are not alone GIVEN, and that which defineth it *Ordinatum* and *Effabile* together, comprehendeth likewiſe the GIVEN, with the defect or want. But that of *Cognitum* and *Ordinatum* together, is not to be received or admitted, becauſe

cause it doth exceed what is defined, for such is not given alone: Therefore those only which have declared that *DATUM* is *Cognitum* and *Porimon* together, seem to have attained the notion of GIVEN, for that which is such is all, and alone comprehended, which two things ought to be in those Definitions that are well given. But the former come neer to those which have thus defined it: *DATUM* is that to which we may find an equal, according to those things we have proposed in the first principles and Hypothesis, of which number *EUCLIDE* is one, making use throughout of the Verb παρισασθαι, which signifies to exhibit or invent, although he leaves *Cognitum* as a consequent of *Porimon*; some one nevertheless might reprove him, for that in the first place he hath not defined *DATUM* in general; but immediately some of the kinds of GIVEN, although in his Elements of GEOMETRY; he hath defined the line simple before the Species or Kinds.

What is the Utility and Profit that ariseth from this Tract of DATA, *or things* GIVEN.

AFter having explained universally, and according to what seemed necessary for our present use, what this Word *DATUM* signifieth: It followes to shew the Utility of this Tract. Now this Tract is such, as that it is not only ordained and instituted for its own respect, but for some other thing; for it is very necessary to a place which is called Resolved, and we have already declared elsewhere how much strength a resolved place doth obtein in Mathematical Disciplines, as also in Opticks and Cannons, which come very neer to them, as well for that Resolve is an invention of the Demonstration, as for that in such like things it serves us much for the invention of the Demonstration, or for that it is much more excellent to meet with a Resolutive power, then to enjoy divers particular Demonstrations.

To what Art or Science this Tract is referred.

NOw seeing the consideration of GIVEN is useful and profitable in all these kinds of Arts, for that it serveth much to RESOLUTION, it may well be said to be recalled, not only to one only Science, but to the Mathematicks universally which treat of Numbers, Time, Swiftnesse, and such like things, which treateth likewise of Reasons, as also of proportions, and in a word of all *Mesietez*: Wherefore for the perfect and demonstrative Knowledge of things GIVEN, being of so great Utility, *EUCLIDE* hath taken pains to frame this Book of things GIVEN, which Author amongst all such as have composed the Elements of Geometry, hath justly deserved the first place and rank, and who having invented the Elements, or rather the Introductions almost of all Mathematical Disciplines, to wit, of all Geometry in 13 Books, of Astronomy, of Musick, and Opticks, he hath left in writing the Elements RESOLUTIVE, in this Treatise of things GIVEN; but as he was a Geometrician, he hath particularly accommodated to Magnitudes, that, that was of the GIVEN; yet nevertheless common in other things, which method hath also been observed by him, when treating universally of Reasons and Proportions, he appropriates them to the Magnitudes mentioned in his Fifth Book of Plains.

Now

Now it hath been declared in general what is the meaning of *DATUM*, to what Science it apperteineth, and how profitable the Contemplation thereof is. We will add to what hath been said, the Description of this Science which treats of things GIVEN; seeing that it is (as appeares by what hath been said) a comprehension in all manners of things GIVEN; and of their accidents and proprieties. But having respect to the proposed Book, we shall declare it to be an Elementary Doctrine of the whole Knowledge of things GIVEN, whence it follows that it will be very profitable, as also the things therein conteined, seeing they refer to things GIVEN.

Now this Book is divided according to the Species or Kinds of the things GIVEN, and in the first section are conteined those things which are given by reason. Secondly, such as are given by position: and lastly, such as are given by Species or Kind; for that which is given by Magnitude is simple, and particularly conteined in the others, and principally in those things given by Specis or Kind. Now he hath begun with those things given by Reason and Position, forasmuch as those that are given by Species are constituted of them. *EUCLIDE* gives yet another division to this Book, for that he divides it into universal Magnitudes, Lines, and Superficies, and into Circular Theoremes, which order he hath also observed in the Definitions and Suppositions of this Book. Moreover, he useth a certain way of instructing, which proceeds not by Composition, but by Resolution, as *Pappus* hath amply set down in his Commentaries on this Book.

THE END OF THE PREFACE OF MARINUS.

DEFINITIONS.

1 PLains or Spaces, Lines, and Angles, to which we may find others equal, are said to be given by Magnitude.

2 A Reason is said to be given, when we may find one of the same or equal thereto.

3 Rectiline figures, whose angles are given, and also the reason of the sides to one another, are said to be given by Species or Kind.

4 Points, Lines, and Angles, which have and keep alwayes one and the same place and situation, are said to be given by Position or Situation.

5 A Circle is said to be given by Magnitude, when the Semidiameter thereof is given by Magnitude.

6 A Circle is said to be given by Position, and by Magnitude when the center thereof is given by Position, and the Semidiameter by Magnitude.

7 Segments of Circles, whose angles and bases are given by Magnitude, are said to be given by Magnitude.

8 Segments of a Circle, whose angles are given by Magnitude, and the bases of the segments by Position and Magnitude, are said to be given by Position and by Magnitude.

9 A Magnitude AB, is greater than another Magnitude C, by a given Magnitude BD; when having taken away the given Magnitude DB, the rest AD, is equal to the other Magnitude C.

10 A Magnitude AB, is lesse than another Magnitude C, by a given magnitude BD, when having added thereto the given magnitude BD, the whole AD, is equal to the other magnitude C.

11 A Magnitude AB, is said to be greater than another magnitude CB, by a given magnitude AD, and in reason, when taking from the same magnitude the given magnitude AD, the rest DB, hath to the other magnitude CB, a given reason.

12 A magnitude AB is said to be lesse than another magnitude BC, by a given magnitude AD, and in reason, when the given magnitude AD being added thereto, the whole DB hath to the other magnitude BC, a given reason.

13 A right line is said to be drawn down from a given point, unto a right line given in position, the right line being drawn in a given angle.

14 A right line is said to be drawn up from a given point, to a right line given in position, the right line being drawn in a given angle.

15 A right line is against another right line in position, when it is drawn parallel thereto thorough a given point.

PROPOSITION 1.

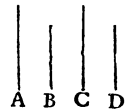
Two magnitudes A and B, being given, the reason they have to one another A to B, is also given.

Demonstration. FOr seeing that the magnitude A is given [a], we may find one equal thereto, which let be C. Again, forasmuch as the magnitude B is given, we may also find one equal to that, and let that be D. Therefore seeing that A is equal to C and B to D, as A is to C [b], so is B to D, and by permutation, [c] as A shall be to B, so C shall be to D. Therefore [d] the reason of A to B is given, for it is the same reason as of C to D, as we have found, and which was first to be demonstrated.

a) 1 def.
b) 7. 5.
c) 16. 5.
d) 2 def.

PROP. 2.

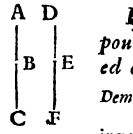
If a given magnitude A, hath to some other magnitude B, a given reason, that other magnitude B, is also given by magnitude.



Demonſtration FOr ſeeing that A is given, we may find one equal thereto, which let be C: And foraſmuch as the reaſon of A to B, is alſo given, we may find [a] one of the ſame. Let it be found, and let the reaſon be of C to D. Now ſeeing that as A is to B, ſo C is to D; and by permutation, as A is to C, ſo B is to D: But A is equal to C, Therefore [b] B ſhall be alſo equal to D. Therefore [c] the magnitude B is given, ſeeing that thereto there hath been found one equal, to wit, D.

a) 2. def.
b) 14. 5.
c) 1. def.

PROP. 3.

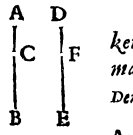


If given magnitudes A B *and* B C, *are compounded, that magnitude* A C, *that is compounded of them ſhall be alſo given.*

Demonſtration FOr ſeeing that A B is given, we may find one equal to it, which let be D E. Again, ſeeing that B C is given, we may alſo find one equal to that, which let be E F. Wherefore ſeeing that D E is equal A B, and E F is equal to B C, the whole A C [a] is equal to the whole D F. Therefore A C is given, ſeeing that D F is propoſed equal thereto.

a) 2 ax. 1.

PROP. 4.

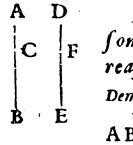


If from a given magnitude A B, *there be taken away a given magnitude* A C, *the remaining magnitude* C B, *is alſo given.*

Demonſtration FOraſmuch as A B is given, we may find one equal thereto, which let be D E. Again, ſeeing that A C is given, we may alſo find one equal to it, which let be D F. Seeing then that the Magnitude A B is equal to the magnitude D E, and the magnitude A C to the magnitude D F; the reſt C B [a] ſhall be equal to the reſt F E. Wherefore C B is given, for to it there hath been found an equal, to wit, F E.

a) 3 ax. 1.

PROP. 5.



If a magnitude A B, *hath a given reaſon to ſome part thereof* A C, *it will have alſo a given reaſon to the part remaining* C B.

Demonſtration LEt D E be expoſed as a given magnitude, and ſeeing that the reaſon of the magnitude A B, to the magnitude A C, is given, [a] we may find one of the ſame, which let be D E to D F; therefore the reaſon of the ſame D E to D F is given. But D E being given, ſo is [b] alſo its part D F; and conſequently, [c] the reſt F E: Therefore [d] ſeeing that D E and F E are given, the reaſon of the ſame D E to F E, is alſo given. And foraſmuch as D E is to D F, as A B is to A C, and by converſion, as D E to F E, ſo A B is to C B. But the reaſon of D E to F E is given, as hath been demonſtrated; therefore the reaſon of A B to C B is alſo given.

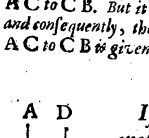
a) 2. def.
b) 2. prop.
c) 4. prop.
d) 1. prop.
19. 5.

SCHO-

SCHOLIUM.

From this it is evident that if a Magnitude hath to ſome part thereof a given reaſon, by diviſion, the reaſon that one part hath to the other, ſhall be alſo given. For ſeeing that as D E *is to* F E, *ſo is* A B *to* C B; *by diviſion, as* D F *to* F E, *ſo* A C *to* C B. *But it hath been demonſtrated that the parts* D F *and* F E *are given, and conſequently, their reaſon is alſo given: In like manner, therefore the reaſon of* A C *to* C B *is given.*

PROP. 6.

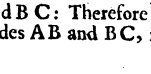


If two magnitudes A B *and* B C, *having to one another a given reaſon, are compounded, the magnitude* A C, *compounded of them, ſhall alſo have a given reaſon to each of them* A B *and* B C.

Demonſtration LEt the given magnitude D E be expoſed, and ſeeing that the reaſon of A B to B C is given; let there be made one and the ſame of the ſaid D E to E F; therefore the reaſon of the ſame D E to E F is given; and therefore [a] the magnitude D E being given, both the one and the other of them D E and F E, is given. Wherefore [b] the whole D F ſhall be alſo given. Therefore [c] the reaſon of the ſame D F to each of them D E and E F, ſhall be given. And foraſmuch then as A B is to B C, ſo is D E to E F; in compounding, [d] as A C is to B C, ſo is D F to E F: Therefore by converſion, as A C to A B, ſo is D F to D E. Therefore as the whole D F is to each of the other magnitudes D E and E F, ſo the whole A C is to each of the Magnitudes A B and B C: Therefore [e] the reaſon of the ſame A C to each of the magnitudes A B and B C, is given.

a) 2. p.
b) 3. p.
c) 1. p.
d) 18. 5.
e) 2. def.

PROP. 7.

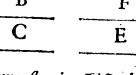


If a given magnitude A B, *be divided according to a given reaſon* A C *to* C B, *each ſegment* A C *and* C B *is given.*

Demonſtration FOr ſeeing the reaſon of A C to C B is given, the reaſon of [a] A B to each of them (A C and C B) is alſo given. But A B is given: Therefore [b] each of the ſegments A C and C B, is alſo given.

a) 6. p.
b) 2. p.

PROP. 8.



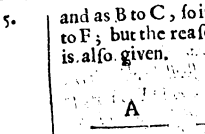
Magnitudes A *and* C, *which have to one and the ſame a given reaſon* B, *ſhall be to one another in a given reaſon,* A *to* C.

Demonſtration FOr let the given magnitude D be expoſed, and ſeeing that the reaſon of A to B is given, Let the ſame be done of the ſaid D to E. Now ſeeing that D is given, [a] E is alſo given. Again, ſeeing that the reaſon of B to C is given, let the ſame be done of E to F. But E is given, and therefore F is alſo given. But ſeeing that D is given, [b] the reaſon of the ſame D to F is given; and ſeeing that as A to B, ſo D to E, and

a) 2. p.
b) 1. p.

c) 22. 5. and as B to C, so is E to F; in reason of equality, c as A is to C, so is D to F; but the reason of D to F is given. Therefore the reason of A to C is also given.

PROP. 9.



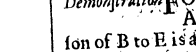
If two or more magnitudes A, B, *and* C, *are to one another in a given reason, and that the same magnitudes* A, B, *and* C, *have to other magnitudes* D, E, *and* F, *given reasons, although they be not the same; those other magnitudes* D, E, *and* F, *shall be also to one another in given reasons.*

Demonstration FOr asmuch as the reason of A to B is given, as also that of A to D, the reason of D to B shall be given: But the reason of B to E is also given; therefore the reason of the same D to E shall be in like manner given. Again, seeing that the reason of B to C is given, and also that of B to E; the reason of E to C shall be given. But the reason of C to F is also given. Therefore a the reason of E to F shall be given. But it hath been demonstrated that the reason of D to E is also given; and therefore b the reason of D to F shall be given. Therefore the magnitudes D, E, and F, are to one another in given reasons.

a) 8. p.

b) 8. p.

PROP. 10.



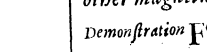
If a magnitude A B, *be greater than another magnitude* B C, *by a given magnitude, and in reason, the magnitude* A C *compounded of both, shall be also greater then that same magnitude, by a given magnitude, and in reason: But if that compounded magnitude be greater then the same magnitude, by a given magnitude, and in reason; either the remainder shall be also greater then that same by a given magnitude, and in reason; or else the same remainder is given with the following, to which the other magnitude hath a given reason.*

Demonstration FOr seeing that A B is greater than B C by a given magnitude, and in reason, let the given magnitude A D be taken away. Therefore a the reason of the remainder D B to B C is given; and in compounding, b the reason of D C to B C is also given. But the magnitude A D is also given; Therefore A C is greater than the same B C by a given magnitude, and in reason.

a) 11 def.

b) 6. p.

Again, Let the magnitude A C be greater than the magnitude B C, by a given magnitude, and in reason: I say that the rest A B, is either greater then the same B C by a given magnitude and in reason, or that the same A B, with that which followeth, to which B C hath a given reason, is given.



For

Forasmuch as the magnitude A C is greater than the magnitude B C, by a given magnitude, and in reason, cut off from it the given magnitude: Now the same given magnitude is either lesse than the magnitude A B, or greater: Let it in the first place be lesse, and let it be A D, Therefore the reason of the remainder D C to C B is given. Wherefore by division, the reason of D B to B C is given. But the magnitude A D is also given; therefore the magnitude A B is greater c then the magnitude B C by a given magnitude, and in reason. Now let the given magnitude be greater than the magnitude A B, and let A E be put equal thereto; therefore d the reason of the remainder E C to C B is given; and by conversion, e the reason of the same B C to B E, is also given. But the same E B with B A is given, for that the whole A E is given: Therefore there is given A B, with that which followes, B E, to which B C hath a given reason.

c) 11 def.

d) 11 def.

e) 5. p.

PROP. 11.

If a magnitude A B, *be greater than a magnitude* B C, *by a given magnitude, and in reason, the same magnitude* A B, *shall be also greater than the magnitude compounded of them by a given magnitude, and in reason, and if the same magnitude be greater then the two others together by a given magnitude, and in reason, that same magnitude shall be also greater then the rest by a given magnitude, and in reason.*

Demonstration FOr seeing that the magnitude A B is greater then B C by a given magnitude, and in reason; let there be taken from it a given magnitude A D: Therefore a the reason of the rest D B to D C, is given, and therefore b the reason of D C to B D shall be also given: Let the same be done of A D to D E, therefore the reason of the same A D to D E is given. But A D is given, therefore c D E is also given, and consequently, d the rest A E, is also given. But seeing that as A D is to D E, so is D C to B D; by permutation, e as A D is to D C, so is D E to D B: Therefore by compounding, f as A C is to C D, so is E B to D B; and by permutation, g as A C is to E B, so is D C to D B. But the reason of D C to D B is given: Therefore also is A C to E B, and consequently, that of E B to A C. But it hath been demonstrated that A E is given, Therefore h A B is greater then A C by a given magnitude, and in reason.

a) 11 def.

b) 6. p.

c) 2. p.

d) 4. p.

e) 16. 5.

f) 18. 5.

g) 16. 5.

h) 11 def.

But now let A B be greater then A C by a given magnitude, and in reason: I say that the same A B is also greater then the rest B C, by a given magnitude, and in reason.

For seeing that A B is greater than A C by a given magnitude, and in reason, Let the given magnitude A B be cut off there-from: Therefore i the reason of the remainder E B to A C, is given, and consequently, also shall be given that of A C to E B. Let the same be done of A D to D E, Therefore the reason of A D to D E is given; and by conversion, k the reason of A D to A E shall be also given, and consequently that of A E to A D. Now A E is given, Therefore the whole A D l shall be also given; and seeing that as the whole A C is to whole E B, so the part cut off A D,

i) 11 def.

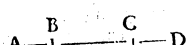
k) 5. p.

l) 2. p.

A D, is to the part cut off E D, so also shall be [m] the remainder D C to the remainder D B. But the reason of A C to E B is given: Therefore also shall be given that of D C to D B. Wherefore by division, [n] the reason of B C to D B is given; and consequently also shall be given that of D B to B C. But it hath been demonstrated that A D is given: Therefore [o] A B is greater then the same B C by a given magnitude, and in reason.

m) 19. 5.
n) Schol. 5. p.
o) 11 def.

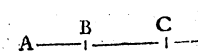
PROP. 12.



If there be three magnitudes A B, B C, *and* C D, *and that the first* A B, *with the second* B C, *to wit* A C, *be given. But the second* B C, *with the third* C D, (*to wit*, B D, *be also given: Either the first* A B *shall be equal to the third* C D, *or the one shall be greater then the other by a given magnitude.*

Demonstration FOrasmuch as each of the magnitudes A C and B D are given, the given magnitudes are either equal to one another, or inequal. Let them be first equal: Therefore A C is equal to B D, take away the common magnitude B C, and there will remain [a] A B, equal to C D. But suppose them to be inequal, as in this second figure, and let B D be greater then A C: Let then B E be put equal to A C. Now seeing that A C is given, B E is also given. But the whole B D is also given, the rest E D [b] shall be so also; and forasmuch as B E is equal to A C, taking away the common magnitude B C [c], there will remain A B equal to C E. But E D is given: Therefore C D is greater then A B by the given magnitude E D.

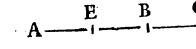
a) 3 ax. 1.



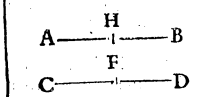
b) 4. p.
c) 3 ax. 1.

SCHOLIUM.

And if the first with the second, to wit, A C, *were greater then the second with the third, to wit,* B D, *as in the other figure,* C E *would be made equal to the same* B D, *and by the same reasons as was above demonstrated, that* A E *is given and equal to* C D; *and therefore* A B *greater then* C D *by a given magnitude.*



PROP. 13.



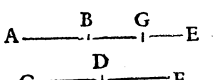
If there be three magnitudes A B, C D, *and* E, *and that the first of them* A B, *hath a given reason to the second* C D; *but the second* C D, *is greater then the third* E, *by a given magnitude, and in reason, also the first* A B, *shall be greater then the third* E, *by a given magnitude, and in reason.*

Demonstration FOr seeing that C D is greater then E by a given magnitude, and in reason, let the given magnitude C F be taken there-

there-from: Therefore the reason of the rest F D to E is given. And forasmuch as the reason of A B to C D is given, let the same be done of A H to C F. Therefore the reason of the same A H to C F is given. But C F is given: Therefore [a] A H is also given. And seeing that as the whole A B is to the whole C D, so the part cut off A H is to the part cut off C F, and so [b] also the rest H B is to the rest F D, the reason of the same H B to F D is also given. But the reason of F D to E is also given: Therefore [c] the reason of H B to E is given. But it hath been demonstrated that A H is given: Therefore [d] A B is greater then the said E by a given magnitude, and in reason.

a) 2. p.
b) 19. 5.
c) 8. p.
d) 11 def.

PROP. 14.



If two magnitudes A B *and* C D, *have to one another a given reason, and that to each of them there be added a given magnitude, to wit,* B E *and* D F; *either the whole* A E, *and* C F, *shall have to one another a given reason, or the one shall be greater then the other by a given magnitude, and in reason.*

Demonstration FOr seeing that each of those magnitudes B E and D F, is given, [a] the reason of the said B E and D F is also given; and if that reason be the same with that of A B to C D, that of the whole A E to the whole C F, [b] shall be the same; and therefore the reason of the said A E to C F is given.

a) 1. p.
b) 12. 5.

Now let the reason of B E to D F be not the same, with that of A B to C D, and let it be as A B to C D, so B G to D F. Therefore the reason of the said B G to D F is given. But the magnitude D F is given, therefore [c] B G is also given; and seeing that the whole B E is given, [d] the rest G E shall be also given. But forasmuch as A B is to C D, as B G is to D F, [e] so also is the whole A G to the whole C F; and therefore the reason of the said A G to C F is given: But the magnitude G E is given: Therefore [f] the magnitude A E is greater then the magnitude C F by a given magnitude, and in reason.

c) 2. p.
d) 4. p.
e) 12. 5.
f) 11 def.

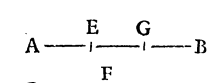
PROP. 15.



If two magnitudes A B *and* C D, *have to one another a given reason, and that from each of them be taken away a given magnitude (to wit, from the magnitude* A B *the magnitude* A E, *and from the magnitude* C D *the magnitude* C F) *the remaining magnitudes* E B *and* F D, *either shall have to one another a given reason, or the one of them shall be greater then the other by a given magnitude, and in reason.*

Demonstration FOr seeing that each magnitude A E and C F is given, the reason of A E to C F is given; and if it be the same with

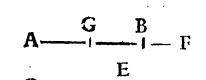
that of A B to C D, that of the remainder E B to the remainder F D, a) 19. 5. [a] shall be also the same; and therefore the reason of the said E B to F D shall be also given. But if it be not the same,



Let it be as A B to C D, so A C to C F. Now the reason of A B to C D is given, therefore also that of A G to C F shall be given. But C F is given, therefore [b] A G is given. But A E is also given, therefore [c] the rest E G is given; and seeing that as A B is to C D, so the part cut off A G is to the part cut off C F, and so also is [d] the rest G B to the rest F D; the reason of the said G B to F D is also given. Therefore seeing that E G is given, E B is greater then F D [c] by a given magnitude, and in reason.

b) 2. p. c) 4. p. d) 4. p. e) 11 def.

PROP. 16.

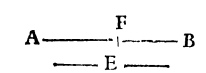


If two magnitudes A B *and* C D, *have to one another a given reason, and that from one of them, to wit,* C D, *there be taken away a given magnitude* D E, *and to the other* A B *there be added a given magnitude* B F, *the whole* A F *shall be greater then the rest* C E, *by a given magnitude, and in reason.*

Demonstration FOr seeing that the reason of A B to C D is given, let the same be made of B O to D E: Therefore [a] the reason of the said B G to D E is given. But D E is given, therefore [b] B G is also given. But B F is also given, therefore [c] the whole G F is given. And seeing that as A B is to C D, so the part cut off B G, is to the part cut off D E; and [d] so also is the remainder A G to the remainder C E; the reason of the said A G to C E is given: But G F is given, Therefore the magnitude A F is greater then the magnitude C E by a given magnitude, and in reason.

a) 2. def. b) 2. p. c) 3. p. d) 19. 5.

PROP. 17.



If there be three magnitudes A B, E, *and* C D, *and that the first* A B *be greater then the second* E, *by a given magnitude, and in reason. But the third* C D *be also greater then the same second* E, *by a given magnitude, and in reason; the first* A B *shall have to the third* C D, *either a given reason, or else the one shall be greater then the other by a given magnitude, and in reason.*

Demonstration FOr seeing that A B is greater then E by a given magnitude, and in reason, let the magnitude A F be taken away: Therefore the reason of the remainder F B to E is given. Again, seeing that C D is greater then the said E by a given magnitude, and in reason, let the given magnitude C G be cut off there-from; and the reason of the remainder G D to E shall be given: Therefore [a] the reason of

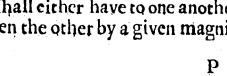
a) 8. p.

F B

F B to G D shall be also given. But to the said F B and G D are added the given magnitudes A F and C G: Therefore the whole A B and C D [b] shall either have to one another a given reason, or the one shall be greater then the other by a given magnitude, and in reason.

b) 14. p.

PROP. 18.



If there be three magnitudes A B, C D, *and* E F, *and that the one of them, to wit,* C D, *be greater then either of the other* A B *or* E F, *by a given magnitude, and in reason; either the two others* A B *and* E F, *shall have to one another a given reason, or the one shall be greater then the other by a given magnitude, and in reason.*

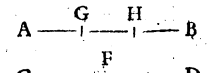
Demonstration FOrasmuch as the magnitude C D is greater then the magnitude A B by a given magnitude, and in reason, let the given magnitude D G be taken there-from: Therefore the reason of the remainder C G to A B is given. Let the same be made of G D to B H, Therefore the reason of the said D G to B H is given. But D G is given, Therefore [a] B H is also given. And seeing that as C G is to A B, so is G D to B H, [b] so also is the whole C D to the whole A H, the reason of the said C D to A H shall be also given.

a) 2. p. b) 12. 5.

Again, seeing that the same C D is greater then E F by a given magnitude, and in reason; let the magnitude D I be cut off there-from: Therefore the reason of the remainder C I to E F is given: Let the same be made of D I to F K, Therefore the reason of the said D I to F K shall be also given. But D I is given, Therefore F K is also given. And seeing that as C I is to E F, so is I D to F K; so also is the whole [c] C D to the whole E K; the reason of the said C D to E K shall be given. But the reason of the same C D to A H is also given: Therefore [d] the reason of the said A H to E K shall be given. And seeing that from the said A H and E K, the given magnitudes B H and F K are cut off, the magnitudes A B and E F [e] are either in a given reason to one another, or the one is greater then the other by a given magnitude, and in reason.

c) 12. 5. d) 8. p. e) 15. p.

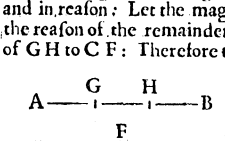
PROP. 19.



If there be three magnitudes A B, C D, *and* E, *and that the first* A B, *be greater then the second* C D, *by a given magnitude, and in reason; and that the second* C D *be greater then the third* E, *by a given magnitude, and in reason; also the first magnitude* A B *shall be greater then the third* E, *by a given magnitude, and in reason.*

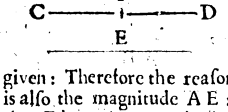
Demonstration FOr seeing that C D is greater then E by a given magnitude, and in reason; let the given magnitude C F be taken there-from: Therefore the reason of the remainder F D to E is given.

Again, seeing that A B is greater then the same C D by a given magnitude, and in reason: Let the magnitude A G be taken there-from: Therefore the reason of the remainder G B to C D is given: Let the same be made of G H to C F: Therefore the reason of the said G H to C F is given. But C F is given: Therefore also G H is given, and then A G is also given, the whole [a] A H shall be also given. But as G B is to C D, so is G H to C F, and so also [b] the remainder H B to the remainder F D: Therefore the reason of the said H B to F D is given. But the reason of the same F D to E is also given: Therefore the reason of H B to E is in like manner given, and so is also the magnitude A E: Wherefore the magnitude A B [c] is greater then E by a given magnitude, and in reason.



a) 3. p.
b) 19. 5.
c) 11 def.

OTHERWISE.

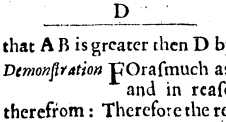


Construction LEt there be three magnitudes A B, C, and D, and let A B be greater then C by a given magnitude, and in reason; but let C be also greater then D, by a given magnitude, and in reason: I say that A B is greater then D by a given magnitude, and in reason.

Demonstration FOrasmuch as A B is greater then C by a given magnitude, and in reason, let the given magnitude A E be cut off therefrom: Therefore the reason of the remainder E B to C is given. But the magnitude C is greater then the magnitude D by a given magnitude and in reason; therefore [d] E B is greater then D by a given magnitude, and in reason: Wherefore let the given magnitude E F be cut off therefrom; and the reason of the remainder F B to D shall be given. But A F [e] is given. Therefore [f] A B is greater then D by a given magnitude, and in reason.

d) 13. p.
e) 3. p.
f) 11 def.

PROP. 20.



If there be two given magnitudes, A B *and* C D, *and that from them there be taken magnitudes* A E *and* C F, *having to one another a given reason; either the remaining magnitudes* E B *and* F D, *shall have to one another given reasons, or else the one shall be greater then the other by a given magnitude, and in reason.*

Demonstration FOr seeing that both the magnitudes A B and C D, are given, the reason of the said A B to C D is [a] also given; and if it be the same as of A E to C F, that of the remainder E B to the remainder F D shall be [b] also the same; and therefore the reason of the said E B to F D shall be also given. But if it be not the same, let it be so as that A E be to C F, as A G to C D. Now the reason of the said A E to C F is given: Therefore the reason of the said A G to C D is given. But C D is given, Therefore [c] A G is also given. But the whole A B is likewise given, Therefore [d] the remainder B G is given. And seeing that as A E

a) 1. p.
b) 19. 5.
c) 3. p.
d) 4. p.

A E is to C F, so is A G to C D, and also the remainder E G to the remainder F D, the reason of the said E G to F D is given. But G B is also given: Therefore the magnitude E B is greater [e] then the magnitude B D by a given magnitude, and in reason.

e) 11 def.

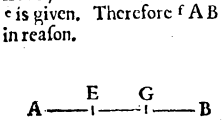
PROP. 21.

If there be two magnitudes given A B *and* C D, *and to them be added other magnitudes* B E *and* D F, *having to one another a given reason: Either the whole* A E *and* C F *shall have to one another a given reason, or else the one shall be greater then the other by a given magnitude, and in reason.*

Demonstration FOr seeing that both the magnitudes A B and C D are given their reason [a] is also given; and if it be the same reason as of B E to D F, the reason of the whole A E to the whole C F shall be also given; for it shall be [b] the same. But if it be not the same, Let it be as B E is to D F, so B G to C D: Therefore the reason of the said B G to C D is given. But C D is given, Therefore [c] also B G shall be given. But the whole A B is given, Therefore also the [d] remainder A G shall be given. And seeing that as B E is to D F, so is B G to C D, and also [e] the whole G E to the whole C F, the reason of the said G E to C F shall be likewise given. But A G is given, Therefore the magnitude A E is greater then the magnitude C F by a given magnitude, and in reason.

a) 1. p.
b) 12, 5.
c) 2. p.
d) 4. p.
e) 12, 5.

PROP. 22.

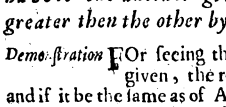


If two magnitudes A B *and* B C, *have to some other magnitude* D, *a given reason, also their compound magnitude* A C, *shall have to the same magnitude* D, *a given reason.*

Demonstration FOr seeing that each magnitude A B and B C, hath a given reason to D, the reason [a] of A B to B C is given; and by compounding, [b] the reason of A C to B C is given. But that of B C to D is also given, Therefore [c] the reason of the said A C to D shall be likewise given.

a) 8. p.
b) 6. p.
c) 8. p.

PROP. 23.

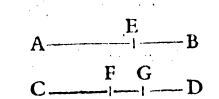


If the whole A B *be to the whole* C D *in a given reason, and that the parts* A E *and* E B, *be to the parts* C F *and* F D *in given reasons, although they be not the same, the whole (to wit,* A B, A E, *and* B E,) *shall be to the whole (to wit,* C D, C F, *and* F D,) *in given reasons.*

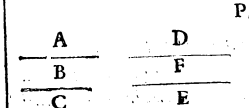
Demon-

Demonſtration FOr ſeeing that A E is to C F in a given reaſon, let the ſame be made of A B to C G; therefore the reaſon of the ſaid A B to C G is given; and conſequently, alſo that [a] of the reſt E B to the reſt F G. But the reaſon of F D to the ſame E B is alſo given: Therefore the reaſon of F D to F G [b] is likewiſe given; and therefore [c] that of F D to the remainder G D is alſo given. But the reaſon of A B to each of the magnitudes C D and C G is given: Therefore [d] alſo the reaſon of C D to C G is given, and again [e] that of C D to the remainder G D. But the reaſon of F D to D G is given, Therefore alſo [f] that of the ſame C D to F D, and conſequently that of [g] C D to the remainder F C; and therefore alſo the reaſon of C F to F D ſhall be given. But the reaſon of E B to F D is propoſed to be given; therefore the reaſon of C F to E B ſhall be given. Again, for that the reaſon of A B to C D is given; and alſo that of C D to each of thoſe F C and F D, the reaſon of the ſame A B to each of the ſaid F C and F D [h], ſhall be likewiſe given. But the reaſon of the ſaid F D to E B is given: Therefore the reaſon of A B to B E ſhall be alſo given, and conſequently A B to the remainder [i] A E. Wherefore by diviſion [k] the reaſon of A E to E B ſhall be likewiſe given. But the reaſon of E B to F D is given. Therefore alſo that of A E to F D. In like manner, ſeeing that the reaſon of C D to A B is given; and that of A B to each of his parts A E and E B; alſo the reaſon of the ſaid C D to each of the ſaid A E and E B, [l] ſhall be given: Wherefore each of the magnitudes A B, C D, A E, E B, C F, and F D, is to each of the others in a given reaſon.

a) 19. 5.
b) 8. p.
c) 5. p.
d) 8. p.
e) 5. p.
f) 8. p.
g) 5. p.
h) 8. p.
i) 5. p.
k) Sch. 5. p.
l) 8. p.



P. R O P. 24.



If of three right lines A, B, *and* C, *proportional*, A *to* B, *as* B *to* C, *the firſt* A *hath to the third* C *a given reaſon, it will alſo have to the ſecond* B *a given reaſon.*

Demonſtration FOr, Let there be expoſed another right line D, and ſeeing that the reaſon of A to C is given: Let the ſame be made of D to F; therefore the reaſon of D to F is given. But D is given, therefore F is alſo given; betwixt the two right lines D and F, let there be taken [a] a mean proportional E. Therefore the rectangle made under D and F is equal [b] to the ſquare of E. But the ſame rectangle of D and F is [c] given: (for all the angles of that rectangle are given, being right angles, and the reaſons that the ſides have to one another are alſo given;) therefore the ſquare of E is given, and conſequently the ſame right line E is alſo given (for one equal thereto may be found [d] ſeeing that the rectangle of D and F is given.) But D is given, therefore [e] the reaſon of D to E is given, and as A is to C, ſo D is to F. But as A is to C [f] ſo the ſquare of A is to the rectangle of A and C, and alſo as D is to F, ſo the ſquare of D is to the rectangle of D and F. Therefore as the ſquare of A is to the rectangle of A and C, ſo the ſquare of D is to the rectangle of D and F. But the rectangle of A and C is equal to the ſquare of B, (ſeeing that

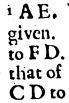
a) 13. 6.
b) 17. 6.
c) 3 def.
d) 14. 2.
e) 1. p.
f) 1. 6.

that A, B, and C, are proportional) and that of D and F to the ſquare of E, Therefore as the ſquare of A is to the ſquare of B, ſo the ſquare of D is to the ſquare of E: Wherefore [g] as A is to B, ſo D is to E. But the reaſon of D to E is given, Therefore [h] alſo the reaſon of A to B is given.

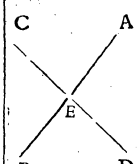
g) 12. 6.
h) 2. def.

O T H E R W I S E.

Demonſtration FOraſmuch as the reaſon of A to C is given, and that as A is to C, ſo the ſquare of A and C, the reaſon of the ſaid ſquare of A to the rectangle made of A and C, is alſo given. But to that rectangle made of A and C the ſquare of B is equal (ſeeing that A, B, and C, are proportional;) therefore the reaſon of the ſquare of A to the ſquare of B is given; and by conſequence, the reaſon of the line A to the line B is given; for to each of them A and B, we have exhibited an equal to the proper ſquare of each one.



P R O P. 25.



If two lines AB *and* CD, *given by poſition do interſect, the point* E *in which they interſect one another, is given by poſition.*

Demonſtration FOr if it change its place, the one or the other of the lines A B and C D, would change its poſition: But ſo it is that by Suppoſition it changeth not: Therefore [a] the point E is given by poſition.

a) 4. def.

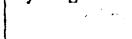
P R O P. 26.



If the extremities A *and* B, *of a right line* A B, *be given by poſition, that ſame right line* A B *is given by poſition and by magnitude.*

Demonſtration FOr if the point A remaining in its place, the poſition, or the magnitude of the right line A B ſhall change, the point B will fall elſwhere. But ſo it is, that by Suppoſition it doth not fall elſwhere. Therefore the right line A B is given by poſition, and by magnitude.

P R O P. 27.

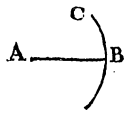


If one of the extremities A *of a right line* A B, *given by poſition and magnitude be given, the other extremity* B *ſhall be alſo given.*

Demonſtration FOr if the point A remaining in its place, the point B ſhall change and fall in ſome other place, either the poſition of the right line A B, or its magnitude would change: But ſo it is that according to the Suppoſition, neither the one nor the other doth change. Therefore the point B is given.

O T H E R-

OTHERWISE.

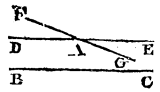


Construction ON the center A, with the distance A B, describe the circumference B C.

a) 6 def. b) 25. p.

Demonstration THerefore [a] that circumference B C is given by position. But the right line A B is also given by position; therefore the point [b] B is given.

PROP. 28.

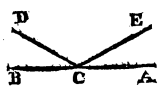


If by a given point A, *there be drawn a right line* D A E, *against another right line* B C, *given by position, the right line* D A E *so drawn, is given by position.*

Demonstration FOr if it be not given, the point A remaining in its place, the position of the right line D A E may change: Let it then change if it be possible, and fall elsewhere, remaining parallel to B C, and let it be the line F A G: Therefore B C is parallel to the said line F A G. But [a] the same B C is also parallel to D A E: Therefore [b] D A E is parallel to the said line F A G, which is absurd; seeing they joyn together and meet in A: Therefore the position of the right line D A E falls not elsewhere. Wherefore the said line D A E is given by position.

a) 13 def. b) 30. 1.

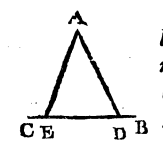
PROP. 29.



If to a right line A B, *given by position, and to a point* C *given therein, there be drawn a right line* C D, *which shall make a given angle* A C D, *the line drawn* C D, *is given by position.*

Demonstration FOr if it be not given by position, the point C remaining in its place, the position of the line C D observing the magnitude of the angle A C D, will fall elsewhere. Let it fall elsewhere then if it be possible, and let it be C E. Therefore the angle A C D is equal to the angle A C E, the greater to the lesser, which is absurd. Therefore the position of the right line C D, shall not fall elsewhere; and therefore the said line C D is given by position.

PROP. 30.



If from a given point A, *be drawn to a right line* B C, *given by position, a right line* A D, *making a given angle* A D B, *that line drawn* A D *is given by position.*

Demonstration FOr if it be not given, the point A remaining in its place, the position of the right line A D keeping the magnitude of the angle A D B will change.

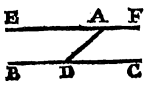
change. Let it change then, and let it be the right line A E: Therefore the angle A D B is equal to the angle A E B, the greater [a] to the lesser, which is absurd. Therefore the position of the right line A D doth not change; and therefore the said line A D is given by position.

a) 16. 1.

OTHERWISE.

Construction BY the point A let there be drawn the line E A F, parallel to the right line B C.

Demonstration THen seeing that by the given point A, and against the right line B C, given by position, there is drawn the right line E F, those lines E F and B C are parallels. But on the same lines doth also fall the right line A D.



Therefore [b] the angle F A D is equal to the given angle A D B; and therefore it is also given. Wherefore to the right line E F given by position, and to the given point A therein, there is drawn the right line A D, making the given angle F A D. Therefore [c] the said line A D is given by position.

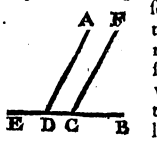
b) 29. 1.

c) 29. p.

OTHERWISE.

Construction IN the line B C E, let there be taken the given point C, and by the same let there be drawn the line C F, parallel to the said D A.

Demonstration FOrasmuch as A D and F C are parallels, and that on them there doth fall the right line B C E, the angle F C B is equal [d] to the given angle A D B; and therefore it is also given. And seeing that the right line B C is given by position, and that to a given point C therein, there is drawn the right line F C, making the given angle F C B, that same line F C [e] is given by position. But by the given point A, opposite to the line F C given by position, there is drawn the line A D. Therefore the said line [f] A D is given by position.



d) 29. 1.

e) 29. p.

f) 28. p.

OTHERWISE.



Construction IN the right line B C assume some point at F, and draw A F.

Demonstration FOrasmuch as each point A and F is given, the right line A F is given [g] by position. But the line B C is also given by position. Therefore * the angle A F D is given. But by Supposition, the angle A D F is given: Therefore D A F (which is the residue [h] of two right angles) is given; and seeing that to the right line A F given by position, and to the given point therein A there is drawn the right line D A, making the given angle D A F, [i] that same line D A is given by position.

g) 26. p.

h) 32. 1.

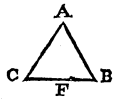
i) 29. p.

SCHOLIUM.

* *EUCLIDE supposeth here that two right lines being given by position, and inclining to one another do make a given angle, which some do demonstrate after this manner.*

Demonſtration FOraſmuch as the two right lines given by poſition, do incline to one another; the inclination of thoſe lines is given. But the angle is the inclination of the lines: Therefore the angle which makes the right lines given by poſition, and inclining to one another, is given.

Another thus demonſtrateth it.



Conſtruction LEt there be two right lines inclining to one another, as A B and C B, given by poſition, and in the line A B let there be taken a given point A, and in B C alſo ſome point, as C; and let the right line A C be drawn.



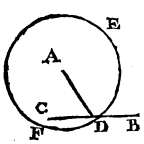
Demonſtration SEeing that as well the point B, as each of the points A and C, is given, [k] the three right lines A B, B C, and A C, are given by magnitude. Wherefore of three direct lines equal unto them, a triangle may be conſtituted: Let there then be made the triangle F D E, having the ſide F D equal to the ſide A B, the ſide F E equal to the ſide A C, and the baſe D E equal to the baſe B C.

Seeing then the angles comprized of equal right lines are equal, we have found the angle F D E equal to the angle A B C; and therefore the ſame [l] angle A B C is given.

k) 26. p.

l) 1 def.

PROP. 31.



If from a given point A *there be drawn to a right line given by poſition* B C, *a right line* A D, *given by magnitude, that line* A D *ſhall be alſo given by poſition.*

Conſtruction FRom the center A, with the diſtance A D, let the circle D E F be deſcribed.

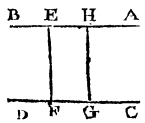
Demonſtration FOraſmuch as the center A is given by poſition, and the ſemidiameter A D by magnitude, the circle D E F [a] is given by poſition. But the right line B C is alſo given by poſition: Therefore the point of interſection D [b] is given, and ſeeing that the point A is alſo given: [c] the right line A D is given by poſition.

a) 6 def.

b) 25. p.

c) 26. p.

PROP. 32.



If unto parallel right lines A B *and* C D, *given by poſition, there be drawn a right line* E F, *making the given angles* B E F *and* E F D, *the line drawn* E F *ſhall be given by magnitude.*

Conſtruction FOr let there be taken in the line C D a given point G, and from that point let be drawn G H parallel to F E.

Demonſtration FOraſmuch as the lines E F and H G are parallels, and that on them doth fall the line C D; [a] the angle E F D is equal

a) 29. 1.

equal to the angle F G H. But the angle E F D is given, therefore the angle F G H is alſo given. And foraſmuch as to the right line C D given by poſition, and to the point G given in the ſame, there is drawn the right line G H, making the given angle F G H, [b] the ſaid line G H is given by poſition. But A B is alſo given by poſition, Therefore [c] the point H is given. But the point G is alſo given: Therefore [d] the line G H is given by magnitude, and is [e] equal to E F. Wherefore [f] the ſaid line E F is given by magnitude.

b) 29. p.

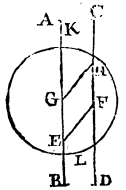
c) 25. p.

d) 26. p.

e) 34. 1.

f) 1 def.

PROP. 33.



If unto parallel right lines A B *and* C D, *given by poſition, there be drawn a right line* E F, *given by magnitude, that line* E F *ſhall make the given angles* B E F *and* D F E.

Conſtruction FOr let there be taken in the right line A B the point G, and by that point let there be drawn the line G H, parallel to E F.

Demonſtration THerefore E F is equal to the ſaid [a] G H. But E F is given by magnitude, Therefore G H is alſo given by magnitude. But the point G is given, and therefore if on that point, with the diſtance G H, there be deſcribed a circle, [b] that circle ſhall be given by poſition: Let it be then deſcribed, and let it be H K L, the ſaid circle H K L is therefore given by poſition. But the line C D which doth cut the circumference K H L in H, is alſo given by poſition. Therefore the ſaid point of interſection H [c] is given. But the point G is given: Therefore [d] the right line G H is given by poſition. But the right line C D is alſo given by poſition: Therefore [e] the angle G H F is given. But to that angle [f] the angle F E D is equal: Therefore the angle E F D is given; and therefore alſo the angle B E F; for that it is the reſidue of the ſumme of two [g] right angles.

a) 34. 1.

b) 6 def.

c) 25. p.

d) 26. p.

e) Sch. 30. p.

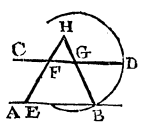
f) 29. 1.

g) 29. 1.

OTHERWISE.

Conſtruction LEt there be taken in the right line C D the point G, and let G D be put equal to E F, then from the center G, with the diſtance G D, let there be deſcribed the circle D B H, and draw G B.

Demonſtration FOraſmuch as the center D is given by poſition, and the ſemidiameter G D by magnitude, the circle B D H [h] is given by poſition. But the line A B is alſo given by poſition: Therefore [i] the point B is given. But the point G is alſo given, Therefore [k] the right line G B is given by poſition. But the right line C D is alſo given by poſition: Therefore [l] the angle B G D is given. Wherefore if E F be parallel to B G, the angle E F D [m] ſhall be given, and conſequently alſo the other angle B E F. But the right lines B G and E F being not parallels, let them meet in the point H. Foraſmuch as E B is parallel to F G, and E F is equal to G D, that is to ſay to B G; alſo F H [n] ſhall be equal to G H (for E H and B H being cut proportionally [o] by the parallel F G, as E F is to F H, ſo is B G to G H; and by permutation, as E F is to B G, ſo



h) 6 def.

i) 25. p.

k) 26. p.

l) Sch. 30. p.

m) 29. 1.

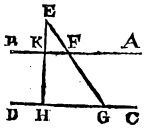
n) 14. 5.

o) 2. 6.

is F H to G H:) Therefore p the angle H F G is equal to the angle H G F, but the said angle H G F is given (for that it is equal q to the given angle B G D:) Therefore the angle H F G is also given. But to that angle the angle B E F is equal; and therefore is given, as also the remaining angle E F G.

p) 5. 1.
q) 15. 1.

PROP. 34.



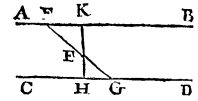
If from a given point E, *there be drawn unto parallel right lines* A B *and* C D, *given by position, a right line* E F G, *that right line* E F G *shall be divided in a given reason (to wit, as* E F *to* F G.

Construction FOr from the point E let there be drawn the line E H, perpendicular to the line C D.

Demonstration FOr asmuch as from the given point E there is drawn to the line C D the right line E H, making the given angle E H G a the said line E H is given by position, but both the one and the other lines A B and C D is also given by position. Therefore b the points of intersection K and H, are given. But the point E is also given: Therefore c each line E K and K H is given. Wherefore d the reason of the said E K to K H is given. But as E K is to K H, so is E F to F G; (for in the triangle G E H the line K F being parallel to H G, the sides F H and E G are cut proportionally:) Therefore the reason of the said E F to F G is given.

a) 30. p.
b) 25. p.
c) 26. p.
d) 1. p.

OTHERWISE.



Construction TO the parallel right lines given by position, A B and C D, let there be drawn from the point E the right line F E G: I say that the reason of G E to E F is given.

Demonstration FOr from the point E let there be drawn to C D the perpendicular E H, and produced to the point K; seeing therefore that from the point E to the right line C D, given by position, there is drawn the line E H, making the given angle E H G, a the said line E H is given by position. But each line A B and C D is also given by position: Therefore b each point of intersection H and K is given. But the point E is also given, Therefore c each of the lines E H and E K is given by magnitude; and therefore d the reason of the said E H to E K is given. But e as E H is to E K, so is E G to E F (for the opposite angles at the point E being equal, and the lines A B and C D parallels, the triangles E H G and E K F are equiangled; and therefore as E H is to E G, so is E K to E F; and by permutation as E H to E K, so is E G to E F.) Therefore the reason of the said lines E G to E F is given.

a) 30. p.
b) 25. p.
c) 26. p.
d) 1. p.
e) 4. 6.

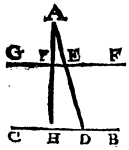
PROP. 35.

If from a given point A, *to a right line* B C, *given by position, there be drawn a right line* A D, *which let be divided in* E, *in a given reason (to wit, as* A E *to* E D, *and that*

that by the point of section E, *there be drawn a right line* F E G, *opposite to the right* B C, *given by position, the line* F G *drawn shall be given by position.*

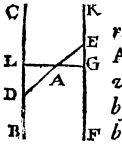
Construction FOr from the point A, let there be drawn the line A H, perpendicular to the line B C.

Demonstration FOr seeing that from the given point A there is drawn to B C given by position, the right line A H, making the given angle A H D, a the said line A H is given by position. But B C is also given by position: Therefore b the point H is given. But the point A is also given: Therefore c the line A H is given by magnitude and by position. And seeing that d as A E is to E D, so is A K to K H, and that the reason of A E to E D is given, also the reason of A K to K H is given; and by compounding, e the reason of A H to A K is given But A H is given by magnitude: Therefore f also A K is given by magnitude. But A K is also given by position, and the point A is given: Therefore g the point K is also given, and seeing that by the said given point K there is drawn the line F G, opposite to the right line B C given by position; the said line F G h is given by position.



a) 30. p.
b) 25. p.
c) 26. p.
d) 2. 6.
e) 6. p.
f) 2. p.
g) 27. p.
h) 28. p.

PROP. 36.



If from a given point A, *there be drawn to a right line* B C *given by position, a right line* A D, *and to it be added a right line* A E, *having to the same* A D *a given reason, and that by the extremity* E *of the added line* A E, *there be drawn a right line* F E K, *opposite to the line* B C, *given by position, that same line* F E K *shall be given by position.*

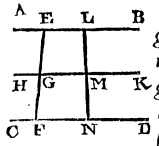
Construction FOr from the point E let there be drawn to the line B C, the perpendicular A L, and let it be prolonged to the point G.

Demonstration FOr asmuch as from the given point A, there is drawn to the right line B C, given by position, the right G L, which makes the given angled G L D, a that line G L is given by position. But B C is also given by position, Therefore b the point L is given; and seeing that the point A is also given the line c A L is given. But forasmuch as the reason of A E to A D is given, and that d as the said A E is to A D, so is A G to A L; (because the triangles A L D and A G E are equiangled) the reason of A G to A L is also given. But A L is is given by magnitude: Therefore c A G is given by magnitude. But it is also given by position, and the point A is given: Therefore f the point G is also given. And seeing that by the same given point G, there is drawn the line F K, opposite to the right line B C, given by position, g the said line F K is given by position.

a) 30. p.
b) 26. p.
c) 26. p.
d) 4. 6.
e) 2. p.
f) 27. p.
g) 28. p.

PROP.

PROP. 37.



If unto parallel right lines A B *and* C D, *given by position, there be drawn a right right line* E F, *divided in the point* E, *in a given reason (to wit, of* E G *to* G F;) *but if by the point of section* G, *there be drawn opposite to the right lines* A B *or* C D, *given by position, a right line* H G K, *that line drawn shall be given by position.*

Construction FOr let there be taken in the line A B the given point L, and from that point let there be drawn the line L N, perpendicular to C D.

Demonstration SEeing that from the given point L, there is drawn to the right line C D, the line L N, making the given angle L N D, the said L N [a] is given by position. But C D is also given by position: Therefore the point N [b] is given. But the point L is also given: Therefore [c] the line L N is given; and seeing that the reason of F G to G E is given, and that * as F G is to G E, so is N M to M L, the reason of the said M N to M L is given; and in compounding, [d] the reason of L N to L M is also given. But L N is given by magnitude, therefore M L is [e] given by magnitude. But it is also given by position, and the point L is given: Therefore the point M [f] is also given. And considering that by the said point M there is drawn the right line K H, opposite to the right line C D, given by position, the said line K H is also given by position.

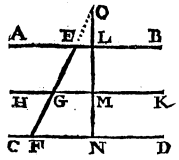
a) 30. p.
b) 25. p.
c) 26. p.
d) 6. p.
e) 2. p.
f) 27. p.

SCHOLIUM.

* *EUCLIDE supposeth here, that as* F G *is to* G E, *so* N M *is to* M L; *but by another it is thus demonstrated.*

The lines E F *and* L N *are parallels or not parallels: Let them in the first place be parallels, and forasmuch as by Construction the lines* E L, F N, E F, *and* L N, *are parallels,* E N *shall be a parallelogram; and therefore the side* E F *is equal to the side* L N. *Again, seeing that* M G *is parallel to* F N, *and* G F *to* M N, G N *shall be also a parallelogram; and therefore the side* G F *is equal to the side* M N. *Wherefore the equal sides* E F *and* L N, *shall have to the equal sides* F G *and* M N, [g] *one and the same reason. Therefore as* E F *is to* F G, *so is* L N *to* M N; *and in dividing,* [h] *as* G E *to* G F, *so is* L M *to* M N.

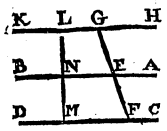
g) 7. 5.
h) 17. 5.



Now suppose that the lines E F *and* L N *be not parallels but that they meet in the point* O. *Forasmuch as in the triangle* O F N *there is drawn* H K, *parallel to* F N *one of the sides;* [i] *the sides* O F *and* O N *are divided reciprocally; and therefore as* F G *is to* G O, *so is* N M *to* M O. *Again, seeing that in the triangle* O G M *there is drawn* E L, *parallel to the side* G M, *the sides* O G *and* O M *are divided proportionally: Wherefore* [k] *as* O E *is to* E G, *so is* O L *to* L M; *and by compounding,* [l] *as* O G *is to* E G, *so is* O M *to* L M; *but it hath been demonstrated that as* F G *is to* G O, *so is* N M *to* M O; *therefore in reason of equality,* [m] *as* F G *is to* G E, *so is* N M *to* M L. PROP.

i) 2. 6.
k) 2. 6.
l) 18. 5.
m) 22. 5.

PROP. 38.



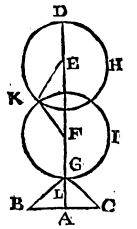
If unto parallel right lines A B *and* C D, *there be drawn a right line* E F, *and that to it there be added some other right line* E G, *which hath a given reason to the same* E F, *but if by the extremity* G, *of the added line* E G, *there be drawn a right line* H K, *against the parallels given by position* A B *and* C D, *the line drawn* H K *shall be also given by position.*

Construction FOr, Let there be taken in the line A B, the given point N, and from thence let there be drawn to C D the perpendicular N M, and let it be prolonged to the point L.

Demonstration FOrasmuch as from the given point N there is drawn to the right line C D, given by position, the right line N M making a given angle N M F, the said angle N M F [a] is given by position. But the line C D is also given by position: Therefore [b] the point M is given. But the point N is also given: Therefore [c] the line N M is given, and for that the reason of E G to E F is given, and that [d] as E G is to E F, so is L N to N M, the reason of L N to N M is also given: But N M is given, therefore L N is [e] also given. But the point N is given: Therefore [f] the point L is also given. Seeing then that by the given point L there is drawn the right line H K, opposite to the line A B given by position, [g] the said line A K is also given by position.

a) 30. p.
b) 25. p.
c) 26. p.
d) Sch. 37. p.
e) 2. p.
f) 27. p.
g) 28. p.

PROP. 39.



If all the sides of a triangle A B C *are given by magnitude, the triangle is given by Kind.*

Construction FOr, Let there be exposed the right line D G given by position, ending in the point D; but being infinite towards the other part G, and therein let there be taken D E, equal to A B.

Demonstration NOw seeing the said A B is given by magnitude, D E is so also; but the same D E is also given by position, and the point D is given: Therefore [a] the point E is given.

Again, Let E F be put equal to B C; and seeing that B C is given by magnitude, E F shall be so also. But the said E F is in like manner given by position, and the point E is given: Therefore [b] the point F is given.

Furthermore, Let F G be taken equal to A C. Now forasmuch as the said A C is given by magnitude, F G is so also. But F G is also given by position, and the point F is given: Therefore the point G is also given. Now from the center E, with the distance E D, let there be described the circle D H K, [c] and that circle shall be given by position. Again, on the center F, and distance F G, let there be described the circle G L K.

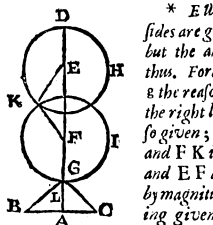
a) 27. p.
b) 27. p.
c) 6. def.

There-

d) 6 def. c) 25. p. f) 26. p.

Therefore [d] the said circle G L K is given by position; and therefore [c] the point of intersection K is given. But each of the points E and F is given: Therefore each line [f] E K, E F, and F K, is given by position and magnitude. Therefore the triangle E K F is given * by kind; but it is equal and alike to the triangle A B C; and therefore the triangle A B C is also given by kind.

SCHOLIUM.



g) 1. p.

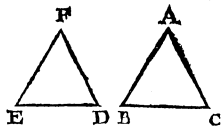
h) Sch. 30. p.

* *EUCLIDE supposeth here that a triangle whose sides are given by magnitude and position, is given by kind; but the antient Interpreters demonstrate it in a manner thus. Forasmuch as the right lines* K E *and* E F *are given,* [g] *the reason which they have to one another is given. Also the right lines* E F *and* F K *being given, their reason is also given; and in like manner, the reason of the said* E K *and* F K *is given. Again, seeing that the same lines* K E *and* E F *are given by position,* [h] *the angle* K E F *is given by magnitude: Moreover, the right lines* E F *and* F K *being given by position, the angle* E F K *is given by magnitude, as is also the residue* E K F, *and so in the triangle* E K F *are all the angles given, and also the reasons of the sides: Therefore* [i] *the said triangle* E K F *is given by kind.*

i) 5 def.

PROP. 40.

If the angles of a triangle A B C, *are given by magnitude, the triangle is given by kind.*



Construction LEt there be exposed the right line D E, given by position and by magnitude; and let there be constituted at the point D the angle E D F, equal to the angle C B A; but in the point E the angle D E F, equal to the angle B C A; therefore the third angle B A C is equal to the third angle D F E.

Demonstration FOr each of the angles constituted in the points A, B, and C, is given: Therefore each of those which are posited in the points D, F, and E, is also given; and seeing that to the right line D E given by position, and to the point D given therein, there is drawn the right line D F, which makes the given angle E D F, [a] the line D F is given by position; and by the same reason, the line E F is given by position: Therefore [b] the point F is given by position. But each of the point D and E is given: Therefore [c] each of the lines D F, D E, and E F, is given by magnitude. Wherefore the triangle D F E is given by kind; and is alike to the triangle A B C: Therefore the triangle A B C is given by kind.

a) 29. p. b) 25. p. c) 26. p.

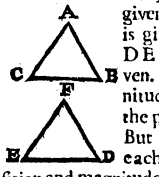
PROP. 41.

If a triangle A B C, *hath one angle* B A C *given, and that the two sides* B A *and* A C, *which do constitute it, have to*

to one another a given reason, the triangle is given by kind.

Construction FOr, Let there be exposed the right line D F given by magnitude and position. But thereon and at the given point F, let there be constituted the angle D F E equal to the angle B A C.

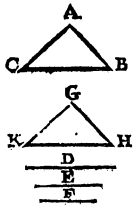
Demonstration NOw the angle B A C is given: Therefore also the angle D F E is given, And seeing that to the right line D F given by position, and from the give point F therein, is drawn a right line F E, making the given angle D F E, [a] the said line F E is given by position. But seeing that the reason of A B to A C is given, let the same be made of D F to F E, then let D E be drawn. Therefore the reason of D F to F E is given. But D F is given: Therefore [b] F E is given by magnitude. But the same F E is also given by position, and the point F is given. Therefore [c] the point E is also given. But each of the points D and F is given: Therefore [d] each of the right lines D F, F E, and D E, is given by position and magnitude. Wherefore [e] the triangle D E F is given by kind. And seeing that the two triangles A B C and D E F have an angle equal to an angle, that is to say, the angle B A C to the angle D F E, and the sides which constitute those equal angles, proportional; [f] the triangle A B C is alike to the triangle D E F. But the triangle D E F is given by kind: Therefore the triangle A B C is given by kind.



a) 29. p. b) 2. p. c) 27. p. d) 26. p. e) 39. p. f) 6. 6.

PROP. 42.

If the sides of a triangle A B C, *be to one another in given reasons, the triangle* A B C *is given by kind.*



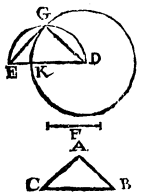
Construction FOr, Let there be exposed the right line D, given by magnitude, and seeing that the reason of B C to A C is given, let the same be made of D to E.

Demonstration NOw D is given, therefore [a] E is also given. Again, seeing that the reason of A C to A B is given, let the same be made of E to F. Now E is given, therefore [b] F is also given. Now of three right lines, equal to the three given right lines D, E, and F, (and of which three lines, two of them, in what manner soever they be taken, are greater then the other:) Let there be constituted the triangle G H K, in such sort as D may be equal to H K, but E is equal to K G, and G H equal to F; therefore each of the said lines H K, K G, and G H, is given by magnitude: Wherefore [c] the triangle H G K is given by kind. And seeing that as B C is to C A, so is D to E, and that D is equal to H K, and E to K G, as B C is to C A, so H K is to K G. Again, seeing that as C A is to A B, so is E to F, and that E is equal to K G, and F to G H; as C A is to A B, so is K G to G H. But it hath been demonstrated that as B C is to C A, so is H K to K G: Therefore by reason of equality, as B C is to A B, so is H K to G H. Therefore [d] the triangle A B C is also given by kind.

a) 2. p. b) 2. p. c) 39. p. d) 5. 6.

PROP. 43.

If the sides B C *and* D A, *about one of the acute angles of a rectangled triangle* A B C, *have to one another a given reason, that triangle is given by kind.*



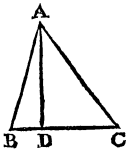
Construction LEt there be exposed the right line DE given by magnitude and position, and on it let there be described the semicircle D G E: Therefore [a] the semicircle D G E is given by position.

Demonstration FOr the line D E being given, and divided in two equal parts, the center of the said circle is given by position, and the semidiameter by magnitude. And forasmuch as the reason of B C to B A is given, let the same be made of D E to F: Therefore the reason of D E to F is given. But D E is given, therefore F [b] is also given. Now B C is greater then [c] A B: Therefore E D is [d] also greater then F. Let D G be fitted equal to F, and let E G be drawn; then on the center D, with the distance D G, let the circle G K be described. Now that circle [e] is given by position, seeing that the center D is given, and the semidiameter D G also given by magnitude. But the semicircle D G E is also given by position: Therefore [f] the point of intersection G is given. But the points D and E are also given, therefore [g] each of the right lines D E, D G, and E G, is given by position and magnitude. Wherefore [h] the triangle D G E is given by kind. And seeing that the triangles A B C and D G E have an angle equal to an angle, to wit, the right angle B A C to the right angle [i] D G E, and the sides about the angles C B A and E D G proportional. But each of the others A C B and D E G lesse then a right angle: Those triangles A B C and D E G [k] are alike. But the triangle D G E is given by kind: Therefore the triangle A B C is also given by kind.

a) 6 def.
b) 2. p.
c) 19. 1.
d) 14. 5.
e) 6 def.
f) 25. p.
g) 26. p.
h) 39. p.
i) 31. 3.
k) 7. 6.

PROP. 44.

If a triangle A B C, *hath one angle* B *given, and that the sides* B A *and* A C, *about another angle* B A C, *have to one another a given reason, the triangle* A B C *is given by kind.*



Construction NOw the given angle B is either acute or obtuse, (for it was a right angle in the fore-going prop.) Let it be in the first place acute, and from the point A let A D be drawn perpendicular to B C.

Demonstration THerefore the angle A D B is given: But the angle B is also given; and therefore the third angle B A D is given: Wherefore [a] the triangle A B D is given by kind; and therefore [b] the reason of B A to A D is given. But the reason of the same B A to A C is also given: Therefore [c] the reason of A D to A C is given, and the angle A D C is a right angle: Wherefore the triangle [d] A C D is given by kind: Therefore the

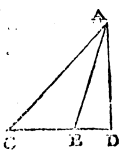
a) 40. p.
b) 3 def.
c) 8. p.
d) 43. p.

[e] the angle C is given. But the angle B is also given; and therefore the other angle B A C is given: Therefore [f] the triangle A B C is given by kind.

e) 3. def.
f) 40. p.

Construction NOw let the angle A B C be obtuse, and on the side C B prolonged, let there be drawn the perpendicular A D.

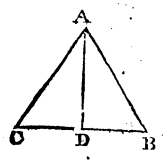
Demonstration FOrasmuch as the angle A B C is given, the angle A B D which follows it, shall be given. But the angle A B D is also given: Therefore the third angle D A B is given. Wherefore [g] the angle A B D is given by kind; and therefore [h] the reason of D A to A B is given. But the reason of A B to A C is also given: Therefore [i] the reason of D A to A C is given, and the angle D is a right angle: Therefore the triangle D A C is given by kind, and therefore the angle A C B is given. But the angle A B C is also given: Therefore the third angle B A C is given. Wherefore the triangle A B C is given by kind.



g) 40. p.
h) 3 def.
i) 8. p.

PROP. 45.

If a triangle A B C, *hath one angle* B A C *given, and that the line compounded of the two sides* A B *and* A C, *about the said given angle* B A C, *hath to the other side* B C *a given reason, the triangle* A B C *is given by kind.*



Construction FOr, Let the angle B A C be divided into two equal parts by the line A D, therefore [a] the angle C A D is given.

a) 7. p.

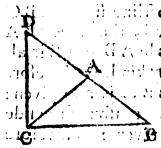
Demonstration SEeing that as A B is to A C, so [b] is B D to C D; by compounding, [c] as the line compounded of C A B is to C A, so is B C to C D, and by permutation, as the line compounded of C A B is to C B, so is C A to C D. But the reason of the line compounded of C A B to B C is given; therefore the reason of C A to C D is also given, and the angle C A D is given. Therefore [d] the triangle A C D is given by kind, and therefore the angle C is given. But the angle B A C is also given: Therefore the third angle B is given: Wherefore [e] the triangle A B C is given by kind.

b) 3. 6.
c) 18. 5.
d) 44. p.
e) 40. p.

OTHERWISE.

Construction LEt B A be prolonged directly unto the point D, in such sort as that A D may be equal to A C, and let C D be joyned.

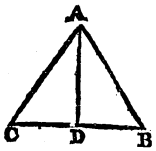
Demonstration FOrasmuch as the reason of the line compounded of C A B to C B is given, and that A D is equal to A C, the reason of the whole line B D to B C is given. But the angle A D C is also given, for it is the half of the the given angle B A C (for that the said angle B A C [f] is equal to the two internal angles A C D and A D C, which are [g] equal to one another, being the sides A C and A D are equal:) Wherefore the triangle B D C [h] is given by kind; and therefore the angle B is given. But



f) 32. 1.
g) 5. 1.
h) 44. p.

the angle B A C is alſo given: Therefore the remaining angle A C B is
i) 40. p. given: Wherefore i the triangle A B C is given by kind.

PROP. 46.



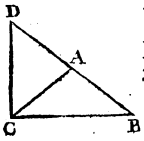
If a triangle A B C *hath one angle* B *given, and that the line* C A B *compounded of the two ſides* A C *and* A B, *about another angle* B A C, *hath to the other ſide* B C *a given reaſon, the triangle* A B C *is given by kind.*

Conſtruction FOr let the angle B A C be divided into two equal parts, by the line A D.

Demonſtration THerefore (as hath been ſhewn in the foregoing Prop.) the compound line C A B is to C B, as A B is to B D. But the reaſon of the ſaid compound line C A B to C B is given: Therefore alſo the reaſon of A B to B D is given. But the angle B is alſo given: There-
a) 41. p. fore the triangle A B D a is given by kind; and therefore b the angle
b) 7. def. B A D is given. But the angle B A C is double to that of B A D; and therefore it is alſo given. Therefore the third angle C is given. Wherefore the triangle A B C is given by kind.

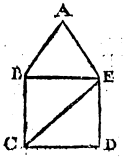
OTHERWISE.

Conſtruction LEt B A be prolonged directly, and let A D be put equal to A C, and let C D be joyned.



Demonſtration FOraſmuch as the reaſon of the line compounded of C A B to C B is given, and that A D is equal to A C, the reaſon of B D to B C is given; and the angle B is alſo given: Therefore the triangle C D B c is given by kind; and
c) 41. p. therefore d the angle D is given: Therefore the angle
d) 3 def. B A C which is double to B D C, is alſo given: Wherefore the other angle A C B is given; and therefore the triangle A B C is given by kind.

PROP. 47.



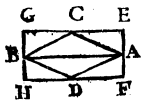
Rectiline figures as A B C D E, *given by kind, are divided into triangles given by kind.*

Conſtruction FOr let the right lines E B and E C be drawn.

Demonſtration FOraſmuch as the rectline figure A B C D E is given by kind, the angle a B A
a) 3. def. E is given, and the reaſon of the ſide A B to A E is alſo given: Therefore b the triangle B A E is given by kind. Wherefore
b) 41. p. the angle A B E is given. But the whole angle A B C is alſo given:
c) 4. p. Therefore c the remaining angle E B C is given. But the reaſon of the ſide A B to the ſide B E, and alſo that of A B to B C is given: Therefore the

d the reaſon of B C to B E is given, and the angle C B E is alſo given: d) 8. p.
Therefore e the triangle B C E is given by kind. By the ſame diſcourſe it e) 41. p.
may be demonſtrated that the triangle C D E is given by kind. Therefore rectiline figures given by kind divide themſelves into triangles given by kind.

PROP. 48.

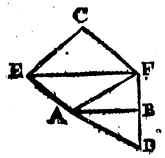


If on one and the ſame right line A B, *are deſcribed triangles as* A C B *and* A B D, *given by poſition, thoſe triangles ſhall have to one another a given reaſon, as* A C B *to* A B D.

Conſtruction FOr from the points A and B, let there be drawn at right angles on the line A B, the lines A E and B G, and prolonged unto the points F and H; but by the points C and D, let there be drawn the lines E C G and F D H, parallel to A B.

Demonſtration FOraſmuch then as the triangle A B C is given by kind,
a) 3. def. a the reaſon of C A to B A is given, and the angle C A B alſo given; but the angle B A E is given: Therefore the remaining angle C A E is alſo given; but the angle C A E is given; and therefore the other angle A C E is alſo given. Wherefore b the triangle A E C is given
b) 40. p. by kind. Now the reaſon of E A to A B c is given; (for d the reaſon of
c) 8. p. E A to A C, and that of A C to A B is given;) and in like manner, the
d) 3 def. reaſon of F A to A B is given. Therefore e the reaſon of E A to A F is
e) 8. p. given; but as A E is to A F, ſo f the parallelogram A H to the parallelo-
f) 1, 6. gram A G; but A C B is g the half of A H, and A D B the half of A G;
g) 41. 1. therefore the reaſon of the triangle A C B to the triangle A D B is given;
h) 15. 5. for it is the ſame reaſon with that of A H to A G h; that is to ſay, of E A to A F, which is given.

PROP. 49.



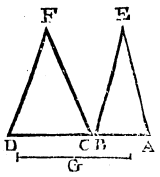
If on one and the ſame right line A B *there be deſcribed any two rectiline figures* A E C F B *and* A D B, *given by kind, they ſhall have to one another a given reaſon (to wit,* A E C F B *to* A D B.)

Conſtruction FOr let the lines F A and F E be drawn: Therefore each of the triangles a A B F
a) 47. p. A F E, and E C F is given by kind.

Demonſtration SEeing that on one and the ſame right line E F there are deſcribed the triangles E C F and E A F, given by kind, the reaſon of E C F to E A F b is given. Therefore, by compounding,
b) 48. p. c the reaſon of A E C F to E A F is given. But the reaſon of the ſaid E A F
c) 6. p. to F A B is given, d being they are triangles given by kind, deſcribed on
d) 48. p. one and the ſame right line A F: Therefore e the reaſon of A E C F to
e) 8. p. F A B is given. Wherefore by compounding, f the reaſon of A E C F B
f) 6. p. to F A B is given. But the reaſon of the ſame F A B to A B D g is given:
g) 48. p. Therefore h the reaſon of A E C F B to A B D is alſo given.
h) 8. p.

PROP.

PROP. 50.

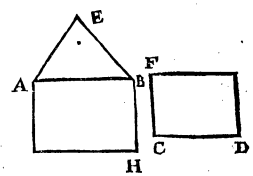


If two right lines A B *and* C D, *have to one another a given reaſon, and that on thoſe lines there be deſcribed rectiline figures* A E B *and* C F D, *alike, and alike poſited, they will have to one another a given reaſon.*

Demonſtration TO the two lines A B and C D, let there be taken a third proportional: Therefore as A B is to C D, ſo is C D to G. But the reaſon of A B to C D is given: Therefore the reaſon of C D to G is alſo given: Wherefore [a] the reaſon of A B to G is given. But [b] as A B is to G, ſo is A E B to C F D: Therefore the reaſon of the ſame A E B to C F D is given.

a) 8. p.
b) Cor. 19, 20. 6.

PROP. 51.



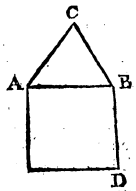
If two right lines A B *and* C D, *have to one another a given reaſon, and that upon them there be deſcribed any rectiline figures* A E B *and* C F D, *given by kind, they will have to one another a given reaſon,* (*to wit, that of* A E B *to* C F D.)

Conſtruction FOr on A B, let the rectangled figure A H be deſcribed alike and alike poſited to D F.

Demonſtration NOw D F is given by kind: Therefore alſo A H is given by kind. But A E B is alſo given by kind, and deſcribed on the ſame line A B: Therefore [a] the reaſon of A E B to A H is given: And ſeeing that the reaſon of A B to C D is given, and that on thoſe lines are deſcribed the rectiline figures A H and D F alike, and alike poſited, the reaſon [b] of the ſaid line A H to D F is given. But the reaſon of A E B to A H is alſo given: Therefore the reaſon [c] of A E B to D F is given.

a) 49. p.
b) 50. p.
c) 8. p.

PROP. 52.



If on a right line A B, *given by magnitude, there be deſcribed a figure* A C B, *given by kind, that figure* A C D *is given by magnitude.*

Conſtruction FOr on the ſame line A B, let the ſquare A D be deſcribed. Therefore A D is given by kind * and by magnitude.

Demonſtration SEeing that on the right line A B, are deſcribed the two rectiline figures A C B

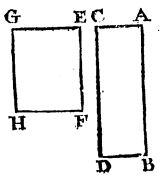
A C B and A D, given by kind, [a] the reaſon of A C B to A D is given: Therefore [b] A C B is given by magnitude.

a) 29. p.
b) 2. p.

SCHOLIUM.

* *The ancient Interpreter hath noted here that every ſquare is given by kind; for that all the angles thereof are given; being all equal and right angles: But alſo the reaſons of the ſides are given; for thoſe ſides being all equal, their reaſons are alſo equal. Moreover, whenſoever a ſquare is expoſed, a ſquare equal thereto may be exhibited; and therefore the ſquare is given by magnitude, as alſo each ſide thereof.*

PROP. 53.

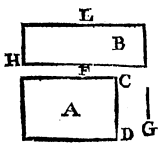


If there be two figures A D *and* E H, *given by kind, and that one ſide* B D, *of the one, hath to a ſide* F H *of the other, a given reaſon; the other ſides ſhall have alſo to the other ſides given reaſons.*

Demonſtration FOr ſeeing that the reaſon of B D to F H is given, and alſo that [a] of B D to B A, [b] the reaſon of the ſaid A B to F H is given. But the reaſon of the ſame F H to F E [c] is alſo given: Therefore [d] the reaſon of A B to E F is given. In like manner alſo the reaſons of the other ſides to the other ſides are given.

a) 3. def.
b) 8. p.
c) 3 def.
d) 8. p.

PROP. 54.



If two figures A *and* B, *given by kind, have to one another a given reaſon, alſo their ſides ſhall be to one another in a given reaſon.*

Conſtruction FOr either the figure A is alike and alike poſited to B, or is not: Let it in the firſt place be alike and alike poſited; and let there be taken the line G, a third proportional to the lines C D and E F.

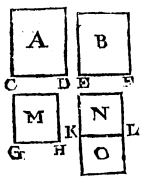
Demonſtration AS C D is to G, [a] ſo is A to B. But the reaſon of A to B is given; therefore alſo the reaſon of C D to G is given. And ſeeing that C D, E F, and G, are proportional, [b] alſo the reaſon of C D to E F is given. But A and B are given by kind: Therefore [c] the other ſides ſhall have given reaſons to the other ſides.

Now let the figure A be not alike to the figure B, and let there be deſcribed on E F the figure E H, alike and alike poſited to A: Therefore the figure E H is given by kind; but the figure B is alſo given by kind: Therefore [d] the reaſon of B to E H is given; and therefore the reaſon of A to the ſame E H [e] is alſo given: But A is alike to E H: Therefore (by what is aboveſaid) the reaſon of C D to E F is given; and in like manner the reaſon of the other ſides to the other ſides is given.

a) Cor. 19, 20. 6.
b) 24. p.
c) 53. p.
d) 49. p.
e) 8. p.

OTHER-

OTHERWISE.



Conſtruction LEt there be expoſed the given line G H: Now either the figure A is alike to the figure B, or not. Let it in the firſt place be alike, and let it be as C D is to E F, ſo is G H to L K; then on G H and L K let the figures M and N be deſcribed alike and alike poſited to the ſaid A and B, which figures M and N ſhall be conſequently given by kind.

Demonſtration THerefore ſeeing that as C D is to E F, ſo is G H to L K, and that on thoſe lines C D, E F, G H, and L K, are deſcribed the figures A, B, M, and N, alike and alike poſited; [f] as A is to B, ſo is M to N. But the reaſon of A to B is given: Therefore the reaſon of M to N is given. But [g] M is given, conſidering that it is a figure given by kind, deſcribed on a right line given by magnitude; therefore N is alſo given.

f) 22. 6.
g) 52. p.

Conſtruction 2. NOw, on L K let the ſquare O be deſcribed: Therefore [h] the figure O is given by kind.

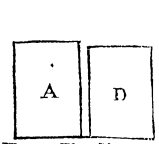
h) Sch. 52. p.

Demonſtration 2. WHerefore the reaſon of K to N is given. But N is given: Therefore K is given; and conſequently, [i] alſo K L. But G H is given: Therefore [k] the reaſon of G H to K L is given. But as G H is to L K, ſo is C D to E F. Therefore the reaſon of C D to E F is given; and therefore the figures A and B being given by kind, [l] the other ſides of the ſame figures ſhall alſo have to the other ſides given reaſons. But if the figures be not alike, the later part of the demonſtration here above muſt be obſerved.

i) Sch. 52. p.
k) 3. p.
l) 53. p.

PROP. 55.

If a Space A, *be given by kind, and by magnitude, the ſides thereof ſhall be given by magnitude.*



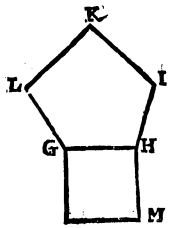
Conſtruction FOr, Let the right line B C given by poſition and by magnitude, be expoſed; and thereon let there be deſcribed the ſpace D, alike and alike poſited to A; therefore the ſaid ſpace D is given by kind.

Demonſtration FOr that it is deſcribed on the line B C, given by magnitude, it is alſo [a] given by magnitude. But the figure A is alſo given: Therefore [b] the reaſon of A to D is given. But thoſe figures A and D are given by kind: Therefore [c] the reaſon of the line E F to the line B C is given. But B C is given: Therefore [d] E F is alſo given. But the reaſon of the ſame E F to F G is given: Therefore [e] F G is given. And by the ſame reaſons it may be demonſtrated that each of the other ſides are given by magnitude.

a) 52. p.
b) 3. p.
c) 54. p.
d) 3. def.
e) 2. p.

OTHER-

OTHERWISE.



Conſtruction LEt the ſpace G H I K L be given by kind and by magnitude: I ſay that the ſides thereof are given by magnitude. For on the right line G H let there be deſcribed the ſquare G M; therefore [f] G M is given by kind.

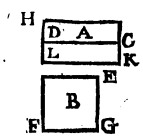
f) Sch. 52. p.

Demonſtration BUt the ſpace G H I K L is alſo given by kind: Therefore [g] the reaſon of the ſame ſpace G K to G M is given. But G K is given by magnitude: Therefore [h] G M is alſo given by magnitude; and ſeeing that G M is the ſquare of the line G H, [i] that line G H is given by magnitude. Wherefore in like manner, each of the other lines H I, I K, K L, and L G, is given.

g) 49. p.
h) 2. p.
i) Sch. 52. p.

PROP. 56.

If two equiangled Parallelograms A *and* B, *have to one another a given reaſon, as one ſide* C D *of the firſt* A, *is to one ſide* F G, *of the ſecond* B; *ſo the other ſide* G E, *of the ſecond* B, *is to that to which* D H *the other ſide of the firſt* A, *hath the given reaſon that the Parallelogram* A *hath to the Parallelogram* B.



Conſtruction FOr let H D be prolonged directly to L, ſo that as C D is to F G, ſo H D may be to D L; and finiſh the Parallelogram D K.

Demonſtration SEeing that as C D is to F G, ſo H D is to D L, and [a] that C D is equal to K L; as L K is to F G, ſo is G E to D L; and thus the ſides about the equal angles D L K and E G F are reciprocally proportional: Wherefore [b] D K is equal to B; and therefore ſeeing the reaſon of A to B is given, and that B is equal to D K, the reaſon of A to D K is given. But as [c] A is to D K (that is to B) ſo is H D to D L; therefore the reaſon of H D to D L is alſo given, and ſeeing that as C D is to F G, ſo G E is to D L, and that the right line H D hath to D L a given reaſon; to wit, that which the ſpace A hath to the ſpace B; as C D is to F G, ſo G E is to that to which H D hath the given reaſon that the ſpace A hath to the ſpace B, that is to ſay, the reaſon of H D to D L.

a) 34. 1.
b) 14. 6.
c) 1. 6.

PROP. 57.

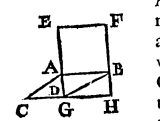
If a given ſpace A D *be applyed to a given right line* A B, *in a given angle* C A B, *the breadth* C A *of the application is given.*

Conſtruction FOr on A B, let there be deſcribed the ſquare A F; therefore [a] the ſame A F is given: Let the lines E A, F B, and C D, be prolonged to the points G and H.

a) Sch. 52. p

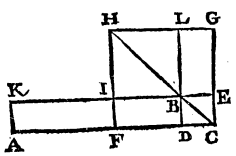
Demonſtration SEeing therefore that each ſpace A D and A F is given, their reaſon is alſo given. But [b] A D is equal to A H: Therefore the reaſon of A F to A H is given: Wherefore the reaſon of F A to A G is given. (for [c] it is the ſame with that of A F to A H.) But E A is equal to A B; therefore the reaſon of A B to A G is given. Now ſeeing that the angle C A B is given, and the angle G A B alſo given, the reſidue C A G is given. But the angle C G A is alſo given, being a right angle: Therefore the remaining angle A C G is given. Wherefore the triangle [d] C A G is given by kind. Therefore the reaſon of C A to A G is given. But the reaſon of A B to the ſame A G is alſo given: Therefore the reaſon of C A to A B is given; and the ſaid A B is given: Wherefore C A is alſo given.

b) 36. 1.
c) 1. 6.
d) 40. p.



PROP. 58.

If a given ſpace A B, *be applyed to a given right line* A C, *wanting by a figure* D E, *given by kind, the breadths of the defects are given.*



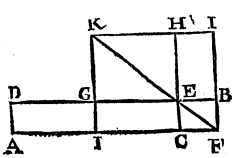
Conſtruction FOr let A C be divided in two equal parts in the point F: Therefore as well A F as F C is given. On the ſaid line F C let there be deſcribed the rectangle figure F G alike and alike poſited to D E. Therefore F G is given by kind.

Demonſtration SEeing the figure F G is deſcribed on the right line F C given by magnitude, the ſaid rectiline F G is [a] alſo given by magnitude. But F G is equal to A B and I L; (for [b] A I and F E being equal, and [c] F B and B G alſo equal, the Gnomon I C L is equal to A B; and therefore their added figure I L, common to both, F G ſhall be equal to A B anp I L:) Therefore the figures A B and I L together are given by magnitude. But A B is given by magnitude: Therefore [d] the remaining figure I L is alſo given by magnitude. But it is alſo given by kind, ſeeing it is [e] alike to D E: Therefore [f] the ſides of the ſame I L are given: Wherefore I B is given; and ſeeing that it is equal [g] to F D, the ſame F D is alſo given. But F C is given, therefore the remainder D C [h] is given; and [i] in a given reaſon to B D, and therefore [k] B D is given.

a) 52. p.
b) 36. 1.
c) 43. 1.
d) 4. p.
e) 24. 6.
f) 55. p.
g) 34. 1.
h) 4. p.
i) 3. def.
k) 2. p.

PROP. 59.

If a given ſpace A B *be applyed according to a given right line* A C, *exceeding it by a figure* C B *given by kind, the breadths of the exceſſes* C E *and* C F *are given.*



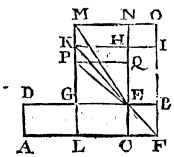
Conſtruction FOr D E being divided into two equal parts in G, let there

there be deſcribed on G E the rectiline figure G H, alike and alike poſited to C B.

Demonſtration NOw ſeeing that C B is alike to G H, thoſe figures C B and G H * are about one and the ſame diameter, and G H is given by kind, as is C B. But it is deſcribed on the given line G E: Therefore [a] the ſame G H is alſo given by magnitude. But A B is given: Therefore A B and G H are given by magnitude. Now thoſe figures A B and G H, are equal to L I, (for A G, L E, and E I, being equal, the Gnomon G F H is equal to A B; and therefore adding G H common to both, L I ſhall be equal to A B and G H;) therefore L I is given by magnitude; but it is alſo given by kind, being it is [b] alike to C B. Therefore [c] the ſides of the ſaid L I are given, ſeeing it is equal to G E: Therefore [d] the remainder C F is given, and in a given reaſon [e] to C E. Wherefore [f] C E is given.

a) 32. p.
b) 24. 6.
c) 55. p.
d) 4. p.
e) 3 def.
f) 2. p.

SCHOLIUM.



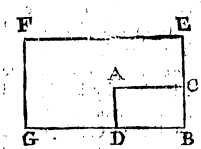
* *EUCLIDE ſuppoſeth here that* C B *and* G H *are about one and the ſame diameter, but we ſhall thus demonſtrate it: Let* C B *and* G H *be two alike Parallelograms diſpoſed as above, that is to ſay, that the equal angles joyn together in* E, *the ſide* C E *meets directly with his homologal ſide* E H, *and the ſide* B E, *his correſpondent ſide* E G; *and let the diameter* F E *be drawn I ſay that the ſaid diameter* F E *prolonged, will paſſe by the point* K; *that is to ſay, the Parallelograms* G H *and* C B, *conſiſt about one and the ſame diameter. For if it be denied, the diameter* E F *being produced, will paſſe above the point* K, *or below it. Let it in the firſt place paſſe above it, and let it cut* G K, *prolonged in the point* M, *and by the point* M, *let there be drawn* M N, *parallel to* K H, *which ſhall meet* E H, *prolonged in the point* N, *and* F B *in* O.

Demonſtration FOraſmuch as the Paralellograms G N and C B are with the Parallelogram L O about one and the ſame diameter, they are [g] alike to one another. Wherefore as F C is to C E, ſo is E G to G M. In like maner, ſeeing the parallelograms C B and G H are alike, as F C is to C E, ſo is E G to G K: Therefore [h] as E G is to G M, ſo is E G to G K. Wherefore [i] G M and G K are equal, a part to the whole, which is abſurd: By the ſame reaſons it may be demonſtrated, that the diameter prolonged will not fall below the point K: Therefore the Paralellograms C B and G E conſiſt about one and the ſame diameter.

g) 24. 6.
h) 11. 5.
i) 9. 5.

PROP. 60.

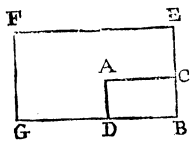
If a Paralellogram A B, *given by kind and by magnitude, be augmented or diminiſhed by a Gnomon* C F D, *the breadths of the Gnomon (conſiſting of the lines* C E *and* D G), *are given.*



Demonſtration FOr ſeeing that A B is given, and the Gnomon C F D alſo given, the whole Parallelogram B F is given: But it is alſo given by kind, ſeeing it is alike to B A: Therefore [a] the ſides of the ſame

a) 55. p.

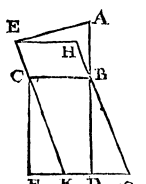
same BF are given; and therefore each of the lines BE and BG is given. But each of the lines BC and BD is given; therefore each of the remaining lines CE and DG is also given.



Construction NOw let the Paralellogram BF, given by kind and by magnitude, be diminished by the given Gnomon CFD: I say that each of the lines CE and DG is given.

Demonstration 2. FOr seeing that BF is given, and the Gnomon CFD given, the remaining figure AB is also given. But it is also given by kind, b) 55. p. seeing it is alike to BF: Therefore b the sides of the said AB are given, and therefore each of the lines CB and BD is given. But each of the lines BE and BG is given: Therefore also each of the remaining lines CE and DG is given.

PROP. 61.



If to one side of a figure ABCE, *given by kind, there be applyed a space Parallelogram* CD, *in a given angle* BCF, *and that the given figure* AC *hath to the Parallelogram* CD *a given reason, the Parallelogram* CD *is given by kind.*

Construction FOr by the point B, let BH be drawn parallel to CE, and by the point E let EH be drawn parallel to CB, and let EC and HB be prolonged to the points K and G.

Demonstration FOrasmuch as the angle BCE is given, and the reason of EC to CB, a) 3 def. a the parallelogram CH is given * by kind. But the figure ABCE is also given by kind, and is described on the same line BC, as the Parallelogram CH given by kind is: Therefore b the b) 49. p. reason of the figure ABCE to the Parallelogram CH is given. But by Supposition, the reason of the said figure ABCE to the Parallelogram CD is also given; and CD is c equal to CG: Therefore d the reason of c) 36. 1. CH to CG is given. Wherefore the reason of the line EC to the line d) 8. p. CK is given; (for e as CH is to CG, so is EC to CK.) But the reason e) 1. 6. of EC to CB is also given: Therefore f the reason of the said CB to f) 8. p. CK is given. And seeing that the angle ECB is given, also the following angle BCK g is given. But the angle BCF is proposed given; and g) 13. 1. 4. p. therefore the remaining angle FCK is given. Also the angle CKF is given, for that h it is equal to the angle BCK: Therefore the other angle h) 29. 1. CFK is given: Wherefore i the triangle FCK is given by kind; and i) 40. p. therefore the reason of FC to CK is given. But the reason of CB to the same CK is also given: Therefore k the reason of FC to CB is given; k) 8. p. and the angle BCF is also given. Wherefore the Parallelogram CD is given by kind.

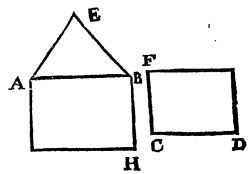
SCHO-

SCHOLIUM.

* *Although it be manifest that a Parallelogrm that hath one angle given, and the reason of the sides about the same angle also given, is given by kind, as* Euclide *doth here declare, so it is notwithstanding that the ancient Interpreter doth thus demonstrate it.*

Seeing that in the Parallelogram CH *the angle* ECB *is given, the angle* CEH *is also given; for the right line* EC *falling on the Parallels* EH *and* CB, *doth make the two internal angles on the same part equal to two right angles. And therefore seeing that the angle* ECB *is given, the other angles are given; and seeing that the reason of* EC *to* CB *is given, and that* BH *is equal to* CE, *and* EH *to* BC, *the reason of the sides to one another is also given.*

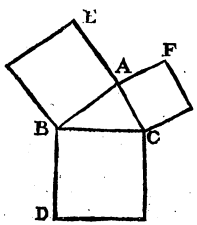
PROP. 62.



If two right lines AB *and* CD, *have to one another a given reason, and that on one of them* AB, *there be described a figure* AEB, *given by kind; but on the other* CD, *a Space Parallelogram* DF, *in a given angle* DCF, *and that the figure* AEB *hath to the Parallelogram* DF *a given reason, the Parallelogram* DF *is given by kind.*

Construction FOr on the line AB let there be described the Parallelogram AH, alike and alike posited to DF.

Demonstration SEeing then that the reason of AB to CD is given, and that on those lines are described the rectiline figures AH and FD, alike and alike posited, a the reason of AH to FD is given. a) 50. p. But the reason of FD to AEB is also given: Therefore b the reason of b) 8. p. AH to AEB is given. But the angle ABH is also given, being equal to the angle FCD, and so the figure AEB is given by kind; and to AB one of the sides thereof, the Parallelogram AH is applyed in a given angle ABH, and the reason of the said figure AEB to the said Parallelogram AH is given: Therefore c the Parallelogram AH is given by kind; c) 61. p. and therefore ED which is alike thereto, is also given by kind.



PROP. 63.

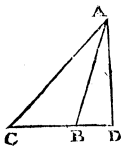
If a triangle ABC *be given by kind, the square* BE, CD, *and* CF, *which is described on each of the sides, shall have a given reason to the triangle* ABC.

Demonstration FOr seeing that on one and the same right line BC, there are described the two rectiline figures ABC and CD,

CD given by kind, [a] the reason of the same ABC to CD is given; and therefore the reason of the squares BE and CF, to the triangle ABC, is also given.

a) 49. p.

PROP. 64.



If a triangle ABC, *hath an obtuse angle* ABC *given, that space of which the side* AC *subtending the obtuse angle* ABC, *is more in power then the sides* AB *and* BC, *that comprehend the said angle, shall have a given reason to the triangle* ABC.

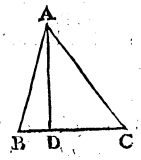
Construction LEt the line CB be prolonged directly, and from the point A let the perpendicular AD be drawn: I say that the space of which the square of the line AC doth exceed the squares of the lines AB and BC, that is to say, [a] the double of the rectangle conteined under CB and BD, shall have a given reason to the triangle ABC.

a) 12. 2.

Demonstration FOr seeing that the angle ABC is given, the angle ABD is also given. But the angle ADB is also given; therefore the other angle BAD is given: Wherefore [b] the triangle ABD is given by kind; therefore [c] the reason of AD to DB is given. But as AD to DB, so [d] the rectangle of AD and BC is to the rectangle of BC and BD. But the reason of AD to BD is given: Therefore also is the reason of the rectangle of AD and BC to the rectangle of BC and BD given: Wherefore the reason of the double of the said rectangle BC and BD to the rectangle of AD and BC is also given. But the said rectangle of AD and BC hath also a given reason to the triangle ABC (to wit, double reason; for the rectangle is [e] double to the triangle) therefore the reason of the double of the rectangle of BC and BD [f] to the triangle ABC is given. But the same double of the rectangle of CB and BD is that space of which the square of the line AC doth exceed the squares of the lines AB and BC: Therefore the same space hath a given reason to the triangle ABC.

b) 40. p.
c) 3. def.
d) 1. 6.
e) 41. 1.
f) 8. p.

PROP. 65.



If a triangle ABC, *hath one acute angle* ACB *given, that space, of which the side subtending the said acute angle is lesse in power then the sides comprehending the same acute angle, shall have a given reason to the triangle.*

Construction FRom the point A let there be drawn the line AD, perpendicular to BC: I say that space of which the square of the line AB is lesse then the squares of the lines AC and CB, that is to say, [a] the double of the rectangle of BC and CD, hath a given reason to the triangle ABC.

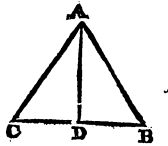
a) 13. 2.

Demonstration FOr seeing that the angle C is given, and the angle ADC also given, the other angle DAC is given: Wherefore the

triangle [b] ADC is given by kind; and therefore the reason of AD to DC is given, and consequently also [c] that of the rectangle of BC and CD to the rectangle of BC and AD: Therefore the reason of the double of the rectangle of BC and CD to the rectangle of BC and AD is given. But the reason of the same rectangle of BC and AD to the triangle ABC is given: (for [d] the rectangle is double to the triangle) Therefore [e] the reason of the double of the rectangle of BC and CD to the triangle ABC is given. And seeing that the same double of the rectangle of BC and CD is that whereof the square of the line AB is lesse then the squares of the lines AC and BC, that space of which the square of the line AB is lesse then the squares of the lines AC and BC, shall have a given reason to the triangle ABC.

b) 40. p.
c) 1. 6.
d) 41. 1.
e) 8. p.

PROP. 66.



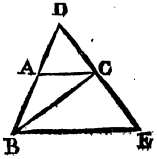
If a triangle ACB, *hath one angle* B *given, the rectangle made of the lines* AB *and* BC, *conteining the same angle, shall have a given reason to the triangle.*

Construction FOr from the point A let AD be drawn perpendicular to CB.

Demonstration THerefore seeing that the angle B is given, and also the angle ACB; the other angle CAD is likewise given. Wherefore the triangle ACB [a] is given by kind; and consequently the reason of AC to AD is given, But as AB is to AD, [b] so the rectangle of AC and CB is to the rectangle of CB and AD: Therefore the reason of the rectangle of AC and CB to the rectangle of CB and AD is given. But the reason of the said rectangle of CB and AD to the triangle ACB is also given; (for that it is double reason, the rectangle being double [c] to the triangle:) Therefore [d] the reason of the rectangle of AC and CB to the triangle ABD is given.

a) 40. p.
b) 1. 6.
c) 41. p.
d) 8. p.

PROP. 67.



If a triangle ABC, *hath one angle* BAC *given, that Space by which the square of the line compounded of the two sides* BA *and* AC, *that contein the same given angle* BAC *doth exceed the square of the other side, it shall have a given reason to the triangle* ABC.

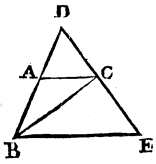
Construction FOr let BA be prolonged in such sort as that AD may be equal to AC, then having drawn DCE infinitely, from the point B let BE be drawn parallel to AC, meeting the said DE in the point E.

Demonstration FOrasmuch as AD is equal to AC, [a] DB is equal to BE; (for the two triangles ADC and BDE are alike) and from the top B is drawn to the base DE, the right line BC: Therefore

a) 4. 6. and 14. 5.

fore * the rectangle of D C and C E, with the ſquare of B C, is equal to the ſquare of B D; but the ſame B D is compounded of B A and A C; therefore the ſquare of the compound of A B and A C is greater then the ſquare of B C, of the rectangle of D C and C E.

Now I ſay that the rectangle of D C and C E hath a given reaſon to the triangle A B C: Foraſmuch as the angle B A C is given, the angle D A C is alſo given. But each of the angles A D C and A C D is given, it being the half of the angle B A C which is given. Therefore [b] the triangle A D C is given by kind; and therefore the reaſon of D A to D C is given. Therefore [c] the reaſon of the ſquare of the ſaid D A to the ſquare of D C is alſo given. And ſeeing that as B A is to A D, [d] ſo is E C to C D, and alſo as B A is to A D, [e] ſo is the rectangle of B A and A D to the ſquare of A D; and as E C is to C D, [f] ſo alſo is the rectangle of E C and C D to the ſquare of C D; by permutation, as the rectangle of B A and A D is to the rectangle of E C and C D, ſo is the ſquare of A D to the ſquare of D C. But the reaſon of the ſaid ſquare of A D to the ſquare of D C is given: Therefore the reaſon of the rectangle of B A and A D to the rectangle of E C and C D is alſo given. But A D is equal to A C: Therefore the reaſon of the rectangle of B A and A C to the rectangle of E C and C D is given. But the reaſon of the rectangle of B A and A C to the triangle A B C [g] is given, becauſe the angle B A C is given: Therefore [h] the reaſon of the rectangle E C and C D to the triangle A B C is given. But the rectangle of E C and C D is that whereof the ſquare of the line compounded of B A and A C is greater then the ſquare of B C: Therefore that ſpace to which the ſquare of the line compounded of B A and A C is greater then the ſquare of B C, ſhall have a given reaſon to the triangle A B C.

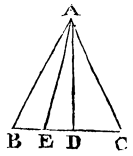


b) 40. p.
c) 50. p.
d) 2. 6.
e) 1. 6.
f) 1. 6.
g) 66. p.
h) 8. p.

SCHOLIUM.

** EUCLIDE ſuppoſeth in this place that when in an Iſoſceles triangle a right line is drawn from the top to the baſe, the ſquare of that line, with the rectangle conteined under the ſegments of the baſes, is equal to the ſquare of either of the other legges, which the ancient Interpreter doth thus demonſtrate.*

Conſtruction LEt A B C be an Iſoſceles triangle, whoſe legges are A B and A C; and from the top A let A D be drawn to the baſe B C: I ſay that the ſquare of A D with the rectangle of B D and D C, is equal to the ſquare of either of the legges A B or A C.



Demonſtration NOw the line A D is perpendicular to B D, or not: Let it in the firſt place be perpendicular: Therefore it will cut the baſe B C into two equal parts in the point D; and therefore the rectangle conteined under B D and D C is equal to the ſquare of the ſaid B D, and adding to them the common ſquare of A D, the rectangle of B D and D C with the ſquare of A D, ſhall be equal to the ſquares of D B and A D. But to thoſe ſquares of A D and D B [i] the ſquare of A B is equal: Therefore the

i) 47. 1.

the ſquare of A B is equal to the rectangle of B D and D C, and the ſquare of A D together.

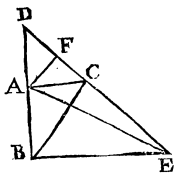
Now ſuppoſe A D not to be perpendicular, but that from the point A there doth fall on B C the perpendicular A E, that being ſo, B C ſhall be cut into two parts equally in the point E, and unequally in D. Wherefore the rectangle of B D and D C, with the ſquare of D E, [k] is equal to the ſquare of B E; and adding the common ſquare of A E, the rectangle of B D and D C, with the ſquares of D E and A E, ſhall be equal to the ſquares of B E and A E. But [l] the ſquare of A D is equal to the two ſquares of D E and A E: Therefore the rectangle of B D and D C, with the ſquare of A D is equal to the ſquares of B E and A E. But to theſe ſquares of B E and A E the ſquare of A B is equal: Therefore the ſquare of A D, with the rectangle of B D and D C, is equal to the ſquare of A B.

k) 5. 2.
l) 47. 1.

OTHERWISE.

Conſtruction HAving done, as in the foregoing Demonſtration, from the point A, let A F be drawn perpendicular to C D, and let A E be drawn.

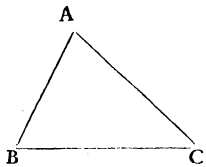
Demonſtration FOraſmuch as the angle B A C is given, the half thereof A C F ſhall be alſo given. But the angle A F C is given; and therefore the triangle A F C is given by kind: Therefore the reaſon of A F to F C is given. But the reaſon of C D to the ſame F C is alſo given, ſeeing that C D is double to F C: Therefore [m] the reaſon of C D to A F is given; and therefore alſo the reaſon of the rectangle of C D and E C, to the rectangle of A F and E C, is given; (for it is the ſame reaſon [n] as that of C D to A F.) But the reaſon of the rectangle of A F and F C to the triangle A C E is given; ſeeing it is double [o] to the ſame triangle. Therefore the reaſon of the rectangle of C D and C E to the triangle A C E is alſo given. But the triangle A C E is equal to the triangle A B C [p], they being both conſtituted on one and the ſame baſe A C, and between the ſame parallels A C and B E: Therefore [q] the reaſon of the rectangle of C E and C D to the triangle A B C is given. But the ſaid rectangle of C E and C D is the ſpace by which the ſquare of the line compounded of A B and A C, is greater then the ſquare of B C: Therefore that ſpace by which the ſquare of the line compounded of A B and A C is greater then the ſquare of B C, hath a given reaſon to the triangle A B C.



m) 8. p.
n) 1. 6.
o) 41. 1.
p) 37. 1.
q) 8. p.

OTHERWISE.

FOr the given angle A is either a right, acute, or obtuſe angle: Let it in the firſt place be ſuppoſed a right angle: Therefore the ſquare of the line compounded of B A C, is greater then the ſquare of B C, by twice the rectangle of B A and A C; (ſeeing that [r] the ſquare of B C is equal to the ſquares of B A and A C; and the ſquare of the line compounded of B A C [s] is equal to thoſe two



r) 47. 1.
s) 5. 2.

squares of B A and A C, and twice the rectangle of the said B A and A C:) Wherefore the reason of double the rectangle of B A and A C to the triangle A B C is given.

Construction NOw let the angle C be supposed acute, and from the point A let there be drawn on C B the perpendicular A D.

Demonstration FOrasmuch as the triangle C A B is an Oxigonium triangle, and the perpendicular A D being drawn, the square of C A and C B are equal [t] to the square of A B with twice the rectangle of C B and C D; adding therefore the common double rectangle of C A and C B, the squares of C A and C B, with the double rectangle of the said C A and C B, that is to say, [u] the alone square of the line compounded of A C B, are equal to the square of A B, with the double of the rectangle of C D and C B, and over and above the double of the rectangle of A C and C B, that is to say, the double of the rectangle conteined under the compound line of A C D and and C B (for the rectangle of A C D and C B is [w] equal to the rectangles of A C and C B, and of C D and C B:) Therefore the square of the line compounded of A C B is greater then the square of A C, by double the rectangle of A C D and C B. And seeing that the angle A C B is given, and the angle B D A also given, the other angle C A D is given: Therefore [x] the triangle C A D is given by kind, and therefore the reason of C D to C A is given; and by consequence the reason of the line compounded of A C D to C A [y] is also given. Wherefore the reason of the rectangle of those lines compounded of A C D and C B [z] to the rectangle of A C and C B is also given. But the reason of the said rectangle of A C and C B to the triangle C A B [a] is given, seeing the angle C is given; therefore the reason of double the rectangle of the line compounded of A C D and C B to the triangle C A B is given.

t) 13. 5.

u) 7. 2.

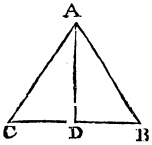
w) 1. 2.

x) 40. p.

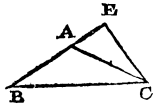
y) 6. p.

z) 1. 6.

a) 66. p.



Lastly, Let the angle B A C be supposed to be obtuse, and having prolonged B A from the point C, let the perpendicular C E be drawn on the said line B A prolonged, and let A F be proposed to be equal to A E.



Demonstration FOrasmuch as the angle B A C is obtuse, and the perpendicular C E being drawn, the squares of A B and A C, and the double of the rectangle under B A and A E, or A F, are all alike equal [b] to the square of B C, and adding the common double rectangle of B A and A C, the squares of the said A B and A C, with the double of the rectangle of the same A B and A C, that is to say, [c] the square of the line compounded of B A C, and the double of the rectangle of B A and A F are together equal to the square of B C, with the double of the rectangle of B A and A C. Let the common double of the rectangle of B A and A F be taken away, and there will remain the square of the line compounded of B A C, equal to the square of B C, with the rectangle of A B and C F; (for the rectangle of A B and A C is equal [d] to the two rectangles of A B and A E, and of A B and C F:) Therefore the square

b) 12. 2.

c) 4. 2.

d) 1. 2.

square of the line compounded of B A C is greater then the square of B C by the double of the rectangle of A B and C F. And forasmuch as the angle B A C is given, the angle C A E [e] is given. But the angle A E C is also given; therefore the other angle A C E is given: Wherefore [f] the triangle A C E is given by kind, and therefore the reason of C A to A E, that is to say, to A F is given. Therefore [g] the reason of the said C A to F G is also given. But the reason of the same C A to C E is given; therefore [h] the reason of C E to C F is also given. Wherefore the reason of the rectangle of E C and A B to the rectangle of F C and A B is given; (for the rectangle is to the rectangle [i] as C E is to C F) and also that of the rectangle of A C and A B, to the rectangle of E C and A B. Therefore [k] the reason of the rectangle of F C and A B to the rectangle of A C and A B is given. But the reason of the rectangle of A C and A B to the triangle A B C [l] is given: Therefore also the reason of the double of the rectangle of F C and A B, to the triangle A B C is given. But the same double of the rectangle of F C and A B is that, whereof the square of the line compounded of B A C is greater then the square of B C, whereof that space of which the square of the line compounded of B A C is greater then the square of B C, hath a given reason to the triangle A B C.

e) 13. 1.

f) 40. p.

g) 5. p.

h) 8. p.

i) 2. 6.

k) 8. p.

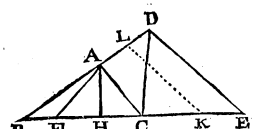
l) 66. p.

OTHERWISE.

Construction LEt the line B A be prolonged to the point D, in such sort as A D may be equal to A C, and let C D be drawn.

Demonstration FOrasmuch as the angle B A C is given, each of the angles A D C and A C D, which is the half thereof shall be also given, and therefore the other angle D A C is also given: Therefore [m] the triangle A C D is given by kind. Wherefore the reason of A C to C D is given. And forasmuch as the angle A D C is given: Let each of the angles D E C and A F C be made [†] equal to the said A D C: Therefore seeing that the angle B D C is equal to the angle D E C, and the angle D B E is common to the triangles D B E and D B C, the other angle B D E is equal to the other angle B C D, and therefore the triangle B D E is equiangled to the triangle B D C. Therefore [n] as E B is to B D, so is B D to C B: Wherefore the rectangle of E B and C B, that is to say, [o] the rectangle of E C and C B, [p] with the square of C B is equal, [q] to the square of B D, that is to say, to the square of the line compounded of B A C; for A D is equal to A C; and therefore the rectangle of E C and C B, with the square of C B, that is to say, the square of the line compounded of B A C is greater then the square of the rectangle of B C and C E: I say therefore that the reason of the said rectangle of B C and C E to the triangle A B C is given. Forasmuch as the angle B D E is equal to the angle B C D, and the angle A D C equal to the angle A C D, the other angle C D E is equal to the other angle A C B: But the angle D E C is also equal to the angle A F C; therefore the remaining

m) 40. p.



n) 4. 6.

o) 5. 2.

p) 5. 2.

q) 17. 6.

angle C A F is equal to the remaining angle D C E. Wherefore the triangle A F C is equiangled to the triangle D C E; and therefore r as C A is to A F, ſo is C D to C E; and by permutation, as A C is to C D, ſo is A F to C E. But the reaſon of A C to C D is given: Therefore alſo the reaſon of A F to C E is given. From the point A let A H be drawn perpendicular to B C: Foraſmuch as the angle A F C is given, and the angle A H F alſo given, the third angle H A F is given: Wherefore ſ the triangle A H F is given by kind; and by conſequence the reaſon of A F to A H is given. But the reaſon of A F to C E is alſo given: Therefore t the reaſon of A H to C E is given; and therefore the reaſon of the rectangle of A H and B C u to the rectangle of B C and C E is alſo given. But the reaſon of the rectangle of A H and B C to the triangle A B C is likewiſe given; (for the rectangle w is double to the triangle) and the rectangle of B C and C E is that whereof the ſquare of the line compounded of B A C is greater then the ſquare of B C. Therefore that ſpace of which the ſquare of the line compounded of B A C is greater then the ſquare of B C by a given reaſon to the triangle A D C.

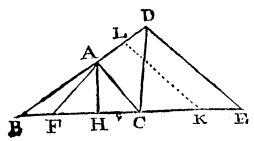
r) 4. 6.

ſ) 40. p.

t) 8. p.

u) 1. 6.

w) 41. 1.



SCHOLIUM.

† *The ancient Interpreter pretending to ſhew the conſtruction of the angle* D E B *equal to the angle* A D C, *ſaith that on the line* B D *and in the point* D, *the angle* B D E *ought to be made, equal to the angle* B C D, *and that the right lines* B C *and* D E *be drawn until they interſect in* E, *in ſuch ſort as he ſuppoſeth the angle* B C D, *to be given, but it is not.*

The ſame Interpreter afterward ſhews how there may univerſally from a given point, be drawn a right line given by poſition to a right line, making an angle equal to a given angle. But we will alſo reject this way, ſeeing we have elſ-where ſhewn another more brief and eaſie. For example, if we would from the point D *draw to the line* B C *given by poſition a right line, making an angle equal to a given angle* A D C, *as is here required, we have no more to do but to aſſume the point* K *in the ſaid line* B C, *and there make the triangle* C K L *equal to the given angle* A D C: *If the line* K L *doth meet with the point* D, *it ſhall be the line required. But if it meet not with it, from the point* D *let there be drawn the line* D E, *parallel to the ſaid* K L, *cutting* B C *protonged in* E, *and the angle* D E C *ſhall be equal to the given angle* A D C, *for on the two parallel lines* L K *and* D E, *there doth fall the line* B E; *and therefore the angle* D E C x *is equal to the angle* L K C, *which hath been made equal to the given angle* A D C; *and by conſequence the ſame angle* D E C *is alſo equal to* A D C.

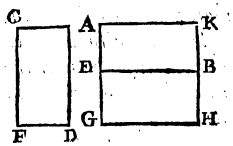
x) 29. 1.

PROP. 68.

If two Parallelograms A B *and* C D, *have to one another a given reaſon, and that a ſide hath alſo a given reaſon to a ſide, the other ſide ſhall have likewiſe a given reaſon to the other ſide.*

Con-

Conſtruction LEt the reaſon of B E to F D be given: I ſay the reaſon of A E to F C is alſo given. For to the right line E B let there be applyed the Parallelogram F H, equal to the Parallelogram C D, and conſtituted in ſuch ſort as A E and E G may make one right line: † Therefore K B and B H will alſo make one right line.



Demonſtration FOraſmuch as the reaſon of A B to C D is given, and that E H is equal to the ſaid C D; the reaſon of A B to E H is given; and therefore the reaſon of A E to E G is alſo given. Seeing therefore that E H is equal and equiangled to C D, as b E B is to F D, ſo is F C to E G. But the reaſon of E B to F D is given: Therefore alſo the reaſon of F C to E G is given. But the reaſon of A E to the ſame E G is alſo given: Therefore the reaſon of A E to F C is given.

a) 1. 6.

b) 14. 6.

SCHOLIUM.

† *EUCLIDE having poſited* A E *and* E G *directly in one right line, preſently concludeth that* K B *and* B H *ſhall alſo make a right line; but we ſhall demonſtrate it thus. Seeing the lines* A E *and* E G *are poſited directly, the angles* A E B *and* B E G b *are equal to two right angles; and ſeeing that* A B *is a Parallelogram, the lines* A K *and* E B *are parallels, on which the line* A E *doth fall; and therefore the two internal angles* A *and* B E A c *are alſo equal to two right angles, and taking away the common angle* B E A, *there will remain the angle* A, *equal to the angle* B E G; *and conſequently their oppoſite angles* E B K *and* H *are alſo equal to one another: Again, ſeeing that* B G *is a Parallelogram, the two lines* B E *and* H G *are Parallels, on which* B H *doth fall; and therefore the two internal angles* H *and* E B H c *are equal to two right angles. But it hath been demonſtrated that* H *is equal to* E B K: *Therefore the two angles* E B K *and* E B H *are alſo equal to two right angles; and therefore* d *the two lines* K B *and* B H *do meet directly according to EUCLIDE.*

b) 13. 1.

c) 29. 1.

d) 14. 1.

OTHERWISE.

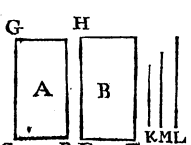
Conſtruction LEt the given right line K be expoſed, and ſeeing that the reaſon of A to B is given, let the ſame be made of K to L; therefore the reaſon of K to L is alſo given.

Demonſtration But K is given; therefore e L is alſo given. Again, ſeeing that the reaſon of C D to E F is given, let the ſame be made of K to M: Therefore the reaſon of K to M is given. But K is given, therefore f M is alſo given; and therefore the reaſon of L to M is given. Now ſeeing that A is equiangled to B, g the reaſon of the ſaid A to B is compounded of that of the ſides, that is to ſay of C D to E F, and of C G to E H. But alſo the reaſon of K to L is compounded of K to M, and of M to L; therefore the reaſon compounded of C D to E F, and of C G to E H, is the ſame with that which is compounded of K

e) 2. p.

f) 1. p.

g) 23. 6.



to

to M, and of M to L (the reason of K to L being the same as of A to B:) But the reason of C D to E F is the same as of K to M: Therefore the other reason of C G to E H is also the same as of M to L. But the said reason of M to L is given: Therefore also the reason of C G to E H is given.

PROP. 69.

If two Parallelograms C B *and* EH, *having the angles* D *and* F *given, and that a side hath also a given reason to a side; in like manner the other side shall have a given reason to the other side.*



Construction LEt the reason of B D to F H be also given: I say that the reason of A B to E F is given. For if C B be equiangled to H E, it is manifest by the precedent Proposition; but if it be not equiangled thereto, let the right line D B be constituted, and in the given point B therein, let the angle D B K be made equal to the angle E F H, and finish the Parallelogram D K.

Demonstration FOrasmuch as each of the angles B K L and B A K is given; † the other angle K B A is given: Wherefore the triangle
a) 40. p. a A B K is given by kind, and therefore the reason of A B to B K is given. But
b) 35. p. reason of C B to E H is supposed to be given, and b C B is equal to D K; therefore the reason of D K to E H is given; and seeing that D K is equiangled to E H, and the reason of the said D K to E H is given, as also
c) 68. p. that of D B to F H, c the reason of B K to F E is given. But the reason of
d) 29. 1. the said B K to B A is also given: Therefore d the reason of A B to F E is given.

SCHOLIUM.

† *EUCLIDE supposeth there that a Parallelogram having one angle given, all the other angles are also given, and as well the Ancient Interpreters as others, do give the reasons why, the angle* F *being given, the other angle* E *shall be also given, it being the remainder of two right angles, for that on the parallel lines* E G *and* F H *there doth fall the line* E F, *which makes* e *the two internal angles (of the same part)* F *and* G, *equal to two right angles. But to those angles* f *the opposite angles* G *and* H *are equal, and therefore they are also given.*

From whence it follows that the angles B D C *and* F *being given by supposition, all the other angles of the two Parallelograms* C B *and* E H, *are also given: Therefore the angle* D B K *having been made equal to the angle* F, *the angle* K *shall be equal to the angle* E, *and given as that is: But the angle* B A L *which is opposite to the given angle* B D C, *is also given; and therefore* B A K *which is the remainder of two right angles, shall be also given; in such sort as in the triangle* A B K, *the two angles* B A K *and* B K A *are given, as EUCLIDE doth declare in this place.*

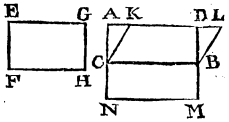
PROP. 70.

If of two Parallelograms A B *and* E H, *the sides about the equal angles, or about the unequal angles (yet nevertheless given*

given angles) have to one another a given reason, to wit (A C *to* E F, *and* C B *to* F H) *also the same Parallelograms* A B *and* E H *shall have to one another a given reason.*

Construction FOr let A B be prolonged to E H, and on the right line C B let the Parallelogram C M be applyed equal to the Parallelogram E H, in such sort as A C may be direct to C N; that is to say, that A C and C N make one right line; and by consequence D B shall be a directly with B M. a) Sch. 68. p.

Demonstration FOrasmuch then as C M is equiangled and equal to E H, the sides about the equal angles shall be reciprocally b proportional: Wherefore as B C is to H F, so is F E to N C. But the reason of B C to H F is given: Therefore the reason of F E to N C is also given. But the reason of A C to the same E F is given: Therefore c the reason of A C to N C is also given. Wherefore the reason of A B to C M is given; (for it is the same d as of A C to C N.) But C M is equal to E H: Therefore the reason of A B to E H is given. b) 14. 6. c) 8. p. d) 1. 6.

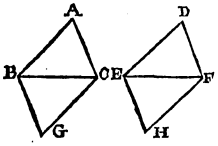


Construction NOw suppose A B not to be equiangled to E H, and on the right line C B, and in the given point C therein: Let there be constituted the angle B C K, equal to the given angle F, and so finish the Parallelogram C L.

Demonstration FOrasmuch as the angle A C B is given, and the angle B C K also given, the remaining angle A C K is given: Therefore the triangle A C K e is given by kind; and therefore the reason of A C to C K is given: But the reason of A C to E F is also given: Therefore the reason of C K to E F is given. But the reason of B C to H F is also given, and the angle B C K is equal to the angle F; therefore (by the first part of this Prop.) the reason of C L to E H is given. But to the said C L, A B is equal: Therefore the reason of A B to E H is given. e) 40. p.

PROP. 71.

If of two triangles A B C *and* D E F, *the sides about the equal angles* A *and* D, *or else about the unequal angles (yet nevertheless given angles) have to one another a given reason (to wit* A B *to* D E *and* A C *to* D F) *the same triangles shall have also to one another a given reason* A B C *to* D E F.



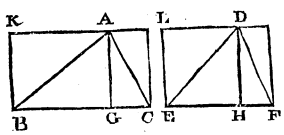
Construction Let the Parallelogram A G and D H be finished.

Demonstration SEeing that the two Parallelograms A G and D H, have the sides about the equal angles A and D, or else about the unequal angles (nevertheless given) have a given reason to one another; the

a) 70. p. b) 34. p.

the reason [a] of the Parallelogram A G to the Parallelogram D H is given. But the triangle A B C is the half of the Parallelogram A G [b] and the triangle DEF the half of the Parallelogram DH. Therefore the reason of the triangle A B C to the triangle D E F is given.

PROP. 72.



If of two triangles A B C *and* D E F, *the bases* B C *and* E F, *are in a given reason*, B C *to* E F, *and that from the angles* A *and* D, *there be drawn to those bases the right lines* A G *and* D H, *making the equal angles* A G C *and* D H F, *or else unequal (yet nevertheless given) which shall have to one another given reasons* A G *to* D H, *those triangles* A B C *and* D E F *shall have also a given reason to one another, to wit*, A B C *to* D E F.

Construction For let the Parallelogram K C and L F be finished.

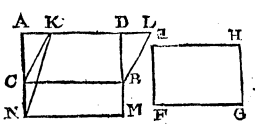
a) 29. 1.

Demonstration FOrasmuch as the angles A G C and D H F are eqnal, or unequal (yet given) and that the angle A G C [a] is equal to the angle K B C. But the angle D H F equal to the angle L E F, the angles at the points B and E are equal, or else unequal (yet given) and for that the reason of A G to D H is given, and A G is equal to K B. But D H is equal to L E, also the reason of K B to L E is given. But the reason of B C to E F is also given, and the angles at the points B and E are equal, or else unequal (yet given:) Therefore [b] the reason of the Parallelogram K C to the Parallelogram L F is given; and therefore the reason of the triangle A B C to the triangle D E F is given, seeing those triangles [c] are the one half of the Parallelograms.

b) 70. p.

c) 41. 1.

PROP. 73.



If of two Parallelograms A B *and* E G, *the sides about the equal angles* C *and* F, *or else about the unequal angles (but nevertheless given) are in such sort to one another, that as the side* C B *of the first, is to the side* F G *of the second; so the other side* E F *of the second, is to some other right line* C N. *But that the other side* A C *hath also to the same right line* C N *a given reason, those Parallelograms will have also to one another a given reason* A B *to* E G.

Construction FOr in the first place, Let the Parallelogram A B be equiangled to E G, and having placed C N directly to A C: Let the Parallelogram C M be finished.

Demon-

Demonstration FOrasmuch then, as C B or N M its equal, is to F G, so is E F to C N, and that the angles N and F are equal (for N is equal to the angle A C B, which is put equal to F) the Parallelograms C M and E G [a] are equal: But as A C to C N, so [b] the Parallelogram A B is to the Parallelogram C M or E G: Therefore seeing that the reason of A C to C N is given, the reason of A B to E G is also given.

a) 14. 6. b) 1. 6.

Construction 2. NOw suppose the Parallelogram A B not to be equiangled to the Parallelogram E G, and let there be constituted at the given point C in the line C B, the angle B C K, equal to the angle E E G, and so finish the Parallelogram C L.

Demonstration 2. SEeing that each of the angles A C B and K C B is given, the remaining angle A C K is also given. But [c] the angle C A K is given, as also the remaining angle A K C: Therefore [d] the triangle A C K is given by kind; and therefore the reason of A C to C K is given. But the reason of the same A C to C N is also given: Therefore [e] the reason of C K to C N is given. And seeing that as C B is to F G; so is E F to the right line C N, to which the other side K C hath a given reason, and that the angle B C K is equal to the angle F, the reason of the Parallelogram C L to the Parallelogram E G is given (by the first part of this Prop.) but the Parallelogram C L is equal to the Parallelogram A B: Therefore the reason of the Parallelogram A B to the Parallelogram E G is given.

c) Sch. 69 p. d) 40. p. e) 8. p.

PROP. 74.

If two Parallelograms (as in the former figure) A B *and* E G, *in equal angles* C *and* F, *or else in unequal angles (yet nevertheless given angles) have a given reason to one another, as one side* C B *of the first shall be to one side* F G *of the second; so the other side* E F *of the second, shall be to that to which the other side* A C *of the first hath a given reason.*

Construction FOr either A B is equiangled or not; suppose it in the first place to be equiangled, and to the right line B C let there be applyed the Parallelogram C M, equal to the Parallelogram E G, and so posited, as that A C and C N may be direct: Therefore [a] D B and B M shall be also direct (that is as one right line.)

a) Sch. 68. p.

Demonstration SEeing that the reason of A B to E G is given, and that C M is equal to E G, the reason of A B to C M is also given; and therefore the reason of A C to C N is given (seeing A B is to C M, [b] as A C is to C N;) and for that C M is equal and equiangled to E G, the sides about the equal angles of the Parallelograms C M and E G, [c] are reciprocally proportional; and therefore as C B is to F G, so is E F to C N. But the reason of A C to C N is given: Therefore as C B is to F G, so is E F to that to which A C hath a given reason.

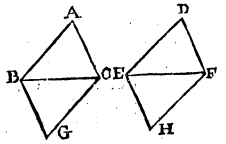
b) 1. 6. c) 14. 6.

Construction 2. NOw suppose A B not to be equiangled to E G, and in the given point C of the line C B, let there be constituted the

angle B C K equal to the angle E F G, and finish the Parallelogram C L.

Demonstration 2. SEeing then that the reason of A B to E G is given, and
d) 36. 1. [d] that A B is equal to C L, also the reason of C L to E G is given, and the angle B C K is equal to the angle F, and therefore
c) Sch. 69 p. C L [c] is equiangled to E G: Therefore (by the first part of this Prop.) as C B is to F G, so is E F to that to the which C K hath a given reason. But the reason of A C to C K is given; (as appears by what hath been demonstrated in the latter part of the precedent Prop.) Therefore as C B is to F G, so is E F to that to which A C hath a given reason.

PROP. 75.



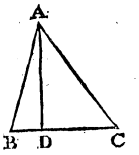
If two triangles A B C *and* D E F, *in equal angles* A *and* D, *or else unequal (yet nevertheless given) have to one another a given reason, as the side* A B *of the first, shall be to the side* D E *of the second, so the other side* D F *of the second, shall be to that right line to the which the other side* A C *of the first hath a given reason.*

Construction For let the Parallelograms A G and D H be finished.

Demonstration FOrasmuch as the reason of the triangle A B C to the triangle D E F is given, also the reason of the Parallelogram A G to the Parallelogram D H is given.

Seeing therefore that the two Parallelograms A G and D H in equal angles, or unequal angles (nevertheless given) have to one another a given
a) 74. p. reason; as [a] A B is to D E, so is D F to that to which A C hath a given reason.

PROP. 76.

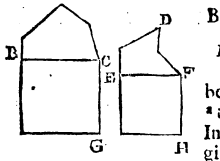


If from the top A *of a triangle* A B C, *given by kind, there be drawn to the base* B C, *a perpendicular line* A D, *that line* A D *shall have to the base* B C *a given reason.*

Demonstration FOr seeing that the triangle A B C is given by kind, the reason of A B to B C is given; and the angle B is also given. But the angle A D B is given; therefore the other angle B A D is given. Where-
a) 40. p. fore [a] the triangle A B D is given by kind; and therefore the reason of A B to A D is given. But the reason of A B to B C is given: Therefore
b) 8. p. [b] the reason of A D to B C is given.

PROP. 77.

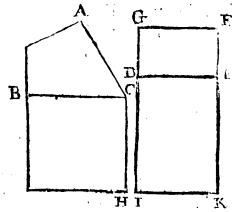
If two figures A B C *and* D E F, *given by kind, have to another a given reason, the reason also shall be given of which you please of the sides of one of the figures, to which you please of the sides of the other figure.*



Construction FOr on the right lines B C and E F, let there be described the squares B G and E H.

Demonstration FOrasmuch as on one and the same right line B C, are described two figures A B C and B G, given by kind, [a] the reason of the said A B C to B G is given: a) 49. p.
In like manner, the reason of D E F to E H is given; and seeing that the reason of A B C to D E F is given, and also that of the same figure A B C to B G; and again the reason of D E F to E H: [b] the reason of B G to E H is given; b) 8. p.
and therefore the reason of B C to E F is also given.

PROP. 78.



If a given figure A B C, *hath a given reason to some rectangled figure* D F, *and that one side* B C *hath a given reason to one side* D E, *the rectangled figure* D F *is given by kind.*

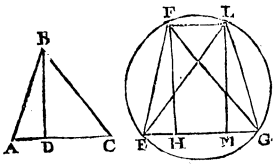
Construction FOr on the right line B C let the square B H be described, and to the right line D E, let the Parallelogram D K be applyed equal to B H, in such manner as that G D and D I may be placed directly; [a] and by con- a) Sch. 68. p
sequence F E and E K also directly.

Demonstration THerefore seeing that on one and the same right line B C are described the two rectiline figures A B C and B H, given by kind, [b] the reason of A B C to B H is given. But the reason of b) 40. p.
the said A B C to D F is also given: Therefore [c] the reason of B H to c) 8. p.
D F is also given. But B H is equal to D K: Therefore the reason of D K to D F is also given. And seeing that B H is equal and equiangled to D K, both the one and the other being rectangles, [d] the sides of those figures d) 14. 6.
are reciprocally proportional; and as B C is to D E, so is D I to C H. But by supposition, the reason of B C to D E is given; therefore also the reason of D I to C H is given; but the reason of D I to D G is also given: (for D I is to D G [e] as D K to D F:) Therefore [f] the reason of D G to e) 1. 6.
C H is given. But C H is equal to B C, seeing that B H is a square; f) 8. p.
therefore the reason of B C to D G is given. But the reason of the same B C to D E is also given; therefore the reason of D E to D G is given, and the angle at D is a right angle: Therefore [g] D F is given by kind. g) Sch. 61. p

PROP. 79.

If two triangles A B C *and* E F G, *have an angle* B *equal*

to an angle F. *But from the equal angles* B *and* F *there be drawn perpendiculars* B D *and* F H, *to the baſes* A C *and* E G; *and that as the baſe* A C *of the firſt triangle* A B C, *is to the perpendicular* B D, *ſo alſo the baſe* E G *of the other triangle* E F G, *is to the perpendicular* F H, *thoſe triangles* A B C *and* E F G *are equiangled.*



Conſtruction FOr about the triangle E F G let there be deſcribed the circle E F L G, then on the right line E G, and in the point E given therein, let there be made the angle G E L, equal to the angle C, and let F L and L G be drawn, and the perpendicular L M.

Demonſtration SEeing then that the angle G E L is equal to the angle C, and the angle E L G is equal to the angle E F G, [a] they being in one and the ſame ſegment of the circle; the third angle E G L is equal to the third angle A. Wherefore the triangle A B C is alike to the triangle E L G, and the perpendiculars B D and L M are drawn: Therefore † as A C is to B D, ſo is E G to L M; but by ſuppoſition as A C is to B D; ſo is E G to F H: Therefore [b] L M is equal to F H. But the ſaid L M is [c] parallel to F H: Therefore [d] F L is alſo parallel to E G; and therefore the angle F L E [e] is equal to the angle L E G. But the angle C is alſo equal to the ſaid angle L E G, and the angle F L E to the angle F G E [f]: Therefore alſo the angle C is equal to the angle F G E. But by ſuppoſition the angle A B C is equal to the angle E F G: Therefore the third angle B A C is equal to the third angle F E G: Wherefore the triangle A B C is equiangled to the triangle E F G.

a) 21. 3.
b) 7. 5.
c) 28. 1.
d) 33. 1.
e) 29. 1.
f) 21. 3.

SCHOLIUM.

† *Now that as* A C *is to* B D, *ſo* E G *is to* L M, *it is by ſome thus demonſtrated. Foraſmuch as the angle* C *is equal to the angle* G E L, *and the angle* B D C *to the angle* L M E, *each being a right angle, the other angle* C B D *is equal to the other angle* E L M: *Therefore* [g] *as* E M *is to* M L, *ſo is* C D *to* D B. *Again, ſeeing the angle* A B C *is equal to the angle* E L G, *and the angle* C B D *to the angle* E L M, *the remaining angle* A B D *is equal to the remaining angle* M L G; *but the angle* A D B *is alſo equal to the angle* L M G; *and therefore the third angle* A *is equal to the third angle* L G M: *Therefore* [h] *as* A D *is to* D B, *ſo is* G M *to* M L. *But it hath been demonſtrated that as* C D *is to* D B, *ſo is* E M *to* M L: *Therefore* [i] *as* A C *is to* B D, *ſo is* E G *to* L M.

g) 4. 6.
h) 4. 6.
i) 14. 5.

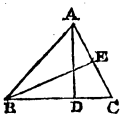
PROP. 80.

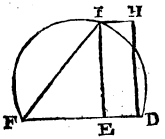
If a triangle A B C *hath one angle* A *given, and that the rectangle conteined under the ſides* A B *and* A C, *comprifing the given angle* A, *hath a given reaſon to the ſquare of the other ſide* B C, *the triangle* A B C *is given by kind.*

Con-

Conſtruction FOr from the points A and B, let there be drawn the perpendiculars A D and B E.

Demonſtration FOraſmuch as the angle B A E is given, and alſo the angle A E B, the triangle A B E is given [a] by kind; and therefore the reaſon of A B to B E is given: Therefore the reaſon of the rectangle of A B and A C to the rectangle of B E and A C is alſo given (for it is the ſame reaſon [b] as of A B to B E.) But the rectangle of A C and B E is equal to the rectangle of B C and A D; for that each of thoſe rectangles is [c] double to the triangle A B C. Therefore the reaſon of the rectangle of A B and A C to the rectangle of B C and A D is alſo given. But the reaſon of the rectangle of A B and A C to the ſquare of B C is given: Therefore [d] alſo the reaſon of the rectangle of B C and A D to the ſquare of B C is given; and therefore the reaſon of the right line B C to the right line A D is given. (For that [e] the rectangle is to the ſquare as A D to B C.) Now let the right line F G given by poſition and magnitude, be expoſed; and thereon let there be deſcribed the ſegment of a circle F I G, capable of an angle equal to the angle A. And ſeeing the ſaid angle A is given, alſo the angle in the ſegment F L G ſhall be given; and therefore [f] the ſame ſegment is given by poſition. From the point G let there be erected at right angles on the line F G, the line G H, which [g] is given by poſition: Let it be ſo made, that as B C is to A D, ſo F G may be to G H; and ſeeing that the reaſon of B C to A D is given, alſo that of F G to G H is given. But F G is given: Therefore [h] G H is given by magnitude. But it is alſo given by poſition, and the point G is given: Therefore the point H is [i] alſo given. Now by the point H let there be drawn H I, parallel to F G, and that line H I ſhall be given by [k] poſition. But the ſegment of the circle F I G is alſo given by poſition. Therefore [l] the point I is given. Let the right lines I F and I G be drawn, and the perpendicular I K: Therefore I K is given by poſition. But the point I is given, as alſo each of the points F and G: Therefore [m] each of the lines F G, F I, and I G is given by poſition and magnitude: Wherefore [n] the triangle F I G is given by kind; and ſeeing that as B C is to A E, ſo is F G to G H, and [o] that to G H, I K is equal, as B C is to A E, ſo is F G to I K, and the angle A is equal to the angle F I G: Therefore [p] the triangle A B C is equiangled to the triangle F I G. But F I G is given by kind: Therefore alſo the triangle A B C is given by kind.





a) 40. p.
b) 1. 6.
c) 41. 1.
d) 8. p.
e) 1. 6.
f) 8. def.
g) 4. def.
h) 2. p.
i) 27. p.
k) 28. p.
l) 25. p.
m) 26. p.
n) 39. p.
o) 34. p.
p) 79. p.

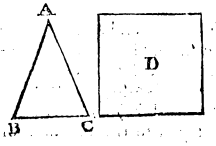
OTHERWISE.

Conſtruction LEt the triangle A B C, whoſe angle A is given, and the reaſon of the rectangle conteined under A B and A C, to the ſquare of B C be given: I ſay that the triangle A B C is given by kind.

Demonſtration FOr ſeeing the angle A is given, that ſpace of which the ſquare of the line compounded of B A C is greater then the ſquare of B C, [q] hath a given reaſon to the triangle A B C. Now let that ſpace be D: Therefore the reaſon of D to the triangle A B C is given

q) 87. p.

given. But the reason of the triangle A B C to the rectangle of A B and A C is given; r seeing the angle A is given: Therefore s the reason of the space D to the rectangle of A B and A C is given. But the reason of the rectangle of A B and A C to the square of B C is also given: Therefore s the reason of the space D to the square of B C is given. Wherefore by compounding, t the reason of the space D, with the square of B C to the said square of B C is given: Therefore the reason of the square of the line compounded of B A C, to the square of B C is given; (for that the space D with the square of B C is equal to the square of the line compounded of B A C;) and therefore v the reason of the said line compounded of B A C to B C is given. But the angle A is also given: Therefore w the triangle A B C is given by kind.



r) 46. p.
s) 8. p.
t) 6. p.
v) Sch. 52. p
w) 46. p.

PROP. 81.

A — D
B — E
C — F

If of three right lines A, B, *and* C, *proportional to three other proportional right lines* D, E, *and* F, *the extreams* A *and* D, C *and* F, *are in a given reason (to wit, as* A *to* D, *and* C *to* F,*) also the meanes* B *and* E *shall be in a given reason, and if one extream hath a given reason to an extream, and the mean to the mean, the other will have also a given reason to the other.*

Demonstration FOrasmuch as the reason of A to D, and of C to F is given, the rectangle of A and D a shall have a given reason to the rectangle of C and F. But the rectangle of A and D is equal b to the square of B; and the rectangle of C and F to the square of E. Therefore the reason of the square of B to the square of E is given; and therefore c the reason of the line B to the line E is also given.

Again, Let the reason of A to D, and B to E, be given: I say that the reason of C to F is also given. For seeing that the reason of A to D, and of B to E is given, also the reason of the square of B d to the square of E is given. But the square of B is equal to the rectangle of A and C, and the square of E to the rectangle of D and F: Therefore the reason of the rectangle of A and C to the rectangle of D and F is given. But the reason of a side A to a side D is given: Therefore e the reason of the other side C to the other side F is also given.

a) 70. p.
b) 17. 6.
c) Sch. 52. p
d) 50. p.
e) 68 p.

PROP. 82.

A
B
C
D
E
F

If there be four right lines A, B, C, *and* D, *proportional, as the first* A, *shall be to that line to which the second* B *hath a given reason, so the third* C, *shall be to that to which the fourth* D *hath a given reason.*

Con-

Construction LEt E be the line to which B hath a given reason, and let it be so as that B may be to E, as D is to F.

Demonstration NOw the reason of B to E is given, Therefore also the reason of D to F is given. And seeing that as A is to B, so is C to D. And again, as B is to E, so is D to F, by reason of equality as A is to E, so is C to F. But E is that line to which B hath a given reason, and F that to which D also hath a given reason: Therefore as A is to that to which B hath a given reason, so C is to that to which D hath a given reason.

PROP. 83.

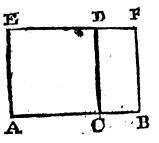
A
B
C
D
E

If four right lines A, B, C, *and* D, *are in such sort to one another, that of any three of them* A, B, *and* C, *and a fourth* E, *taken proportional, to which that line* D, *which remaines of the four lines, hath a given reason, the four lines* A, B, C, *and* E, *are proportionall; as the fourth* D *is to the third* C, *so the second* B *shall be to that to which the first* A *hath a given reason.*

Demonstration FOrasmuch as A is to B as C is to E, the rectangle conteined under A and E a is equal to the rectangle conteined under B and C; and seeing that the reason of D to E is given, also shall be given the reason of the rectangle of A and D to the rectangle of A and E (for b it is the same reason as of D to E.) But the rectangle of A and E is equal to the rectangle of B and C: Therefore the reason of the rectangle of A and D to the rectangle of B and C is given. Wherefore c as D is to C, so is B to that to which A hath a given reason.

a) 16. 6.
b) 1. 6.
c) 74. p.

PROP. 84.



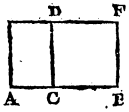
If two right lines A B *and* A E *comprehending a given space* A F *in a given angle* B A E, *and that the one* A B *be greater then the other* A E *by a given line* C B, *also each of the lines* A B *and* A E *is given.*

Demonstration FOr seeing that A B is greater then A E by the given line C B, the remainder A C is equal to A E: Finish the Parallelogram A D. Therefore seeing that A E is equal to A C, the reason of A E to A C is given, and the angle A is also given: Therefore a A D is given by kind. Wherefore the given space A F is applyed to the given right line C B, exceeding it by the given figure A D given by kind; and therefore b the breadth of the excesse is given. Therefore A C is given. But C B is also given: Therefore the whole A B is given. But A E is also given: Therefore each of the right lines A B and A E is given.

a) Sch. 61. p.
b) 59. p.

PROP.

PROP. 85.



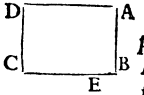
If two right lines A C *and* C D, *do comprehend a given space* A D *in a given angle* A C D, *the line compounded of those lines* A C *and* C D *is given, also each of those lines* A C *and* C D *is given.*

Construction FOr let A C be prolonged to the point B, and let C B be put equal to C D, then by the point B let B F be drawn parallel to C D, and so finish the Parallelogram C F.

Demonstration SEeing then that C B is equal to C D, and the angle D C B is given; for that angle that followes is the given angle; and therefore [a] the Parallelogram D B is given by kind; and again, seeing that the line compounded of A C D is given, and C B is equal to C D, also A B is given. And thus to the right line A B there is applyed the given space A D, deficient by the figure D B given by kind; and therefore [b] the breadths of the defects are also given: Therefore the right lines D C and C B are given. But the compounded line A C D is also given: Therefore [c] each of the lines A C and C D is given.

a) Sch. 61. p.
b) 58. p.
c) 4. p.

PROP. 86.



If two right lines A B *and* B C, *do comprehend a given space* A C, *in a given angle* A B C, *the square of the one* B C *is greater then the square of the other* A B, *by a given space (yet in a given reason) also each of those lines* A B *and* B C *shall be given.*

Demonstration FOr seeing that the square of B C is greater then the square of A B by a givvn space (yet in a certain reason.) Let the given space be taken away, that is to say, the rectangle contained under C B and B E: Therefore [a] the reason of the remainder, [b] which is the rectangle contained under B C and C E to the square of A B is given. And forasmuch as the rectangle † under A B and B C is given, and also that of C B and B E, their [c] reason is given. But as the rectangle under A B and B C is to the rectangle under C B and E B, [d] so A B is to B E; and therefore the reason of A B to B E is given: Wherefore [e] the reason of the square of A B to the square of B E is also given. But the reason of the square of A B to the rectangle under B C and C E is given: Therefore [f] also the reason of the rectangle under B C and C E to the square of B E is given. Wherefore the reason of four times the rectangle under B C and C E to the square of B E is given; and by compounding, [g] the reason of four times the rectangle under B C and C E, with the square of B E to the square of B E is given. But four times the rectangle of B C and C E, with the square of B E, [h] is the square of the compound line B C E: Therefore the reason of the square of the compound line B C E to the square of B E is given: Wherefore [i] the reason of the line compounded of

a) 11 def.
b) 2. 2.
c) 1. p.
d) 1. 6.
e) 50. p.
f) 8. p.
g) 8. p.
h) 8. 2.
i) 54. p.

of B C and C E to B E is given, and by compounding, [k] the reason of the compound of the lines B C, C E, and B E, that is to say, the double of B C to B E is given; and therefore the reason of the onely line B C to B E is also given. But as B C is to B E, [l] so the rectangle under B C and B E is to the square of B E: Therefore the reason of the rectangle under B C and B E to the square of B E is given. But the rectangle of B C and B E is given: Therefore [m] the square of B E is also given, and consequently the line B E is given. Wherefore B C is also given, seeing that the reason of B E to B C is given. But the space A C is given, and also the angle B: Therefore [n] A B is given. Wherefore each of the lines A B and B C is given.

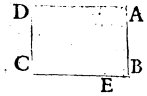
k) 6. p.
l) 1. 6.
m) 2. p.
n) 57. p.

SCHOLIUM.

† *Instead of saying in this place* [what is under, &c.] *we have used this Word Rectangle, it being manifest by what followes that such was the intention of* EUCLIDE, *seeing he makes use in the said Demonstration of the 2d. and 8th. Prop. of the 12th. Element; and also that the space or Parallelogram given being not rectangled, it may be reduced thereto, making on* B C, *and in the given point* B, *a right angle* C B A, *so as that there will be two Parallelograms constituted on one and the same base* B C, *and between the same Parallels, as in the 69th. Prop. by meanes whereof this conclusion is drawn:*

¶ Note this serves also for the next Proposition.

PROP. 87.



If two right lines A B *and* B C, *do comprehend a given space* A C, *in a given angle* B, *the square of the one* B C, *is greater then the square of the other* A B, *by a given space, also each of those lines* A B *and* B C *shall be given.*

Demonstration FOr seeing that the square of B C is greater then the square of A B by a given space: Let the given space be taken away, and let the rectangle be contained under B C and B E: Therefore the remainder, [a] which is the rectangle of B C and C E, is equal to the square of A B. And seeing that the rectangle of B C and B E is given, and also the space or rectangle A C; the reason of the said rectangle of B C and B E to A C is given. But as [b] the rectangle of B C and B E is to the rectangle of A B and B C, so is B E to A B: Therefore the reason of B E to A B is given, and therefore [c] the reason of the square of the said D E to the square of A B is also given. But to that square of A B the rectangle of B C and C E is equal: Therefore the reason of the said rectangle of B C and C E to the square of B E is given; and therefore the reason of the quadruple of the said rectangle of B C and C E to the square of B E is also given; and by compounding, [d] the reason of four times the rectangle of B C and C E, with the square of B E, to the said square of B E is given. But four times the rectangle of B C and C E, with the square of B E, [e] is the square of the compound line B C E: Therefore the reason of the square of that compound line B C E to the square of B E is also given; and therefore the reason [f] of the compound line B C E to B E is given. Wherefore by compounding,

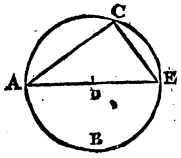
a) 2. 2.
b) 1. 6.
c) 50. p.
d) 6. p.
e) 8. 2.
f) 54. p.

pounding, g the reason of the said compound line BCE and EB, that is to say, twice BC to BE is also given; therefore the reason of the only line BC to BE is given. But the reason of the same BE to AB is also given: Therefore h the reason of AB to BC is given. And seeing that the reason of BC to BE is given, and that as the said BC is to BE, so the square of BC i to the rectangle of BC and BE, the reason of the square of BC to the rectangle of BC and BE is also given. But the said rectangle of BC and BE is given, it being that which was taken away, and which was given. Therefore the square of BC k is given, and therefore the line BC is given. But the reason of the same BC to BA is given, therefore AB is also given.

g) 6. p.
h) 8. p.
i) 1. 6.
k) 2. p.

PROP. 88.

If in a circle A B C, *given by magnitude, there be drawn a right line* A C, *which shall take away a segment* A B C, *which doth comprehend a given angle* A E C, *that line* A C *is given by magnitude.*



Construction FOr let D be the center of the circle; and let the diameter thereof A D E be drawn, and let E C be joyned.

Demonstration FOrasmuch as the angle A C E is given, for a it is a right angle. But the angle A E C is also given, and therefore the other angle C A E is given. Wherefore the triangle A C E b is given by kind; and therefore the reason of E A to A C is given. But A E is given by magnitude, seeing that the circle A B C is given by magnitude. Therefore c A C is also given by magnitude.

a) 31. 3.
b) 40. p.
c) 2. p.

PROP. 89.

If in a circle A B C, *given by magnitude, there be drawn a right line* A C, *given by magnitude, that line* A C *will take away a segment* A B C, *comprehending a given angle.*

Construction FOr having taken the point D for the center of the circle, Let the diameter A D E be drawn, as also the right line E C.

Demonstration FOrasmuch as each of the right lines A E and AC are given, the reason of the line A E to A C a is given; and the angle A C E is a right angle: Therefore b the triangle A C E is given by kind, and therefore the angle A E C is given.

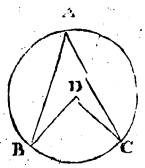
a) 1. p.
b) 43. p.

PROP. 90.

If in the circumference of a circle A B C *given by position, and by magnitude, there be taken a given point* B, *and that from that point* B, *to the circumference of the circle* A B C *there doth bend a right line* B A C, *making a given angle* B A C *the other extremity* C *of the bent line shall be given.*

Construction FOr let the center of the circle be D, and let the right lines B D and B C be drawn.

Demon-

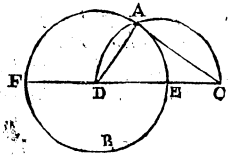


Demonstration FOrasmuch as each point B and D is given, the right line B D, a is given by position; and seeing that the angle B A C is given, the angle B D C is also given. Wherefore to the right line B D given by position, and in the point D given therein, there is drawn the right line C D; which makes the given angle B D C; and therefore b the line D C is given by position. But the circle A B C is given by position and magnitude: Therefore c the right line D C is given by position and by magnitude. But the point D is given: Therefore d the point C is also given.

a) 26. p.
b) 29. p.
c) 6. def.
d) 27. p.

PROP. 91.

If from a given point C, *there be drawn a right line* C A, *which shall touch a circle* A B, *given by position; that line* C A *is given by position and by magnitude.*



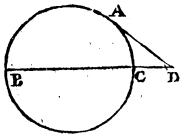
Construction FOr having taken the point D for the center of the circle, let the right lines D A and D C be drawn.

Demonstration FOrasmuch as each point C and D is given, the right line C D a is given by position and by magnitude. But the angle C A D b is a right angle; and therefore the semicircle described on C D shall passe by the point A: Let it then passe by that point, and let the semicircle be D A C: Forasmuch as the same D A C c is given by position and also the circle A B E, d the point A is given. But the point C is also given: Therefore e the right line A C is given by position and by magnitude.

a) 26. p.
b) 18. 3.
c) 6. def.
d) 25. p.
e) 26. p.

PROP. 92.

If without a circle A B C, *given by position, there be taken some point* D, *and from that given point there be drawn a right line* D B, *cutting the circle, the rectangle comprised under the whole line* B D, *and the part* D C, *between the point* D *and the circumference convex* A C *shall be given.*



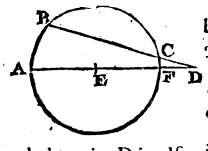
Construction FOr from the point D let the right line D A be drawn, which shall touch the circle in the point A.

Demonstration THerefore D A a is given by position and magnitude; and therefore the square of the said D A is b given. But the said square of D A is equal c to the rectangle of B D and D C: Therefore the said rectangle of B D and D C is also given.

a) 91. p.
b) 52. p.
c) 36. 3.

OTHERWISE.

Construction LEt E be the center of the circle, and by the same center let there be drawn from the point D the right line D A.

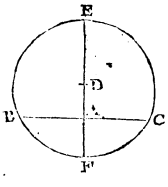


Demonstration FOrasmuch as each point D and E is given, the right line D E is [d] given by position and by magnitude. But the circle A B C is given by position and by magnitude: Therefore each point A and F [e] is given, and the point D is also given; and therefore [d] each line A D and F D is given. Wherefore the rectangle of the lines A D and D F is also given. But the said rectangle of A D and D F is equal to the rectangle of D B and D C: Therefore the rectangle of D B and D C is given.

d) 26. p.
c) 25. p.

PROP. 93.

If in a circle given by position there be taken a given point A, *and by that point* A *there be drawn a right line* B C *to the circle, the rectangle comprised under the segments of the same line* B C *shall be given.*



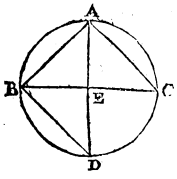
Construction FOr let D be taken for the center of the circle, and having drawn the right line A D prolong it to the points E and F.

Demonstration FOrasmuch as each point A and D is given, the right line A D [a] is given by position. But the circle B E C is also given by position: Therefore each point E and F is also given by position; and the point A is given. Wherefore each line [b] A E and A F is given: Therefore the rectangle of the same lines A E and A F is given; and is equal to the rectangle [b] of A B and A C: Therefore the said rectangle of A B and A C is given.

a) 26. p.
b) 35. 3.

PROP. 94.

If in a circle A B C, *given by magnitude, there be drawn a right line* B C, *which doth take away a segment which doth comprehend a given angle* A B C, *and that the said angle being in the segment is cut into two equal parts, the line compounded of the right lines* B A *and* A C, *which comprehend the given angle* B A C *shall have a given reason to the line* A D, *which doth divide that angle into two equal parts; and the rectangle conteined under the line compounded of those lines* B A *and* A C, *comprehending the given angle* B A C, *and that part* E D *of the intersecting line which is below the segment between the base* B C *and the circumference, shall be given.*



Con-

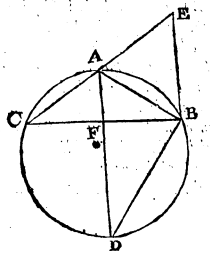
Construction Let B D be drawn.

Demonstration FOrasmuch as in the circle A B C given by magnitude, there is drawn the right line B C, which takes away the segment B A C comprehending the given angle B A C, that line B C [a] is given; and therefore B D is also given: Therefore the reason of B C to B D [b] is given. And seeing that the given angle B A C is cut in two equal parts by the right line A D, as [c] B A is to C A, so is B E to C E; and by compounding, as B A C is to C A, so is B C to C E; and by permutation, as B A C is to B C, so is C A to C E. And seeing that the angle B A E is equal to the angle C A E, and the angle A E C [d] to the angle B D E, the other angle A E C is equal to the other angle A B D; and therefore the triangle A C E is equiangled to the triangle A B D: Therefore [e] as A C is to C E, so is A D to B D. But as A C is to C E, so the line compounded of B A and A C is to B C: Therefore as the compound line B A C is to B C, so is A D to B D; and by permutation, as the compound line B A C is to A D, so is B C to B D. But the reason of B C to B D is given: Therefore the reason of the compound line B A C to A D is also given. Moreover, I say that the rectangle under the compound line B A C and E D is given. For seeing that the triangle A E C is equiangled to the triangle B D E, (for the angle A C E [d] is equal to the angle B D E, and the angle A E C [f] to the angle B E D) as B D is to D E, so is A C to C E. But as A C is to C E, so is also the compound line B A C to B C: Therefore as the compound line B A C is to B C, so is B D to D E. Wherefore the rectangle of the compound line B A C and D E [g] is equal to the rectangle of B C and B D. But the rectangle of B C and B D is given, (for that those lines B C and B D are given:) Therefore the rectangle under the compound line B A C and E D is also given.

a) 88. p.
b) 1. p.
c) 3. 6.
d) 21. 3.
e) 4. 6.
f) 15. 1.
g) 16. 6.

OTHERWISE.

Construction LEt C A be prolonged to the point E, and let A E be put equal to B A, and let B E and B D be joyned.

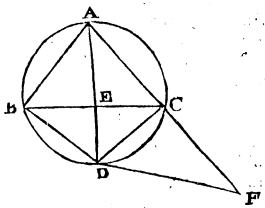


Demonstration FOrasmuch as the angle B A C is double to each of the angles C A D and A E B (for the angle B A C is cut into two equal parts by the line A D, and equal [h] to the two angles A B E and A E B, which [i] are equal) the angle A B E is equal to the angle C A D, that is to say, [k] to the angle C B D; adding therefore the common angle A B C, the whole angle A B D shall be equal to the whole angle F B E. But the angle A C B is [k] equal to the angle A D B: Therefore the third angle A E B is equal to the third angle B A D; and therefore the triangle C E B is equiangled to the triangle A B D: Wherefore as C E is to C B, so is A D to B D. But the right line C E is compounded of the two lines C A and A B: Therefore as the compound line B A C is to C B, so is A D to B D; and by permutation, as the compound line B A C is to A D, so is C B to B D. But the reason of C B to B D is given, seeing that each of those lines is given: Therefore the reason of the compound line B A C to A D is also given. And seeing that the triangle C E B is equiangled to the triangle F B D (for the angle A F C is equal [l] to the angle B F D, and the angle E C B [m] to the angle A D B) as E C is to C B, so is B D to D F. But E C is equal to the compound

h) 32. 1.
i) 5. 1.
k) 21. 3.
m) 21. 3.
n) 16. 6.

pound line B A C: Therefore as the compound line B A C is to C B, so is
n) 16.6. B D to D F. Wherefore n the rectangle of the compound line B A C and D F is equal to the rectangle of C B and B D. But the rectangle of C B and B D is given, considering that each of the lines C B and B D is given: Therefore the rectangle of the compound line B A C and D F is given.

OTHERWISE.

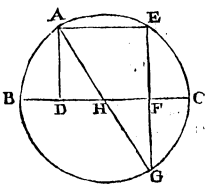


Construction LEt A C be prolonged to F, and let C F be put equal to A B, and let the right lines B D and D F be drawn.

o) 26,29.3. *Demonstration* FOrasmuch as B A is equal to C F, and o B D to D C, the two sides A B and B D are equal to the two sides C D and D F, each to his corresponding side, and the angle A B D is equal to the angle D C F, p seeing that the four sided figure A B D C is within the circle: Therefore the base A D is q equal to the base D F, and the angle D A B to the angle D F C. But the angle B A D is given, (being the half of the given angle B A C.) Therefore the angle D F C is so also. But D A F is also given: Therefore the triangle A D F is given by kind. Wherefore the reason of F A to A D is given. But A F is the compound of B A and A C, for that C F is equal to A B: Therefore the reason of the compound line B A C to A D is given: The same demonstration will serve to shew that the rectangle conteined under the compound line B A C and E D is given also.

p) 22.3. 13. 1.
q) 4. 1.

PROP. 95.



If in the diameter B C *of a circle* A B C *given by position, there be taken a given point* D, *and that from that point* D *there be drawn a right line* D A, *to the circumference of the circle. But from the section of the said line there be drawn a right line* A E, *perpendicular thereto, and by the point* E *where that perpendicular doth meet with the circumference, there be drawn a parallel* E F, *to the first line drawn* A D, *that point* F *in which the parallel meets with the diameter, is given; and the rectangle conteined under the parallel lines* A D *and* E F *is also given.*

Construction LEt the right line E F be prolonged to the point G, and let the right line A G be drawn.

Demonstration FOrasmuch as the angle A E G is a right angle, the right line A G is the diameter of the circle. But B C is also the diameter: Therefore the point H is the center of the circle. Now the point D is given; and therefore
a) 26. p. a the line D H is given by magnitude. But seeing that A D is parallel to E G,
b) 26. p. and A H equal to G H; b D H is equal to F H, and A D to F G; (for the angles
c) 15. 1. A H D and F H G c are equal, and D A H and F G H d are also equal.) But the
d) 29. 1. line D H is given: Therefore F H is also given. But each of those lines D H and
e) 27. p H F is also given by position, and the point H is given: Therefore e the point F is also given And seeing that in the circle A B C given by position, is taken the given point F, and through the same is drawn the right line E F G; the rectangle under
f) 93. p. E F and F G f is given. But F G is equal to A D. Therefore the rectangle comprehended under A D and E F is given. Which Was to be demonstrated.

THE END OF EUCLIDES DATA.

A BOOK OF THE DIVISIONS OF Superficies:

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PRINCE OF

URBIN.

WHen John Dee *of* London, *a man of excellent wit, and ſingular learning, (Moſt Illuſtrious Prince) going away, left with me this little Book of the* Diviſions of Superficies, *as a token or teſtimony of his affection towards me, he added, that nothing could happen more welcome to him from me, than that it might by my means come into the hands of thoſe that are deſirous of learning, eſpecially of the* Mathematicks. *Therefore I being moved both with the honeſt minde and intent of the friendly and moſt learned man, and alſo much taken with the marvelous benefit of the Book, for that I knew there was not any ſuch thing extant among us, I have now moſt willingly endeavoured to ſatisfie his deſire, and as he requeſted me, have not ſuffered this Treatiſe to conſiſt in a Pentagonal diviſion: for what things the Author of the Book hath at large comprehended in many Problems, I have compendiouſly*

diouſly compriſed and diſpatched in two only; yet ſo, as that from them it may clearly appear, in what ſort thoſe ſections might in other Figures be infinitely produced. In which thing truly, (if I miſtake not) very profitable, I have both obeyed the requeſt of a Friend, and alſo promoted their Studies, who are delighted with this moſt Noble and Excellent kinde of Learning. For it can hardly be conceived, how great a help and Ornament, this attempt will bring to him, that endeavours to be a Geometrician, *ſo as he refuſe not to beſtow good pains therein. Therefore this product of common induſtry (ſuch as it is) I reſolved ſhould come forth adorned with your moſt Excellent Name, until I carefully trim up for you ſome more conſiderable teſtimonies of my engagements to you; both for that I have conſecrated all my endeavours to your bounty (to which I am exceedingly obliged) and alſo becauſe I conceived that would be moſt acceptable to Dr.* Dee, *who by the renown of your famous Court was drawn hither, with the pains of a tedious Journey. Farewell, and be pleaſed courteouſly to continue in your defending and cheriſhing of Learning and its Friends.*

Fredericus Commandinus.

TO

TO
FREDERIC COMMANDINE
OF
URBIN.
Iohn Dee of London
Wiſheth Health and all Happineſſe.



Aving now for many years ſet my ſelfe chiefly (my moſt learned *Frederick*) how to preſerve from utter ruine, the moſt famous moniments of our Anceſtors (ſuch as I could) in all the more curious or elegant kinds of Philoſophy; leaſt that either ſo worthy men ſhould be robbed of their due renown, or we longer want the moſt abundant benefit of ſuch Books. I ſay that I ſo beſtowing my pains among other moſt ancient writings of Philoſophers, did at length happen upon this ſmall Book, written, indeed, in a very blinde ilfavoured Character, and alſo by reaſon of its age, hard to be read. But that I might the better ſee, I uſed helps to my ſight: and by often ſtudy and exerciſe therein, I got the knack of reading it. Whereupon being hereby better perſwaded of the excellencie and worth of the Book, I earneſtly wiſhed that the ſame might forthwith be communicated to the Society of Philoſophers. But while I was pondering this in my minde, I found none more worthy than your ſelf (my *Commandine*) in this our age, to enjoy theſe our Labours; who have alſo your own ſelfe revived certain moſt excellent Works of *Archimedes*,

and *Ptolemee* almoſt loſt, and have brought them forth into pnblick view in a moſt magnificent dreſſe. Therefore this little Book, as a perpetual pledge, even of the affection wherewith I ever embrace you, I commit to you and your truſt; and do earneſtly beſeech you, that you will not ſuffer this our common labour to go forth into the World deſtitute of that adornment wherewith you are wont to ſend abroad others. Yea, I ſurely hope (if I well know you and your endeavours) that you will ſome time or other ſo enrich this ſubject, as that you will neither permit it to reſt in a Pentagonal form; not ſuffer the *Solids* themſelves long to want the like Sections by Plains. Verily, if you would but a little put forward theſe things, they will by themſelves go on to the other kindes of Superficies. But that they may be applyed to *Solids* they will require your ſound knowledge, and more than ordinary pains in the Mathematicks. As for the Authors name, I would have you underſtand, that to the very old Copy from whence I writ it, the name of *MACHOMET BAGDEDINE* was put in Ziphers or Characters, (as they call them) who whether he were that *Albategnus* whom *Copernicus* often cites as a very conſiderable Author in Aſtronomie; or that *Machomet* who is ſaid to have been *Alkindus*'s Scholar, and is reported to have written ſomewhat of the art of Demonſtration, I am not yet certain of: or rather that this may be deemed a Book of our *Euclide*, all whoſe Books were long ſince turned out of the Greeke into the Syriack and Arabick Tongues. Whereupon, It being found ſometime or other to want its Title with the *Arabians* or *Syrians*, was eaſily attributed by the tranſcribers to that moſt famous Mathematician among them, *Machomet*: Which I am able to prove by many teſtimonies, to be often done in many Moniments of the Ancients; and certain friends of mine doe know (that I may bring one of many) that we have by this means reſtored one ſmall Book, incomparable in occult and myſticall Phyloſophie, of that moſt ancient and excellent Phyloſopher *Anaxagoras*, every where now through many ages enobled with the name of *Ariſtotle* to *Anaxagoras* himſelfe, and that by moſt ſure proofs. Yea further, we could not yet perceive ſo great acuteneſſe of any *Machomet* in the Mathematicks, from their moniments which we enjoy, as everywhere

rywhere appears in theſe Problems. Moreover, that *Euclid* alſo himſelfe wrote one Book περὶ διαιρέσεων, that is to ſay, *of Diviſions*, as may be evidenced from *Proclus*'s Commentaries upon his firſt of *Elements*: and we know none other extant under this title, nor can we finde any, which for the excellencie of its treatment, may more rightfully or worthily be aſcribed to *Euclid*. Finally, I remember that in a certain very ancient piece of Geometry, I have read a place cited out of this little Book in expreſſe words, even as from amoſt certain work of *Euclid*. Therefore we have thus briefly declared our opinions for the preſent, which we deſire may carry with them ſo much weight, as they have truth in them. And if any man ſhall object to me, that that title of Diviſions doth not denote the ſections of Magnitudes into their parts, but the diviſions of *Genus's* by their differences into *Species*, like as the methodicall diviſions of Points, Lines, Angles, Figures, and the like; ſuch as we have in our demonſtrated work of Mathematicall *Acribologie* ſhewed above five huudred: truly I confeſſe, that this alſo may in probability be ſaid; but yet how truly, is not more manifeſt yet to me, than our conjecture is to him. But whatſoever that Book of *Euclid* was concerning Diviſions, certainly this is ſuch an one as may be both very profitable for the ſtudies of many, and alſo bring much honour and renown to every moſt noble ancient Mathematician; for the moſt excellent acuteneſſe of the invention, and the moſt accurate diſcuſſing of all the Caſes in each Probleme: and ſo much for this, I will now direct my diſcourſe to you, who are herein to be greatly intreated by me, that you would with all poſſible diligence advance your moſt weighty and uſefull labours, which you did yeſterday moſt courteouſly ſhew to me in your Study. For ſo you will make the faireſt way to perpetuate your fame, who have in ſo few years, put forth many Books, ſo happily, ſo neatly, and ſo many of your own: who alone in our time doſt adorn the moſt excellent Princes of Mathematicians, *Archimedes*, *Apollonius*, and *Ptolomeus*, with their due luſtre. So you will reſtore a new and wonderfull livelyneſſe to Mathematicall Learning much decayed: and ſo you ſhall now at laſt oblige me wholly to your ſelfe by many engagements. And I intreat you, that ſo ſoon as this

this Book is printed, you will be pleaſed to ſend a copie or two thereof to the moſt Noble man, and ſingular patron of all good Arts, and ſpecially of the Mathematicall, Sir *William Pickering* Knight, my exceeding great friend, living at *London*. For then they will moſt readily be conveyed to my Library. Now my purpoſe of Travell calls me away, leſt I ſhould be put to undergo a greater trouble of this ſcorching Seaſon round about us, before I can ſhelter my ſelfe in the Roman ſhades. Farewel, therefore, the honour and renown of Mathematicians; farewel my moſt courteous *Commandine*. And I very earneſtly beſeech the great God, that he will vouchſafe to bring your excellent attempts by his ſpecial favour, to their deſired ends.

URBIN.

TO THE READER.

I Am here to advertiſe thee (kinde Reader) that this Author which we preſent to thee, made uſe of Euclid *tranſlated into the Arabick Tongue, whom afterwards* Campanus *made to ſpeake Latine. This I thought fit to tell thee, that ſo in ſearching or examining the Propoſitions which are cited by him, thou mighteſt not ſometime or other trouble thy ſelfe in vain,* FAREWEL.

A BOOK OF THE DIVISION OF SUPERFICIES.

PROP. 1. PROBL. 1.

By a line drawn from an angle of a triangle, to divide that triangle according to a proportion given.



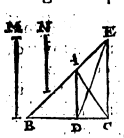
LEt the triangle be A B C, and let it be required to divide the ſame by a line deſcending from the angle A in proporiton as E to F.

Divide B C in D in proportion as E to F, and [a] having drawn the line A D, that line A D divides the triangle as is required [b]. a) 12. 6. b) 1. 6.

PROP. 2. PROB. 2.

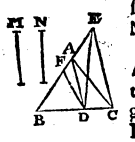
To divide a triangle given according to a proportion given, by a line drawn from a point aſſigned in the ſide of the triangle.

LEt the triangle be A B C, in whoſe ſide B C let the point D be aſſigned, from whence it is required to draw a line dividing the triangle according to the proportion of M to N, draw D A. From that end or extream of B C, towards which I would have the conſequent term of the diviſion, which for Examples ſake let be the point C. Draw a parallel to D A till it meet in the point E with the line B A extended (that they will meet is manifeſt [a].) The proportion of M to N will be either equal to the proportion of B A to A E, or greater, or leſſer. Let firſt the proportion be equal, therefore [b] the proportion of the triangle B A D to the triangle A D E, is as the proportion of M to N. But [c] the triangle A D E is equal to the triangle A D C: Therefore [d] the proportion of the triangle A B D to the triangle A D C, is as the proportion of M to N. Which was to be proved.



a) 29. 17. 1. b) 1. 6. c) 37. 1. d) 7. 5.

Secondly, Let the proportion of M to N be leſſer then the proportion of the line B A to the line A E; therefore divide the line B E according to the proportion of M to N, and let the diviſion fall between B and A [e], it will fall in the point F; and let the line D F be drawn; which line will I ſay e) 8. 5.



say divide the triangle according to the proportion of M to N.

Demonstr. For having drawn the line DE, the triangle ADE [f] shall be equal to the triangle ADC. Putting therefore the triangle AFD common to both, the triangle FDE will be equal to the quadrilateral figure AFDC. Seeing that [g] the triangle BFD is to the triangle FED, as BF is to FE; and by consequence, as M to N; then also the triangle BFD is to the quadrangle AFDC as M to N, and the Proposition is manifest.

f) 37. 1.

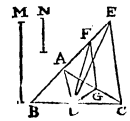
g) 1. 6.

Thirdly, Let the proportion of M to N be greater then the proportion of BA to AE. Divide therefore BE in the point F, which will be between A and E, according to the proportion of M to N; and draw FG parallel to CE, until it meet with the line AC in the point G; then joyn GD, I say the line GD will divide the triangle according to the given proportion.

For draw the lines DF and DE, then is the triangle ADE equal to the triangle ADC, and [h] the triangle ADF is equal to the triangle ADG; therefore [i] the remaining angles, to wit, the triangle FDE and the triangle GDC are also equal.

h) 37. 1.

i) 37. 1.



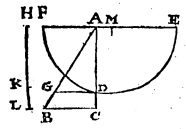
Putting also the triangle ABD common to both the equal triangles AFD and AGD, the triangle BFD will be equal to the quadrilateral figure BAGD. Therefore the triangle FBD is to the triangle FDE, as the quadrilateral figure BAGD is to the triangle GCD. But the triangle FBD is to the triangle FDE, as M is to N, by supposition [k]. Therefore the proportion of the quadrilateral figure BAGD to the triangle GDC is as the proportion of M to N, as was proposed.

k) 1. 6.

PROP. 3. PROBL. 3.

By a line equidistant to a side assigned of a known triangle, to divide that triangle according to a proportion given.

LEt the given proportion be HK to KL, and the triangle ABC, &c. From the angle A (towards which I would have the antecedent in the proportion sought) protract a line AE perpendicular to CA, and equal to it, and let EA be produced to F, until the proportion of EA to AF be as HL to HK, and making a center in the middest of the line FE, as at M, let the semicircle FDE be described, according to the quantity of the line ME, which semicircle will cut the line AC in the point D; because the line AD is lesse then the line AE, and the line AE is equal to the line AC; drawing then DG parallel to BC; I say that the proportion of the triangle AGD to the superficies GBCD, is according to the proportion of HK to KL.

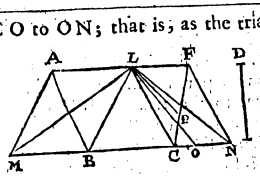


Demonst. For the proportion of the triangle ABC to the triangle AGD, is as AC to AD, a duplicate proportion [a]. But AC is equal to AE; therefore the proportion of the triangle ABC to the triangle AGD, is as the proportion of AE to AD, duplicate, for the proportion of AE to AD duplicate, is as the proportion of AE to AF [b]. Therefore

a) 17. 6.

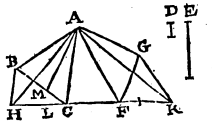
b) 30. 3.

CO to ON; that is, as the triangle LOC to the triangle LON; in compounding, the triangle LCF is to the triangle LPF, as the triangle LCN is to the triangle LON; and by permutation, the triangle LCN is equal to the triangle LCF: Therefore also the triangle LON is equal to the triangle LPF; and the remaining triangle LMO is equal to the pentagon ABCPL. Wherefore as the triangle LMO is to the triangle LON; that is, as MO to ON, that is, as D to E; so is the pentagon ABCPL to the triangle LPE: Therefore the Quadrangle ABCF, &c. Which was to be done.

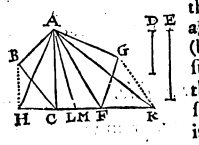


¶ But if the point be given in any other either, angle or side of the Quadrangle, the Proposition is wrought after the same manner.

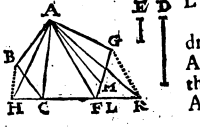
A Pentagon divided from an angle.



Let the pentagon ABCFG be to be divided by a right line drawn from the angle A, in proportion as D to E: Let there be drawn the lines AC and AF, and from B and G, upon CF, extended on both ends; let there be drawn BH parallel to AC, and GK parallel to AF; and having drawn AH and AK, the triangle AHF will be equal to the Quadrangle ABCF, and the triangle AFK will be equal to the triangle AFG; and the whole triangle AHK is equal to the whole pentagon ABCFG: Let HK be cut in L, so as that HL may be to LK, as D to E; the point L then will either be in HF or FK; and if in HF, let the Quadrangle ABCF (by the Rules aforesaid) be divided by AM, issuing from the angle A, so as that the parts thereof may be in proportion as HL to LF: I say the line AM doth divide the Pentagon, as is required.



Demonstr. For (by the reasons aforesaid) the triangle ABM is to the pentagon AMCFG, or as in the other Scheme, the Quadrangle ABCM is to the Quadrangle AMFG, as HL is to LK, required.

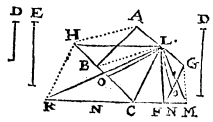


But if L doth fall in FK, in like manner, AM drawn from the angle A, doth divide the triangle AFG in proportion as FL to LK, and sheweth that the pentagon ABCFM is to the triangle AMG, as HL is to LK, that is, as D to E.

To divide a Pentagon from a side.

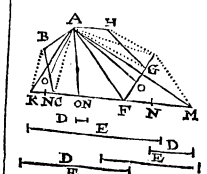
Let the point L be taken in the side AG, from which point a line drawn, doth divide the pentagon in the proportion given, to wit, of D to E. Draw LC and LF, and CB being produced, on the side B, let a triangle (by the Rules before) be made, to wit, the triangle LHC, equal

to the Quadrangle LHCF. Then producing CF on the part C, make the triangle LKF equal to the Quadrangle LHCF, that is, to the pentagon LABCF. And again, it being produced on the part F, make the triangle LFM equal to the triangle LFG; the whole triangle LKM will be equal to the pentagon ABCFG: Therefore let KM be divided in N, so as that KN may be to NM, as D to E. And if the point N fall in KF, divide the pentagon LABCF by LO, so as that the Quadrangle LABO may be to the Quadrangle OCFL, as KN to NF: The Quadrangle LABO will be to the pentagon OCFGL, as KN to NM: In the same manner, it may be demonstrated that the Hexagon LABCFO, is to the triangle LOG, as KN to NM; that is, as D to E. Which ought to be done.



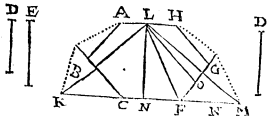
To divide a Hexagon from an angle.

Let the Hexagon be ABCFGH, and the angle A from whence it ought to be divided in proportion as D to E, draw AF, being extended both wayes, make the angle AKF equal to the Quadrangle ABCF, and the triangle AFM equal to the Quadrangle AFGH: Then the whole triangle AKM will be equal to the Hexagon ABCFGH: Then divide KM in the point N, so as that KN may be to NM, as D to E. And if the point N fall in KF, divide the Quadrangle ABCF by a line drawn from the angle A, so that its parts be in proportion as KN to NF. But if the point N fall in FM, divide the Quadrangle AFGH, as FN to NM, and so the Hexagon ABCFGH will be divided as KN to NM; that is, as D to E, required.



To divide a Hexagon from a side.

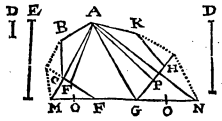
Let there be taken in the side AH the point L, from whence a line drawn, may divide the Hexagon given, according to the proportion of D to E. Draw LF, and having produced CF, make the triangle LKF equal to the pentagon LABCF, and the triangle LFM equal to the quadrangle LFGH, so the whole triangle LKM will be equal to the whole Hexagon ABCFGH. Then divide KM in N, in proportion as D to E, and if the point N fall in KF. Let the pentagon LABCF be divided from the angle L in proportion as KN to NF. And if it fall in FM, let the Quadrangle LFGH be divided according to the proportion of FN to NM, so will the whole Hexagon be divided by a line drawn from L, in proportion as KN to NM, that is, as D to E, required.



To

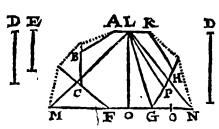
To divide an Heptagon from an angle.

Let the Heptagon ABCFGHK, be divided from the angle A, in proportion as D to E: Let AG be drawn, and make the triangle AMG equal to the pentagon ABCFG, and the triangle AGN equal to the quadrangle AGHK, and the whole triangle AMN equal to the whole Heptagon ABCFGHK: Let MN be cut in the point O, in proportion as D to E. And if O fall in MG: Let the pentagon ABCFG be divided as MO is to OG, with the line AP; and if it fall in GN, divide the quadrangle AGHK as GO to ON, and so the Heptagon will be divided in proportion as MO to ON.



To divide an Heptagon from a point in a side.

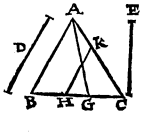
In the last place, Let the point L be in the side AK, and from L a line be drawn to divide the Heptagon according to the proportion of D to E. Let there be drawn the line LG, and constitute the triangle LMG, equal to the Hexagon LABCFG; and let the triangle LGN be made equal to the quadrangle LGHK, and so as that the whole triangle LMN will be equal to the Heptagon AECFGHK. Again, Let MN be cut according to the given proportion in O, and if O fall in MG, divide the Hexagon in proportion as MO to MG. But if it fall in GN (as on the left hand) divide the Quadrangle according to the proportion of GO to ON. And the whole Heptagon will be divided as MO is to ON, that is, as D to E given. In like manner one may proceed to other Figures, of how many angles or sides soever. Which was to be done.



PROBLEME II.

To divide a right lined figure GABC, *according to a proportion given* E *to* F, *by a right line parallel to another line given* D.

Divide BC in G, so as that BG may be to GC, as E to F; then either the line D is parallel to one side of the triangle, or to none of them. First, suppose it parallel to the side AB, and find a mean proportional CH, and by H let HK be drawn parallel to BA: I say HK divides the triangle, as is required.



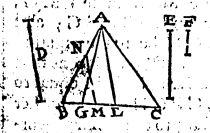
Demonstr. For having drawn AG, the triangle ABG will be to the triangle AGC, as BG to GC, that is, as E to F; and in compounding, the triangle ABC is to the triangle AGC, as BC to CG. But BC is to CG, as the triangle ABC is to the triangle KHC; [a] for the triangles ABC and KHC are like, and BC to CG hath dupli- [a] 19. 6.

duplicate proportion of BC to EH. Wherefore the triangle KHC is equal to the triangle AGC, and the remaining Quadrangle ABHK is equal to the triangle ABG: Therefore the Quadrangle ABHK is to the triangle KHC, as the triangle ABG is to the triangle AGC, to wit, as E to F.

In like manner, it will be demonstrated, if the given line be parallel to the side BC or CA required.



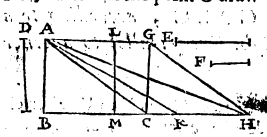
If the given line D be parallel to no side of the triangle, Let AL be drawn parallel to D: Then the point G will fall either betwixt L and C, or betwixt B and L; if betwixt L and C: Let there be taken between LC and CG, a mean proportional, and draw MN parallel to AL: Then the triangle NMC (by what is before demonstrated) will be equal to the triangle AGC, and the Quadrangle ALMN will be equal to the triangle ALG. Wherefore adding the triangle common to both, to wit, ABL, the Quadrangle ABMN will be equal to the triangle ABG. Therefore the Quadrangle ABMN is to the triangle NMC, as E to F.



But if the point G fall betwixt B and L, then also find a mean proportional, as BM, between LB and BG; and let MN be drawn parallel to AL. By the same reason, the triangle NBM will be equal to the triangle ABG, and the Quadrangle ANML is equal to the triangle AGL. Therefore adding the triangle ALC common to both, the Quadrangle ANMC will be equal to the triangle AGC: Therefore the triangle ABC is divided in the proportion given, by a parallel to D, as was required.

A Quadrangle ABCG, *divided by a line parallel to a line given* D, *in proportion as* E *to* F.

Where D is parallel to a side of a Quadrangle, as here to AB: Joyn AC, and from the point G draw GH parallel to AC, which will meet with BC extended in H; then let AH be drawn: Therefore (by what is before said) the triangle ACH is equal to the triangle ACG; and adding to both the common triangle ABC, the triangle ABH will be equal to the Quadrangle ABCG: Let BH be divided in K, so as that BK may be to KH, as E is to F, and let AK be joyned, the side then of the Quadrangle is parallel to BA, or not parallel, and if it be parallel, howsoever the point K doth fall, let [b] (to the line AB) there be applyed the superficies ABML, equal to the triangle ABK, so as that LM be parallel to the said line AB: I say then that LM performes the Propositon.



b) 10. of the former book.

Demonstr. For because the triangle ABH is equal to the Quadrangle ABCG, and the triangle ABK is equal to the Quadrangle ABML: The triangle AKH remaining, will be equal to the remaining Quadrangle LMCG: Therefore as the Quadrangle ABML is to the Quadrangle